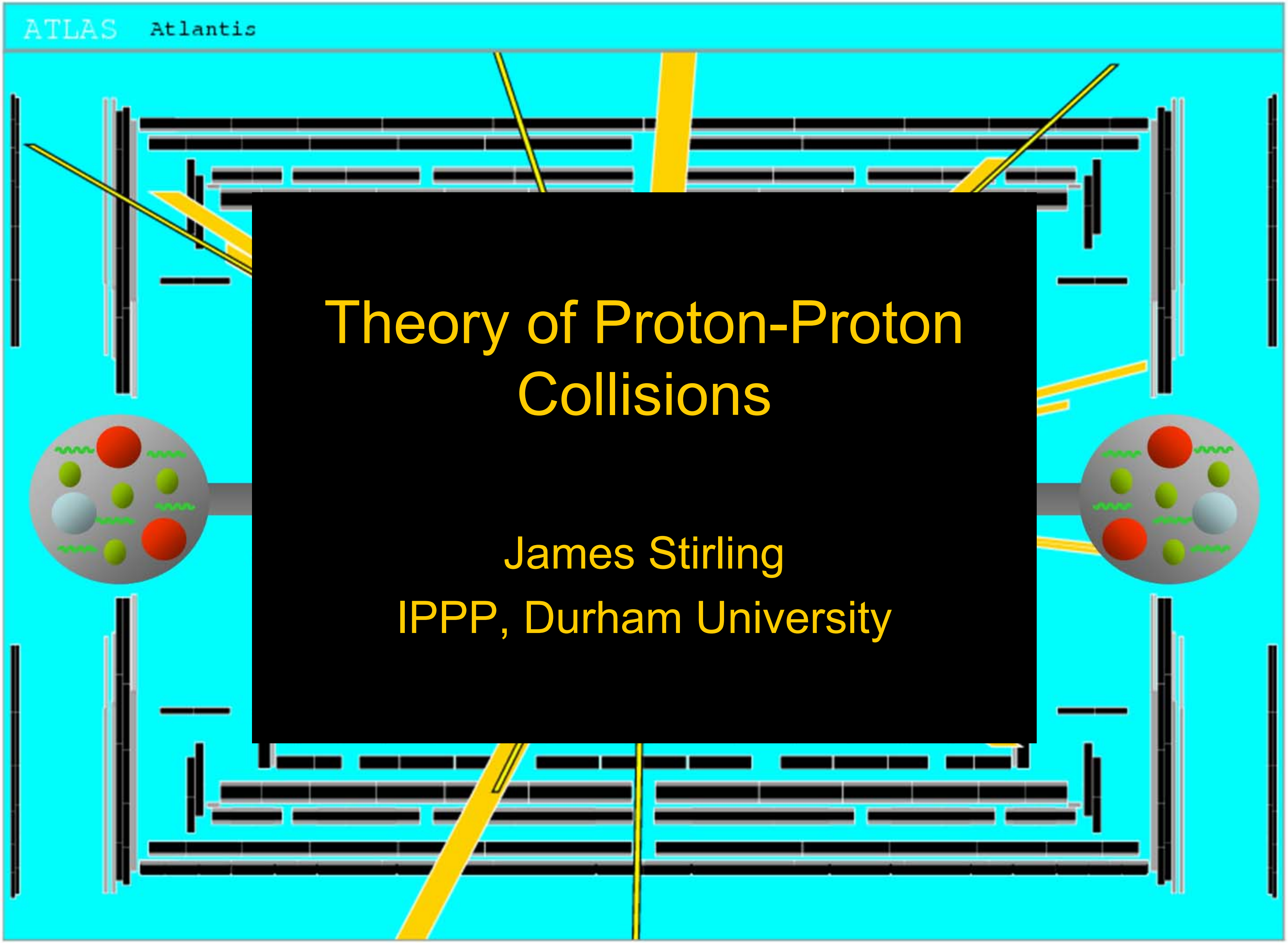
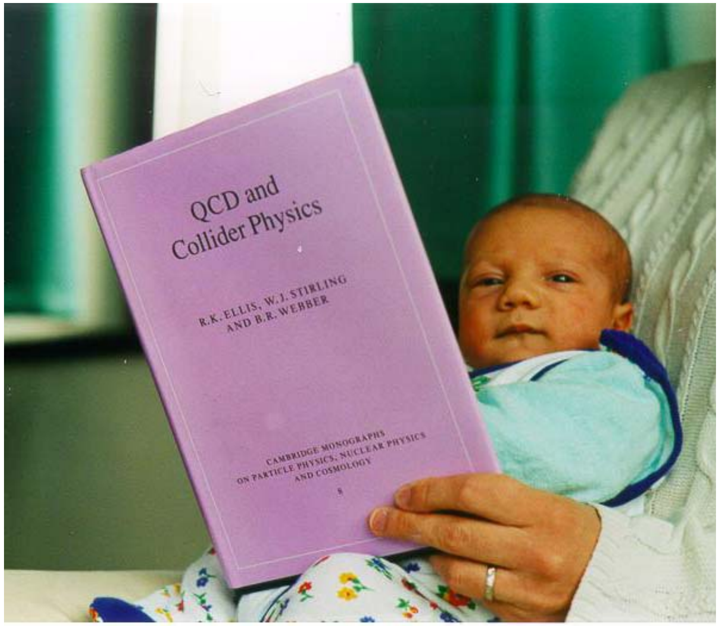
Theory of Proton-Proton Collisions

James Stirling

IPPP, Durham University



references



"QCD and Collider Physics"

RK Ellis, WJ Stirling, BR Webber
Cambridge University Press (1996)

also

"Handbook of Perturbative QCD"

G Sterman et al (CTEQ Collaboration)
www.phys.psu.edu/~cteq/

… and

“Hard Interactions of Quarks and Gluons: a Primer for LHC Physics ”

JM Campbell, JW Huston, WJ Stirling (CSH)

www.pa.msu.edu/~huston/seminars/Main.pdf

to appear in Rep. Prog. Phys.

REVIEW ARTICLE

Hard Interactions of Quarks and Gluons: a Primer for LHC Physics

J. M. Campbell
Department of Physics and Astronomy
University of Glasgow
Glasgow G12 8QQ
United Kingdom

J. W. Huston
Department of Physics and Astronomy
Michigan State University
East Lansing, MI 48840
USA

W. J. Stirling
Institute for Particle Physics Phenomenology
University of Durham
Durham DH1 3LE
United Kingdom

Abstract. In this review article, we will develop the perturbative framework for the calculation of hard scattering processes. We will undertake to provide both a reasonably rigorous development of the formalism of hard scattering of quarks and gluons as well as an intuitive understanding of the physics behind the scattering. We will emphasize the role of logarithmic corrections as well as power counting in α_s in order to understand the behaviors of hard scattering processes. We will include “rules of thumb” as well as “official recommendations”, and where possible will seek to dispel some myths. We will also discuss the impact of soft processes on the measurements of hard scattering processes. Experiences that have been gained at the Fermilab Tevatron will be recounted and, where appropriate, extrapolated to the LHC.

1. Introduction

Scattering processes at high energy hadron colliders can be classified as either hard or soft. Quantum Chromodynamics (QCD) is the underlying theory for all such processes, but the approach and level of understanding is very different for the two cases. For hard processes, e.g. Higgs or high p_T jet production, the rates and event properties

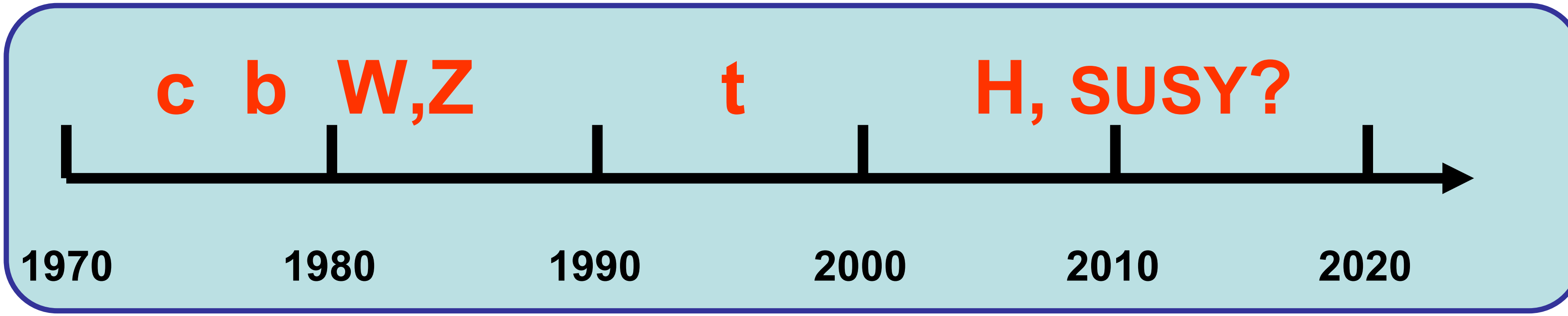
past, present and future proton/antiproton colliders…

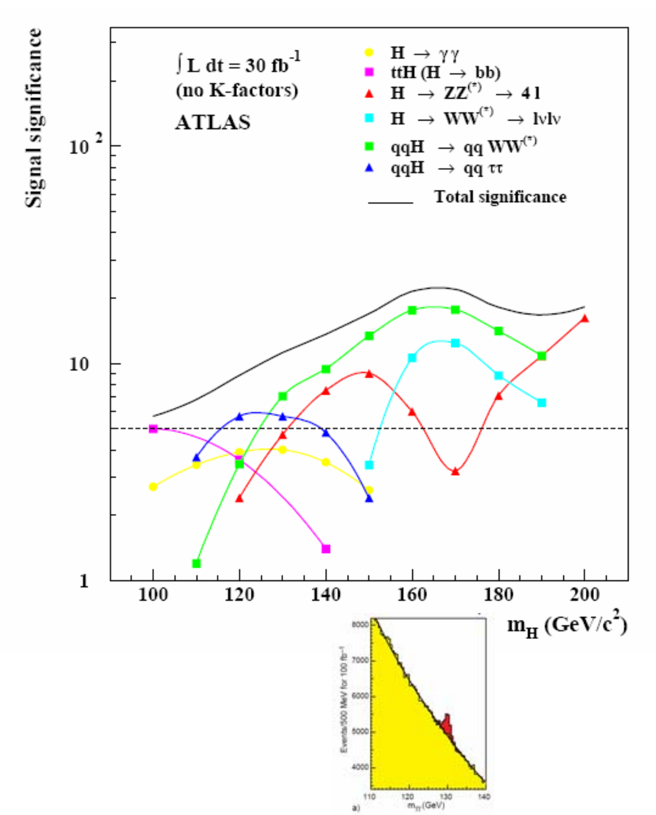
Tevatron (1987→)
Fermilab
proton-antiproton collisions
$\sqrt{S}$ = 1.8, 1.96 TeV

Sp$\bar{p}$S (1981 → 1990)
CERN
proton-antiproton collisions
$\sqrt{S}$ = 540, 630 GeV

LHC (2007 →)
CERN
proton-proton and heavy ion collisions
$\sqrt{S}$ = 14 TeV

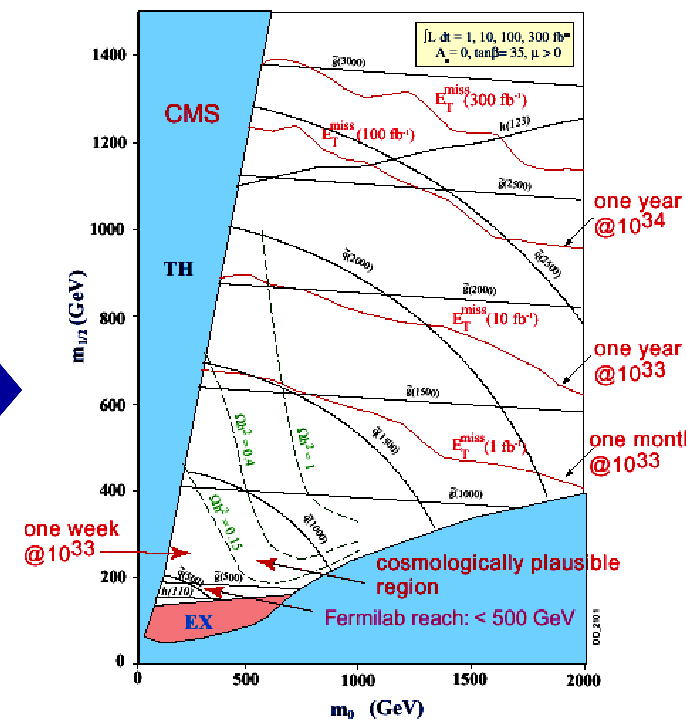
discoveries





Higgs discovery at LHC

SUSY discovery at LHC



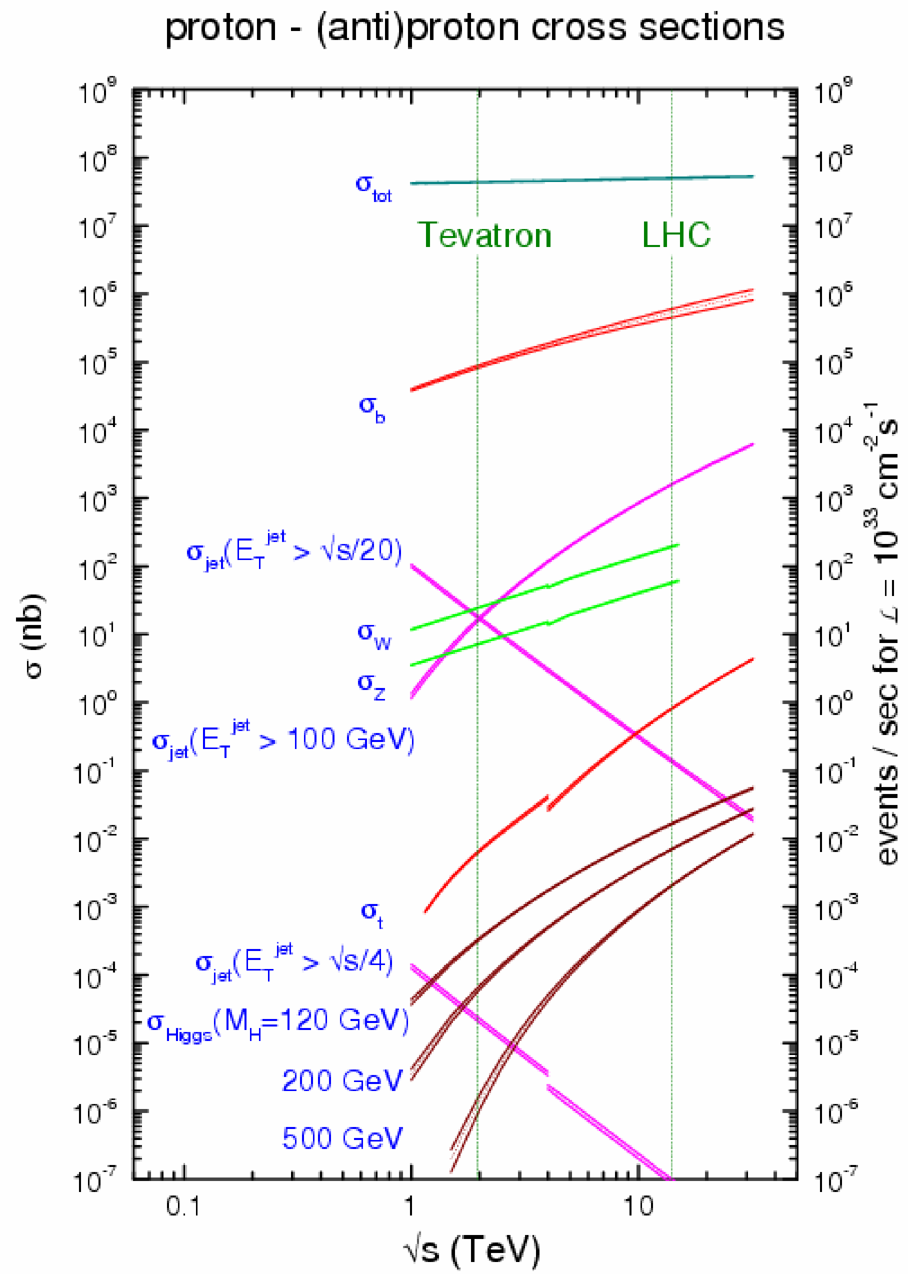
What can we calculate?

Scattering processes at high energy hadron colliders can be classified as either **HARD** or **SOFT**

Quantum Chromodynamics (QCD) is the underlying theory for **all** such processes, but the approach (and the level of understanding) is very different for the two cases

For **HARD** processes, e.g. W or high-E_T jet production, the rates and event properties can be predicted with some precision using perturbation theory

For **SOFT** processes, e.g. the total cross section or diffractive processes, the rates and properties are dominated by non-perturbative QCD effects, which are much less well understood



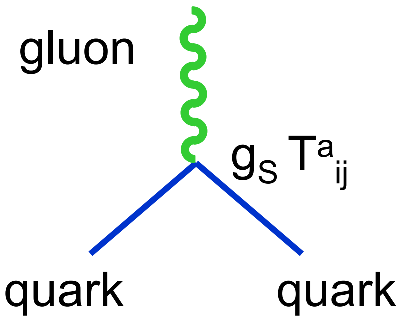
Outline

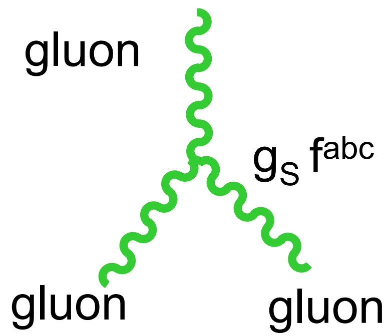
- Basics: QCD, partons, pdfs
  - basic parton model ideas for DIS
  - scaling violation & DGLAP
  - parton distribution functions

- Application to hadron colliders
  - hard scattering & basic kinematics
  - the Drell Yan process in the parton model
  - order α_S corrections to DY, singularities, factorisation
  - examples of other hard processes and their phenomenology
  - parton luminosity functions
  - uncertainties in the calculations

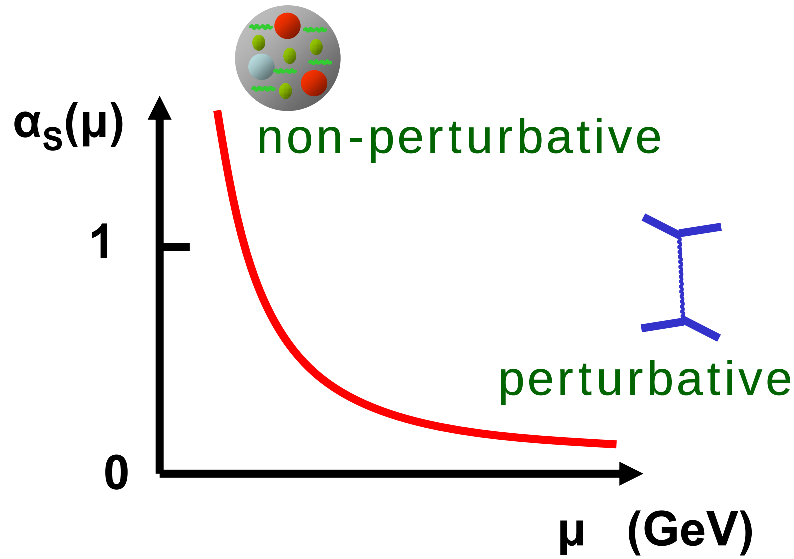
- Beyond fixed-order inclusive cross sections
  - Sudakov logs and resummation
  - parton showering models (basic concepts only!)
  - the role of non-perturbative contributions: intrinsic k_T,
  - underlying event/minimum bias contributions
  - theoretical frontiers: exclusive production of Higgs

Basics of QCD





$\alpha_S = g_S^2/4\pi$



- renormalisation of the coupling

$$\frac{\mu^2}{\alpha_s(\mu^2)}\frac{\partial\alpha_s(\mu^2)}{\partial\mu^2} = -\frac{\alpha_s(\mu^2)}{4\pi}\beta_0 - (\frac{\alpha_s(\mu^2)}{4\pi})^2\beta_1 - (\frac{\alpha_s(\mu^2)}{4\pi})^3\beta_2 + \dots$$

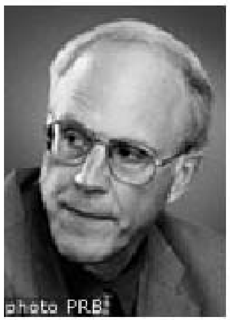
$$\beta_0 = 11 - \frac{2}{3}n_f \,, \qquad \beta_1 = 102 - \frac{38}{3}n_f \,, \qquad \dots$$

- colour matrix algebra

$$[T^a, T^b] = if^{abc}T^c \,, (T^aT^a)_{ij} = C_F\delta_{ij} = 4/3\delta_{ij} \,, \mathrm{Tr}(T^aT^b) = T_F\delta^{ab} = 1/2\delta^{ab} \,, \dots$$

The Nobel Prize in Physics 2004

"for the discovery of asymptotic freedom in the theory of the strong interaction"



David J. Gross

1/3 of the prize

USA

Kavli Institute for Theoretical Physics, University of California
Santa Barbara, CA, USA

b. 1941

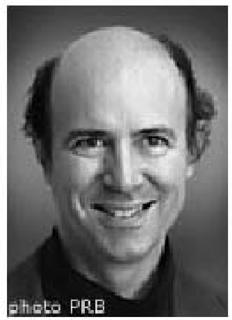
H. David Politzer

1/3 of the prize

USA

California Institute of Technology
Pasadena, CA, USA

b. 1949



Frank Wilczek

1/3 of the prize

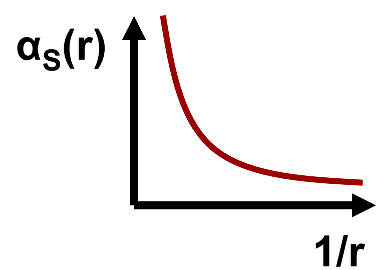
USA

Massachusetts Institute of Technology (MIT)
Cambridge, MA, USA

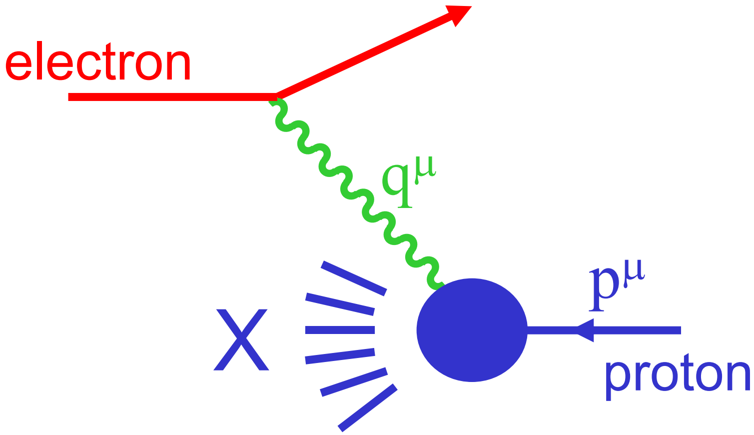
b. 1951

Asymptotic Freedom

“What this year's Laureates discovered was something that, at first sight, seemed completely contradictory. The interpretation of their mathematical result was that the closer the quarks are to each other, the *weaker* is the 'colour charge'. When the quarks are really close to each other, the force is so weak that they behave almost as free particles. This phenomenon is called ‘asymptotic freedom’. The converse is true when the quarks move apart: the force becomes stronger when the distance increases.”



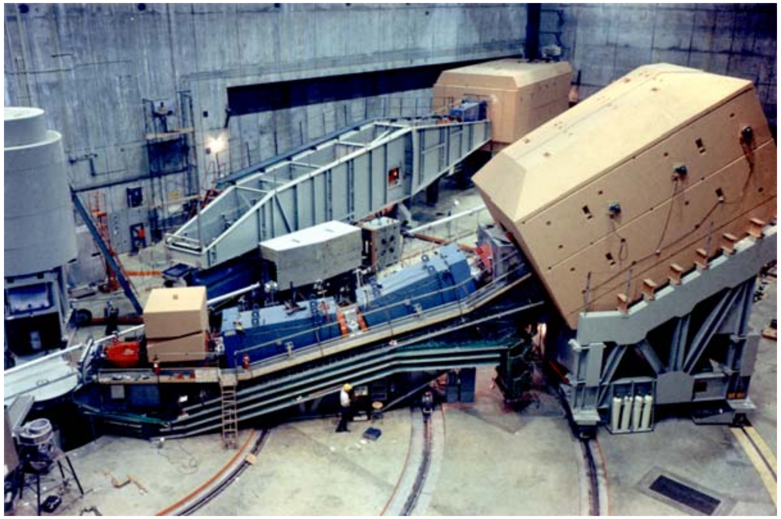
deep inelastic scattering and the parton model



- variables

$Q^2 = -q^2$ (resolution)

$x = Q^2 / 2p \cdot q$ (inelasticity)



- structure functions

$d\sigma / dx dQ^2 \propto \alpha^2 Q^{-4} F_2(x, Q^2)$

- (Bjorken) scaling

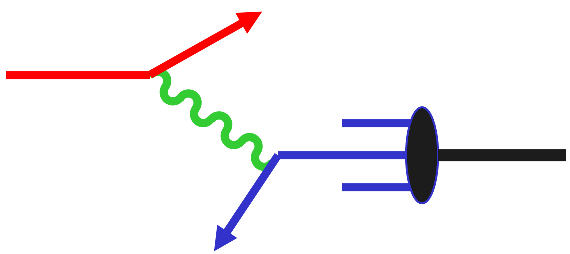
$F_2(x, Q^2) \rightarrow F_2(x)$ (SLAC, ~1970)

Bjorken 1968

the parton model (Feynman 1969)

infinite momentum frame

- photon scatters incoherently off massless, pointlike, spin-1/2 quarks
- probability that a quark carries fraction ξ of parent proton's momentum is $q(\xi)$, $(0 < \xi < 1)$



$$F_2(x) = \sum_{q,\bar{q}} \int_0^1 d\xi \;\; e_q^2 \, \xi \, q(\xi) \, \delta(x-\xi) = \sum_{q,\bar{q}} \;\; e_q^2 \, x \, q(x)$$

$$= \frac{4}{9} \, x u(x) + \frac{1}{9} \, x d(x) + \frac{1}{9} \, x s(x) + \; ...$$

- the functions u(x), d(x), s(x), … are called parton distribution functions (pdfs) - they encode information about the proton's deep structure

extracting pdfs from experiment

- different beams (e,μ,ν,…) & targets (H,D,Fe,…) measure different combinations of quark pdfs
- thus the individual q(x) can be extracted from a set of structure function measurements
- gluon not measured directly, but carries about 1/2 the proton's momentum

$$F_2^{ep} = \frac{4}{9}(u+\bar{u}) + \frac{1}{9}(d+\bar{d}) + \frac{1}{9}(s+\bar{s}) + \ldots$$

$$F_2^{en} = \frac{1}{9}(u+\bar{u}) + \frac{4}{9}(d+\bar{d}) + \frac{1}{9}(s+\bar{s}) + \ldots$$

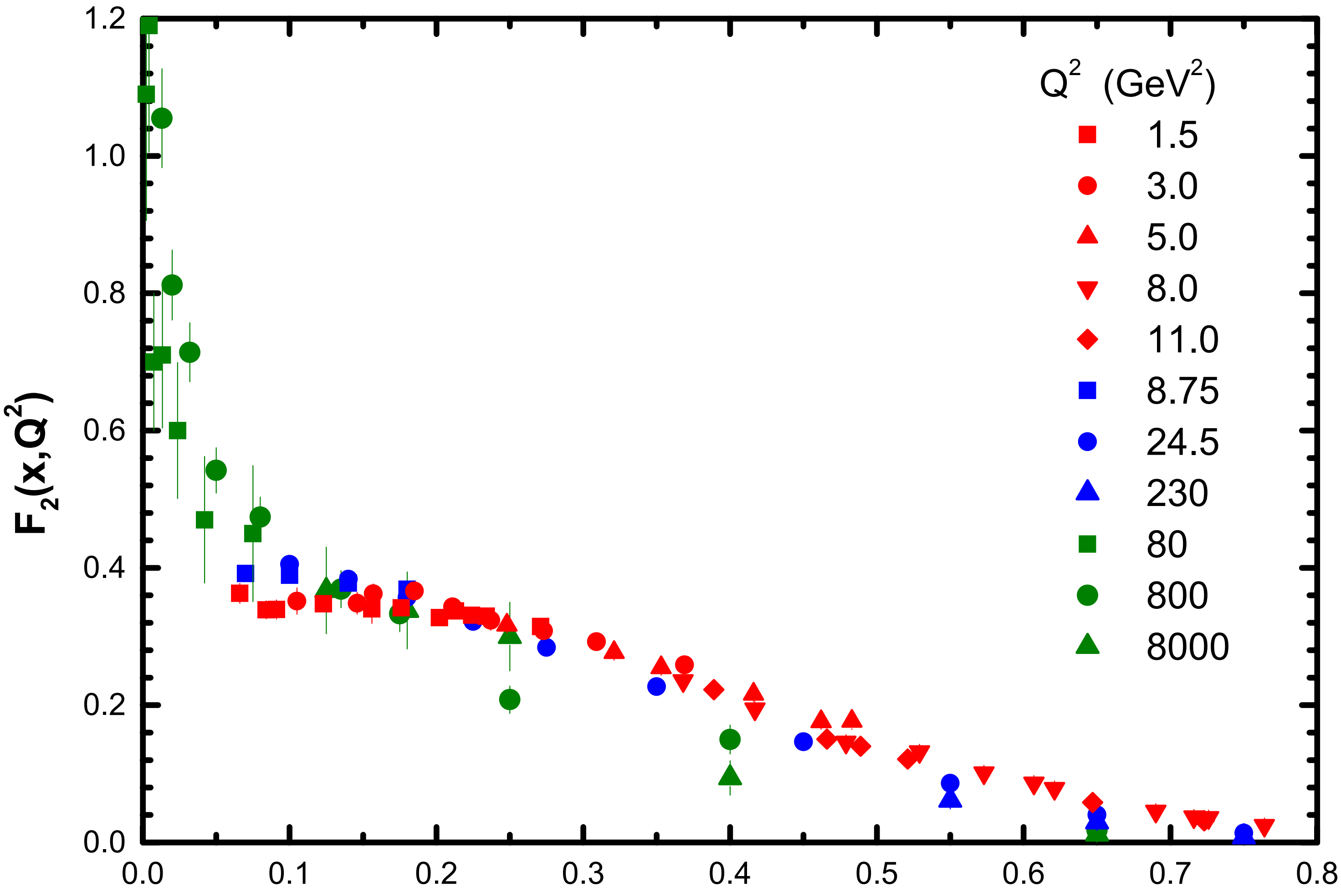
$$F_2^{\nu p} = 2\left[d + s + \bar{u} + \ldots\right]$$

$$F_2^{\nu n} = 2\left[u + \bar{d} + \bar{s} + \ldots\right]$$

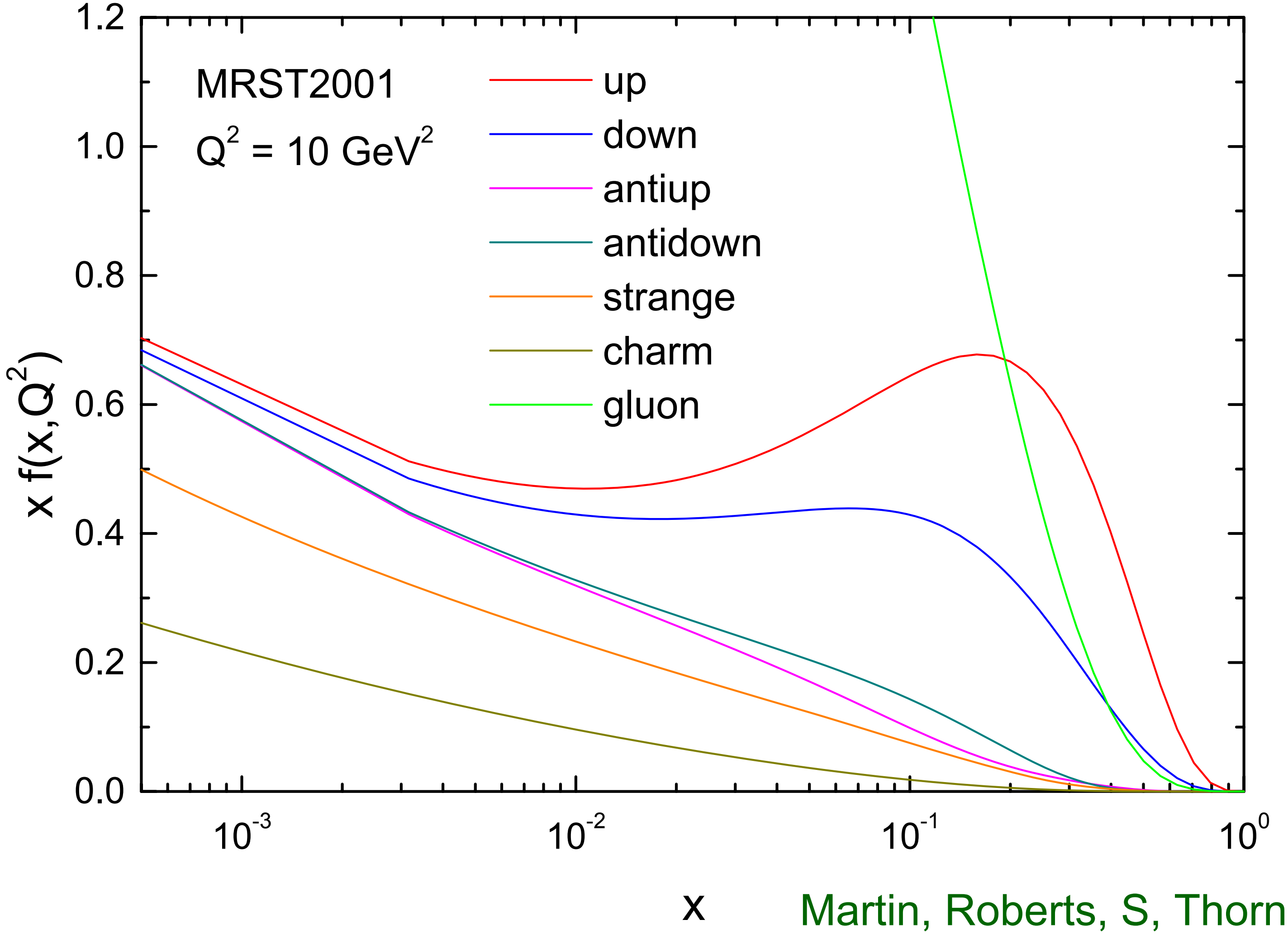
$$s = \bar{s} = \frac{5}{6} F_2^{\nu N} - 3F_2^{eN}$$

$$\sum_q \int_0^1 dx\, x\left(q(x) + \bar{q}(x)\right) = 0.55$$

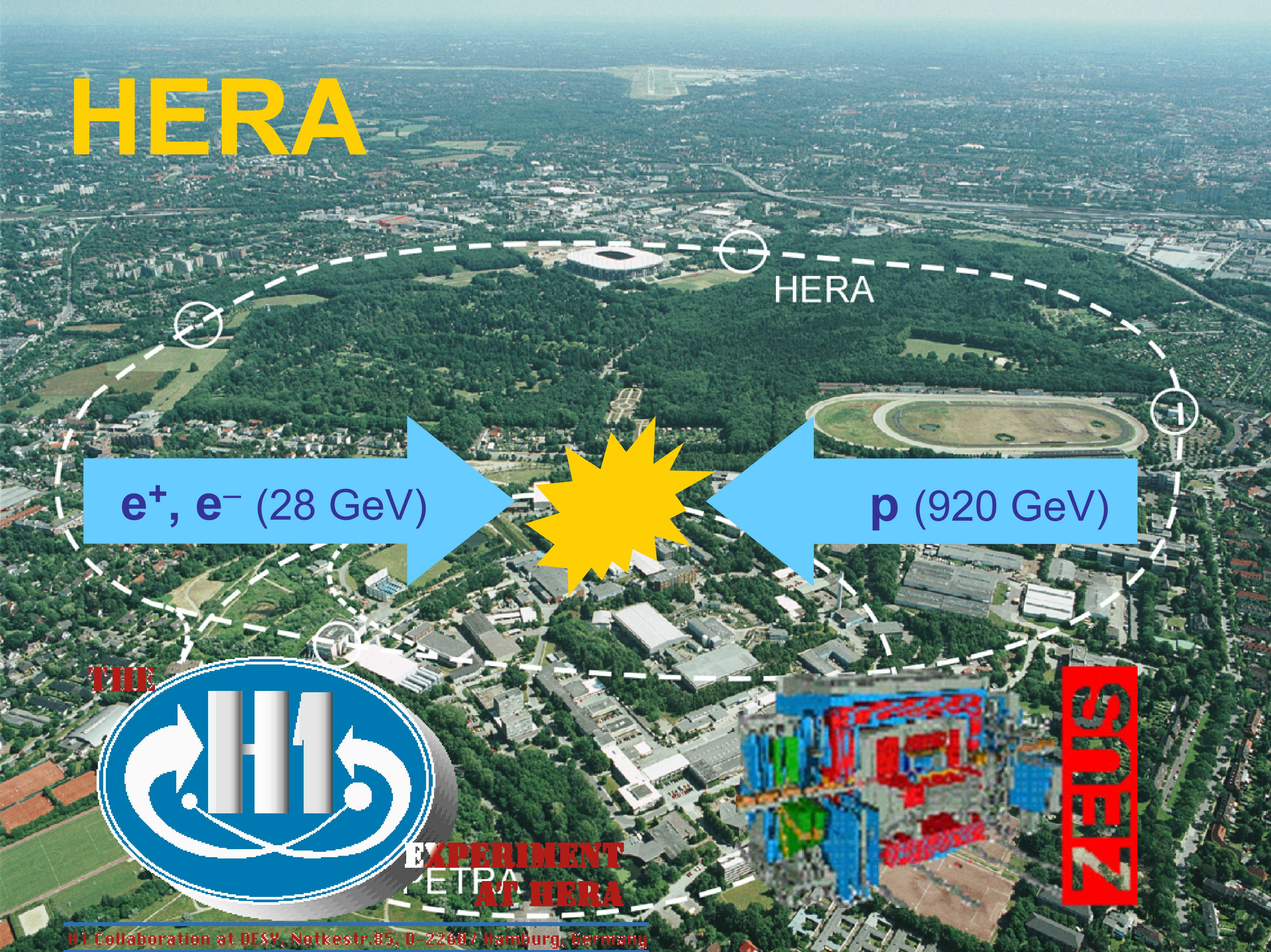
35 years of Deep Inelastic Scattering



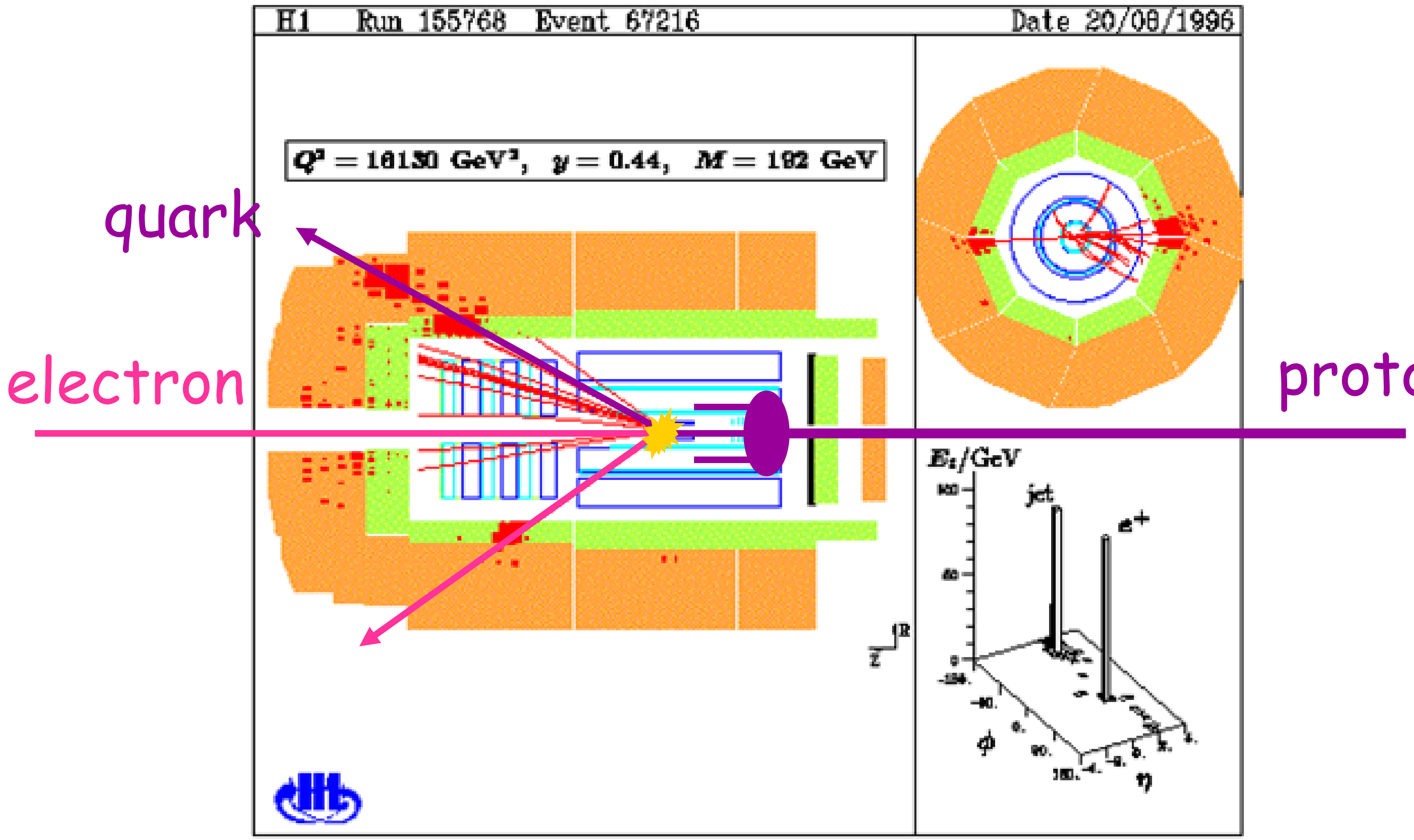
(MRST) parton distributions in the proton



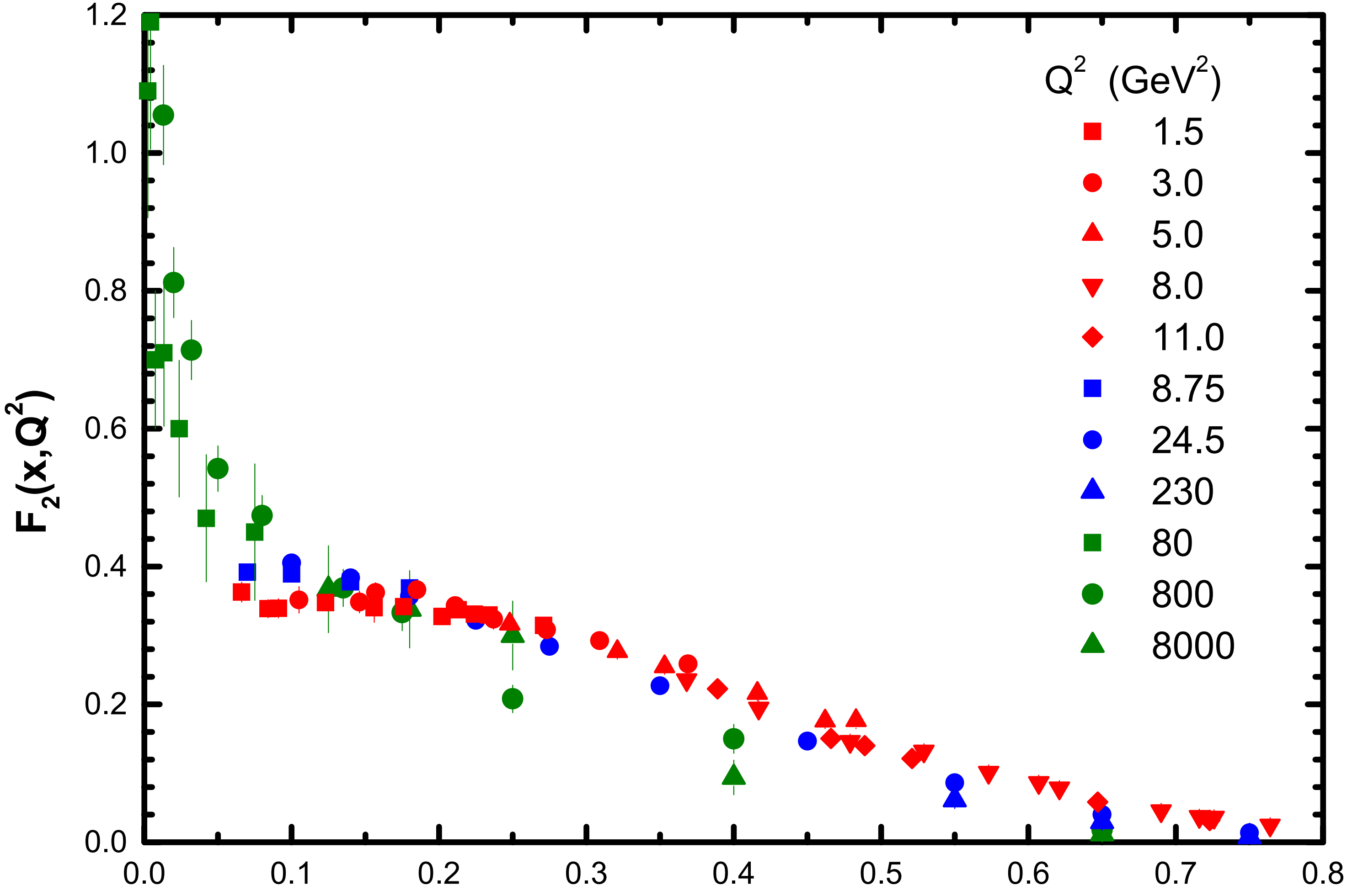
Martin, Roberts, S, Thorne


HERA
HERA
e+, e− (28 GeV)
p (920 GeV)
THE
H1
EXPERIMENT
AT HERA
PETRA
ZEUS
H1 Collaboration at DESY, Notkestr.85, D-22607 Hamburg, Germany

a deep inelastic scattering event at HERA

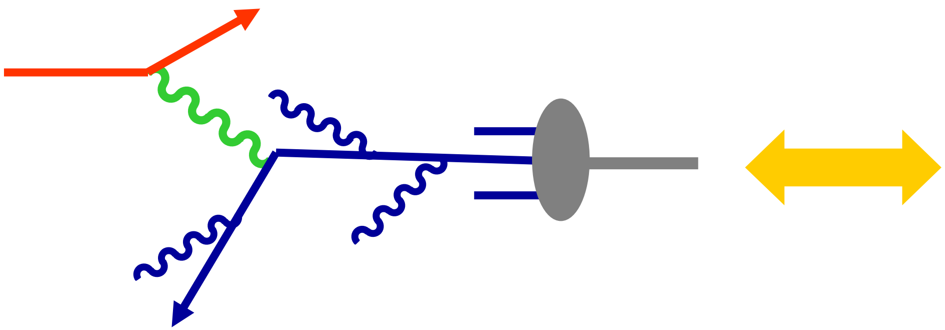


35 years of Deep Inelastic Scattering

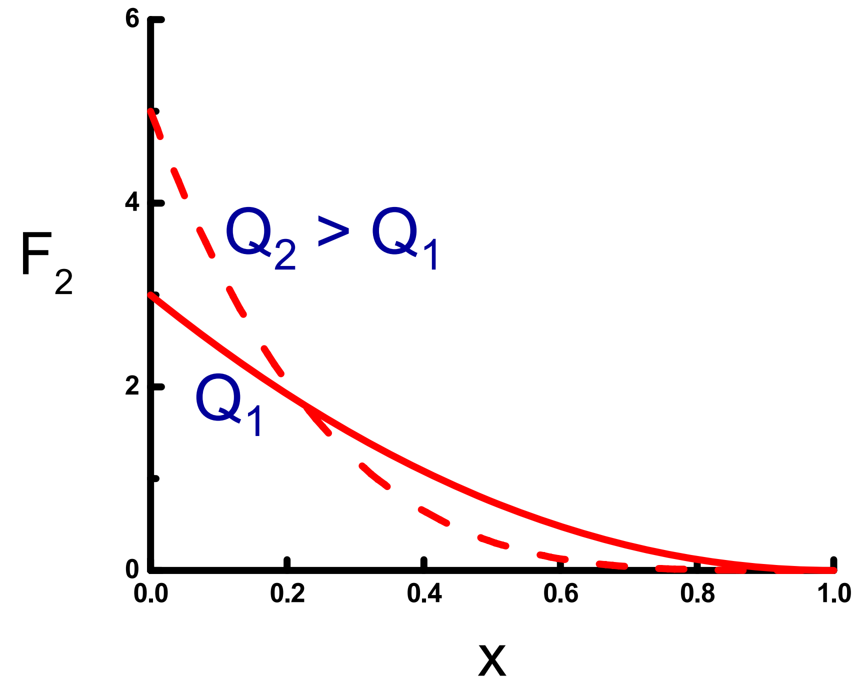


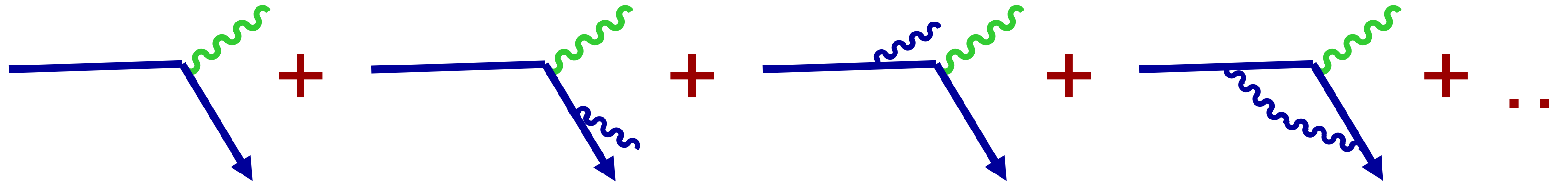
scaling violations and QCD

The structure function data exhibit systematic violations of Bjorken scaling:



quarks emit gluons!





$$\hat{F}_2 = e_q^2\delta(1-x)+e_q^2\frac{\alpha_S}{2\pi}x\left[P(x)\ln(Q^2/\kappa^2)+C(x)\right]$$

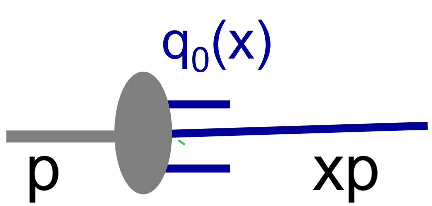
where the logarithm comes from ('collinear singularity') and

$$\int_0^{\sim Q^2}\frac{dk_T^2}{k_T^2} \to \int_{\kappa^2}^{\sim Q^2}\frac{dk_T^2}{k_T^2} \to \ln(Q^2/\kappa^2)$$

$$P(x) = C_F\left[\frac{1+x^2}{(1-x)_+}+\frac{3}{2}\delta(1-x)\right]$$

$$\int_0^1 dx = \frac{f(x)}{(1-x)_+} = \int_0^1 \frac{f(x)-f(1)}{1-x}$$

then convolute with a 'bare' quark distribution in the proton:



$$F_2(x,Q^2) = x\sum_q e_q^2\Big[q_0(x)+\frac{\alpha_S}{2\pi}\int_x^1\frac{dy}{y}q_0(y)$$
$$\left\{P(x/y)\ln(Q^2/\kappa^2)+C(x/y)\right\}\Big]$$

next, factorise the collinear divergence into a ‘renormalised’ quark distribution, by introducing the factorisation scale μ^2 :

$$q(x,\mu^2) = q_0(x) + \frac{\alpha_S}{2\pi}\int_x^1 \frac{dy}{y} q_0(y)\left\{P(x/y)\ln(\mu^2/\kappa^2) + \overline{C}(x/y)\right\}$$

then

$$\frac{1}{x}F_2(x,Q^2) = x\sum_q e_q^2 \int_x^1 \frac{dy}{y} q(y,\mu^2) \left\{\delta(1-\frac{x}{y}) + \frac{\alpha_s}{2\pi}\left(P(x/y)\ln(Q^2/\mu^2) + C_q(x/y)\right)\right\}$$

finite, by construction

note arbitrariness of $C_q = C - \overline{C}$ → ‘factorisation scheme dependence’

we can choose $\overline{C}$ such that $C_q = 0$, the DIS scheme, or use dimensional regularisation and remove the poles at N=4, the $\overline{\text{MS}}$ scheme, with $C_q \neq 0$

$q(x,\mu^2)$ is not calculable in perturbation theory,* but its scale (μ^2) dependence is:

$$\mu^2 \frac{\partial}{\partial \mu^2} q(x,\mu^2) = \frac{\alpha_S(\mu^2)}{2\pi}\int_x^1 \frac{dy}{y} q(y,\mu^2) P(x/y)$$

Dokshitzer
Gribov
Lipatov
Altarelli
Parisi

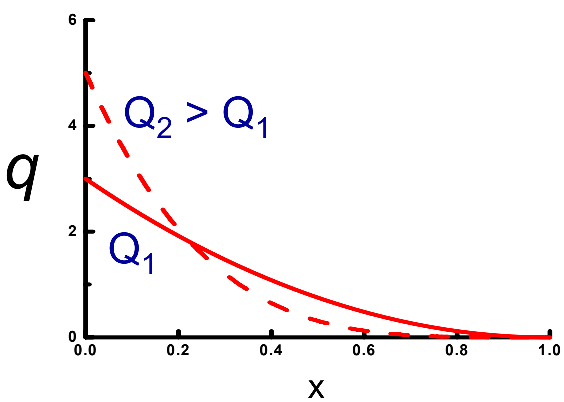
*lattice QCD?

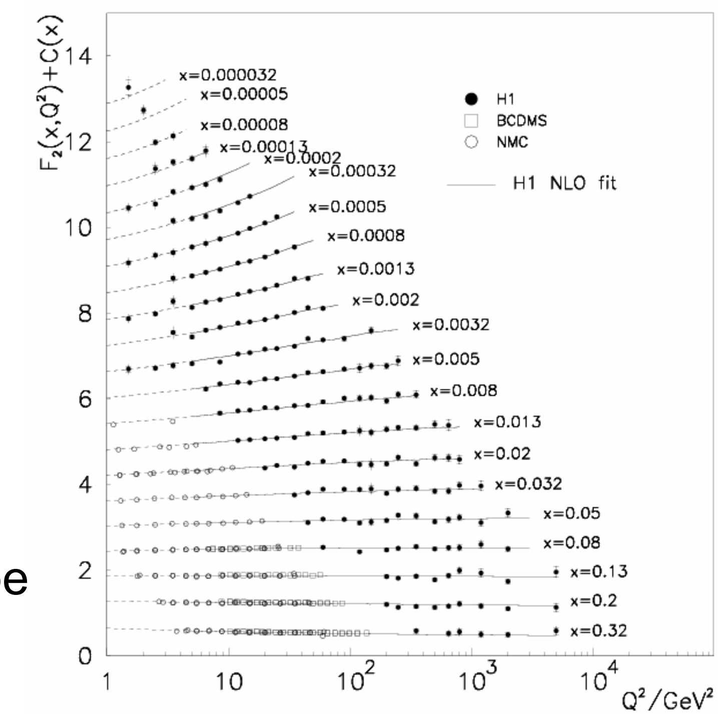
note that we are free to choose $\mu^2 = Q^2$ in which case

$$\frac{1}{x}F_2(x,Q^2) = x\sum_q e_q^2 \int_x^1 \frac{dy}{y} q(y,Q^2) \left\{ \delta(1-\frac{x}{y}) + \frac{\alpha_s}{2\pi} C_q(x/y) \right\}$$

coefficient function, see QCD book

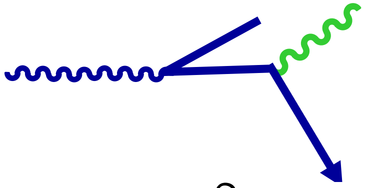
… and thus the scaling violations of the structure function follow those of $q(x,Q^2)$ predicted by the DGLAP equation:





the rate of change of F_2 is proportional to α_S (DGLAP), hence structure function data can be used to measure the strong coupling!

however, we must also include the gluon contribution



$$\frac{1}{x}F_2(x,Q^2) = x\sum_q e_q^2 \int_x^1 \frac{dy}{y} q(y,Q^2)\left\{\delta(1-\frac{x}{y}) + \frac{\alpha_s(Q^2)}{2\pi}C_q(x/y)\right\}$$
$$+ \; x\sum_q e_q^2 \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dy}{y} g(y,Q^2) C_g(x/y)$$

coefficient functions - see QCD book

… and with additional terms in the DGLAP equations

$$\mu^2 \frac{\partial q_i(x,\mu^2)}{\partial \mu^2} = \frac{\alpha_S(\mu^2)}{2\pi}(P^{qq} * q_i + 2n_f P^{qg} * g)$$
$$\mu^2 \frac{\partial g(x,\mu^2)}{\partial \mu^2} = \frac{\alpha_S(\mu^2)}{2\pi}(P^{gq} * \sum_i q_i + P^{gg} * g)$$

$q_i = u, \bar{u}, d, \bar{d}, \ldots$

$* =$ convolution integral

note that at small (large) *x*, the gluon (quark) contribution dominates the evolution of the quark distributions, and therefore of F_2

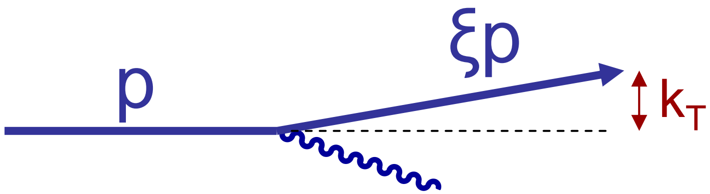
$$P^{qq} = \frac{4}{3}(\frac{1+x^2}{1-x})_+$$
$$P^{qg} = \frac{1}{2}(x^2 + (1-x)^2)$$
$$P^{gq} = \frac{4}{3}\left(\frac{1+(1-x)^2}{x}\right)$$
$$P^{gg} = 6\left(\frac{1-x}{x} + x(1-x) + (\frac{x}{1-x})_+\right) - \left(\frac{1}{2} + \frac{n_f}{3}\right)\delta(1-x)$$

splitting functions

DGLAP evolution: physical picture

Altarelli, Parisi (1977)

- a fast-moving quark loses momentum by emitting a gluon:

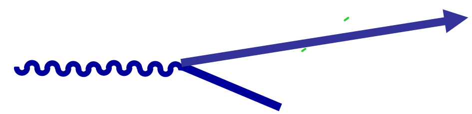


$$d\mathcal{P} \simeq \frac{\alpha_S(k_T^2)}{2\pi} \frac{dk_T^2}{k_T^2} P^{qq}(\xi)\, d\xi$$

- … with phase space $k_T^2 < O(Q^2)$, hence

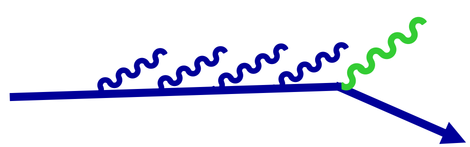
$$d\mathcal{P} \simeq \frac{\alpha_S}{2\pi} \ln Q^2\, P^{qq}(\xi)\, d\xi$$

- similarly for other splittings



$$\sim \; P^{gq}$$

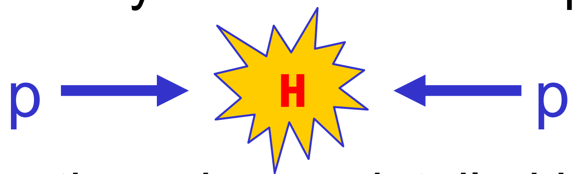
- the combination of all such probabilistic splittings correctly generates the leading-logarithm approximation to the all-orders in pQCD solution of the DGLAP equations



basis of parton shower Monte Carlos!

parton distribution functions

- the bulk of the information on pdfs comes from fitting DIS structure function data, although hadron-hadron collisions data also provide important constraints (see later)
- pdfs are useful in two ways:
  - they are essential for predicting hadron collider cross sections, e.g.



$$\sigma_H \propto [g(x \simeq M_H/\sqrt{s}, M_H^2)]^2$$

  - they give us detailed information on the quark flavour content of the nucleon
- no need to solve the DGLAP equations each time a pdf is needed; high-precision approximations obtained from 'global fits' are available 'off the shelf", e.g.

```
SUBROUTINE MRST(X,Q,U,UBAR,D,DBAR...,BBAR,G)
```

input | output

how pdfs are obtained

- choose a factorisation scheme (e.g. MSbar), an order in perturbation theory (see below, e.g. LO, NLO, NNLO) and a 'starting scale' Q_0 where pQCD applies (e.g. 1-2 GeV)
- parametrise the quark and gluon distributions at Q_0, e.g.

$$f_i(x, Q_0^2) = A_i x^{a_i} [1 + b_i \sqrt{x} + c_i x] (1 - x)^{d_i}$$

- solve DGLAP equations to obtain the pdfs at any x and scale $Q > Q_0$; fit data for parameters $\{A_i, a_i, \ldots \alpha_S\}$
- approximate the exact solutions (e.g. interpolation grids, expansions in polynomials etc) for ease of use

pdfs from global fits - summary

Formalism
LO, NLO or NNLO DGLAP
MSbar factorisation
Q_0^2
functional form @ Q_0^2
sea quark (a)symmetry
etc.

Data
DIS (SLAC, BCDMS, NMC, E665, CCFR, H1, ZEUS, …)
Drell-Yan (E605, E772, E866, …)
High E_T jets (CDF, D0)
W rapidity asymmetry (CDF,D0)
νN dimuon (CCFR, NuTeV)
etc.

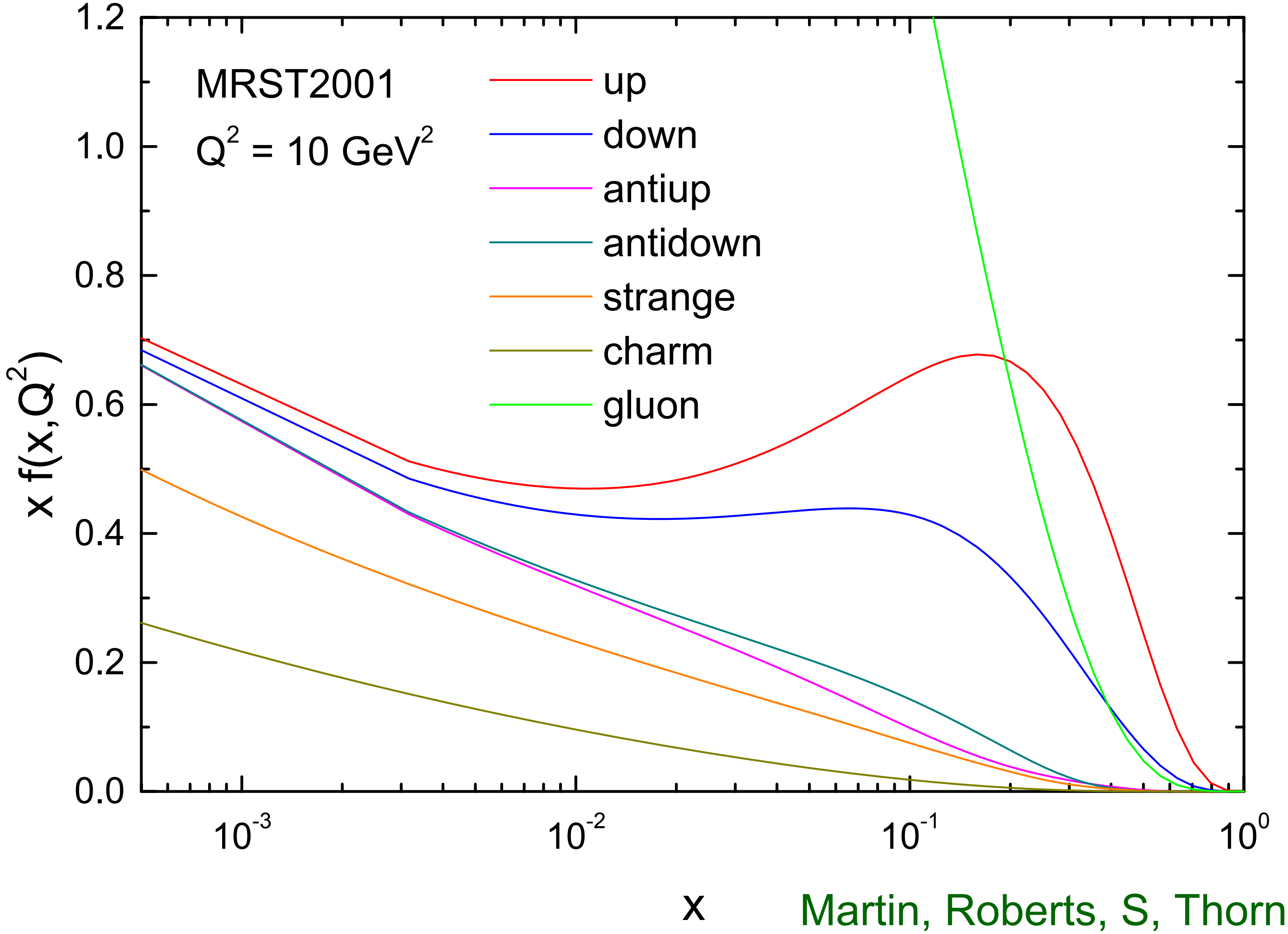
$$f_i(x,Q^2) \pm \delta f_i(x,Q^2)$$

$$\alpha_S(M_Z)$$

Who?
CTEQ, MRST, Alekhin, H1, ZEUS, …

http://durpdg.dur.ac.uk/hepdata/pdf.html

(MRST) parton distributions in the proton



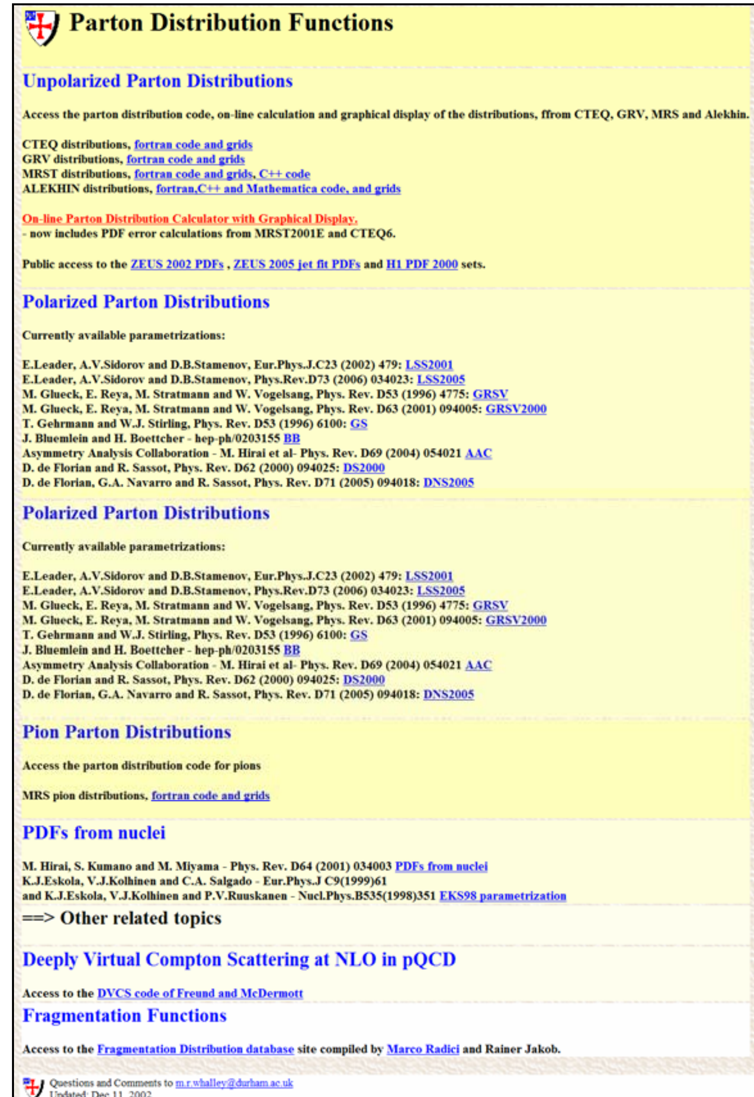
Martin, Roberts, S, Thorne

where to find parton distributions

HEPDATA pdf website
http://durpdg.dur.ac.uk/hepdata/pdf.html

- access to code for MRST, CTEQ etc

- online pdf plotting



Parton Distribution Functions

Unpolarized Parton Distributions

Access the parton distribution code, on-line calculation and graphical display of the distributions, ffrom CTEQ, GRV, MRS and Alekhin.

CTEQ distributions, fortran code and grids
GRV distributions, fortran code and grids
MRST distributions, fortran code and grids, C++ code
ALEKHIN distributions, fortran,C++ and Mathematica code, and grids

On-line Parton Distribution Calculator with Graphical Display,
- now includes PDF error calculations from MRST2001E and CTEQ6.

Public access to the ZEUS 2002 PDFs , ZEUS 2005 jet fit PDFs and H1 PDF 2000 sets.

Polarized Parton Distributions

Currently available parametrizations:

E.Leader, A.V.Sidorov and D.B.Stamenov, Eur.Phys.J.C23 (2002) 479: LSS2001
E.Leader, A.V.Sidorov and D.B.Stamenov, Phys.Rev.D73 (2006) 034023: LSS2005
M. Glueck, E. Reya, M. Stratmann and W. Vogelsang, Phys. Rev. D53 (1996) 4775: GRSV
M. Glueck, E. Reya, M. Stratmann and W. Vogelsang, Phys. Rev. D63 (2001) 094005: GRSV2000
T. Gehrmann and W.J. Stirling, Phys. Rev. D53 (1996) 6100: GS
J. Bluemlein and H. Boettcher - hep-ph/0203155 BB
Asymmetry Analysis Collaboration - M. Hirai et al- Phys. Rev. D69 (2004) 054021 AAC
D. de Florian and R. Sassot, Phys. Rev. D62 (2000) 094025: DS2000
D. de Florian, G.A. Navarro and R. Sassot, Phys. Rev. D71 (2005) 094018: DNS2005

Polarized Parton Distributions

Currently available parametrizations:

E.Leader, A.V.Sidorov and D.B.Stamenov, Eur.Phys.J.C23 (2002) 479: LSS2001
E.Leader, A.V.Sidorov and D.B.Stamenov, Phys.Rev.D73 (2006) 034023: LSS2005
M. Glueck, E. Reya, M. Stratmann and W. Vogelsang, Phys. Rev. D53 (1996) 4775: GRSV
M. Glueck, E. Reya, M. Stratmann and W. Vogelsang, Phys. Rev. D63 (2001) 094005: GRSV2000
T. Gehrmann and W.J. Stirling, Phys. Rev. D53 (1996) 6100: GS
J. Bluemlein and H. Boettcher - hep-ph/0203155 BB
Asymmetry Analysis Collaboration - M. Hirai et al- Phys. Rev. D69 (2004) 054021 AAC
D. de Florian and R. Sassot, Phys. Rev. D62 (2000) 094025: DS2000
D. de Florian, G.A. Navarro and R. Sassot, Phys. Rev. D71 (2005) 094018: DNS2005

Pion Parton Distributions

Access the parton distribution code for pions

MRS pion distributions, fortran code and grids

PDFs from nuclei

M. Hirai, S. Kumano and M. Miyama - Phys. Rev. D64 (2001) 034003 PDFs from nuclei
K.J.Eskola, V.J.Kolhinen and C.A. Salgado - Eur.Phys.J C9(1999)61
and K.J.Eskola, V.J.Kolhinen and P.V.Ruuskanen - Nucl.Phys.B535(1998)351 EKS98 parametrization

==> Other related topics

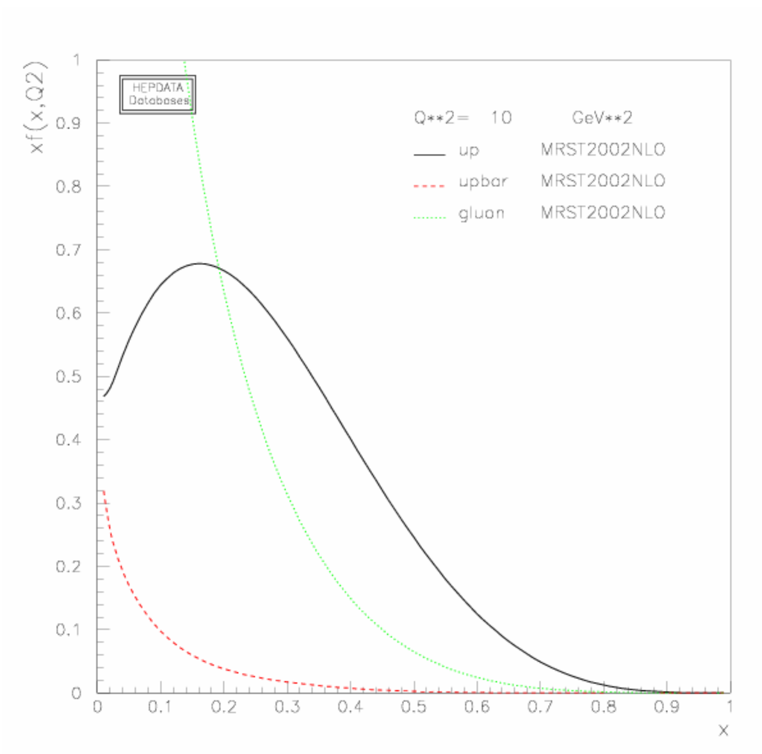
Deeply Virtual Compton Scattering at NLO in pQCD

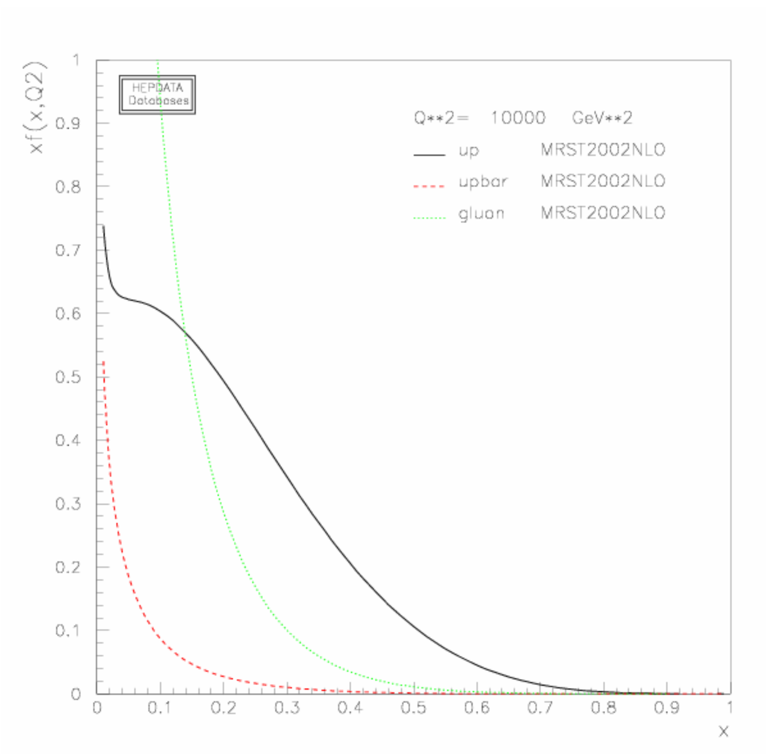
Access to the DVCS code of Freund and McDermott

Fragmentation Functions

Access to the Fragmentation Distribution database site compiled by Marco Radici and Rainer Jakob.

Questions and Comments to m.r.whalley@durham.ac.uk
Updated: Dec 11, 2002


HEPDATA Databases
Q**2= 10 GeV**2
up MRST2002NLO
upbar MRST2002NLO
gluon MRST2002NLO
xf(x,Q2)
x


HEPDATA Databases
Q**2= 10000 GeV**2
up MRST2002NLO
upbar MRST2002NLO
gluon MRST2002NLO
xf(x,Q2)
x

beyond lowest order in pQCD

going to higher orders in pQCD is straightforward in principle, since the above structure for F_2 and for DGLAP generalises in a straightforward way:

$$\frac{\partial \mathbf{q_i(x, Q^2)}}{\partial \log \mathbf{Q^2}} = \frac{\alpha_S}{2\pi} \int_x^1 \frac{\mathbf{dy}}{\mathbf{y}} \Big\{ \mathbf{P_{q_i q_j}(y, \alpha_S)}\ \mathbf{q_j}\Big(\frac{\mathbf{x}}{\mathbf{y}}, \mathbf{Q^2}\Big) + \mathbf{P_{q_i g}(y, \alpha_S)}\ \mathbf{g}\Big(\frac{\mathbf{x}}{\mathbf{y}}, \mathbf{Q^2}\Big) \Big\}$$

$$\frac{\partial \mathbf{g(x, Q^2)}}{\partial \log \mathbf{Q^2}} = \frac{\alpha_S}{2\pi} \int_x^1 \frac{\mathbf{dy}}{\mathbf{y}} \Big\{ \mathbf{P_{g q_j}(y, \alpha_S)}\ \mathbf{q_j}\Big(\frac{\mathbf{x}}{\mathbf{y}}, \mathbf{Q^2}\Big) + \mathbf{P_{gg}(y, \alpha_S)}\ \mathbf{g}\Big(\frac{\mathbf{x}}{\mathbf{y}}, \mathbf{Q^2}\Big) \Big\}$$

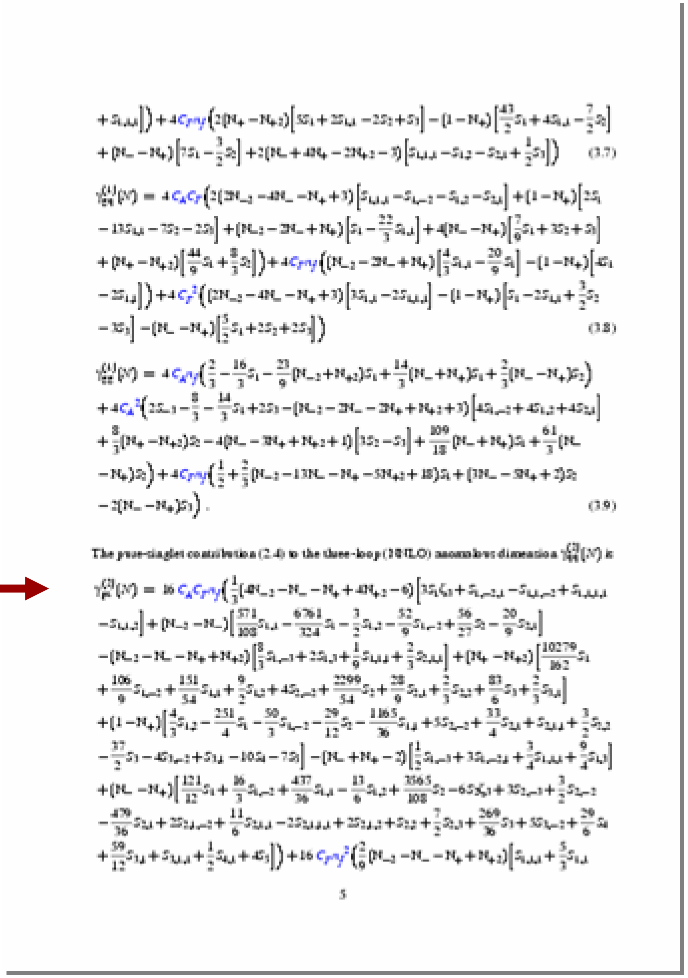
1972-77 → $P^{(0)}$; 1977-80 → $P^{(1)}$; 2004 → $P^{(2)}$

DGLAP: $$P(x, \alpha_S) = P^{(0)} + \alpha_S P^{(1)}(x) + \alpha_S^2 P^{(2)}(x) + \ldots$$

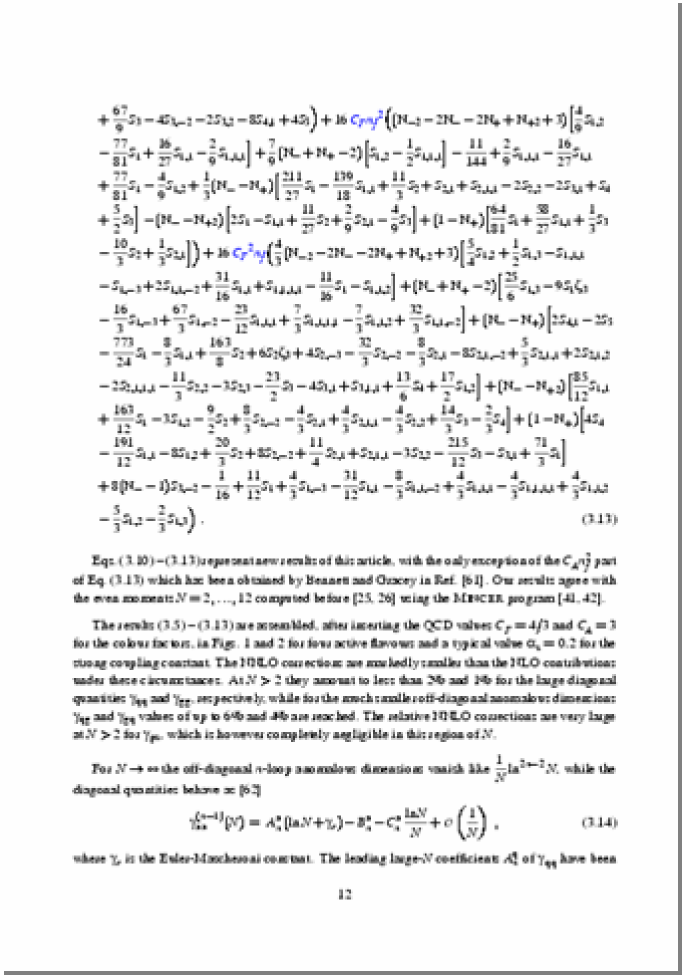
see above — see book — see next slide!

The calculation of the complete set of $P^{(2)}$ splitting functions by Moch, Vermaseren and Vogt (hep-ph/0403192,0404111) completes the calculational tools for a consistent NNLO pQCD treatment of Tevatron & LHC hard-scattering cross sections!

Moch, Vermaseren and Vogt,
hep-ph/0403192, hep-ph/0404111



7 pages later…



anomalous dimensions (moments): $\gamma_N = \int_0^1 dx\, x^{N-1}\, P(x)$

- and for the structure functions…

$$\frac{1}{x}F_2(x,Q^2) = x\sum_q e_q^2 \int_x^1 \frac{dy}{y} q(y,Q^2) \left\{ \delta(1-\frac{x}{y}) + \frac{\alpha_s(Q^2)}{2\pi} C_q^{(1)}(x/y) \right\}$$
$$x\sum_q e_q^2 \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} g(y,Q^2) C_g^{(1)}(x/y) + \mathcal{O}(\alpha_s^2(Q^2))$$

… where up to and including the $O(\alpha_S{}^3)$ coefficient functions are known

- terminology:
  - LO: $P^{(0)}$
  - NLO: $P^{(0,1)}$ and $C^{(1)}$
  - NNLO: $P^{(0,1,2)}$ and $C^{(1,2)}$

- the more pQCD orders are included, the weaker the dependence on the (unphysical) factorisation scale, $\mu_F{}^2$
  - and also the (unphysical) renormalisation scale, $\mu_R{}^2$; note above has $\mu_R{}^2 = Q^2$

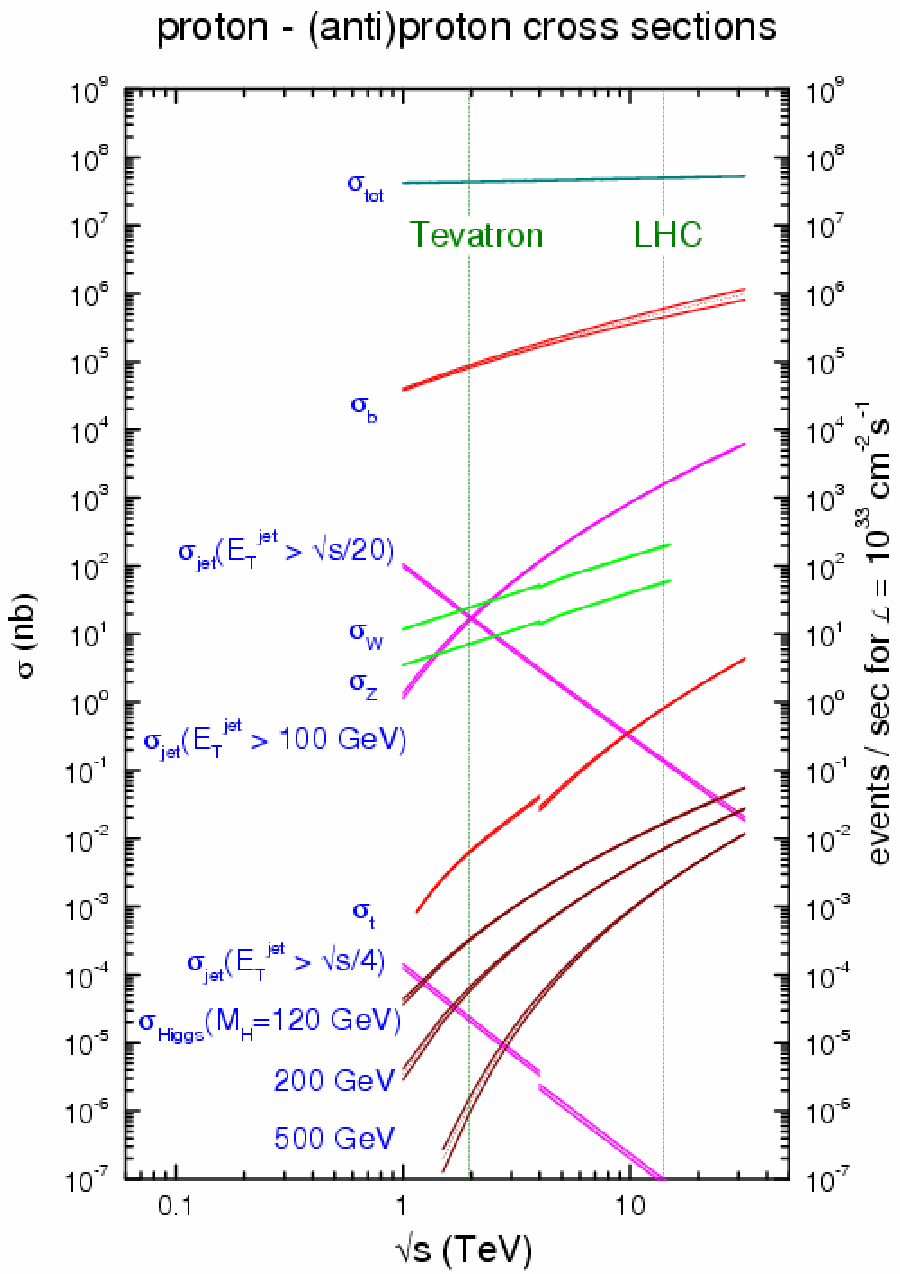
What can we calculate?

Scattering processes at high energy hadron colliders can be classified as either **HARD** or **SOFT**

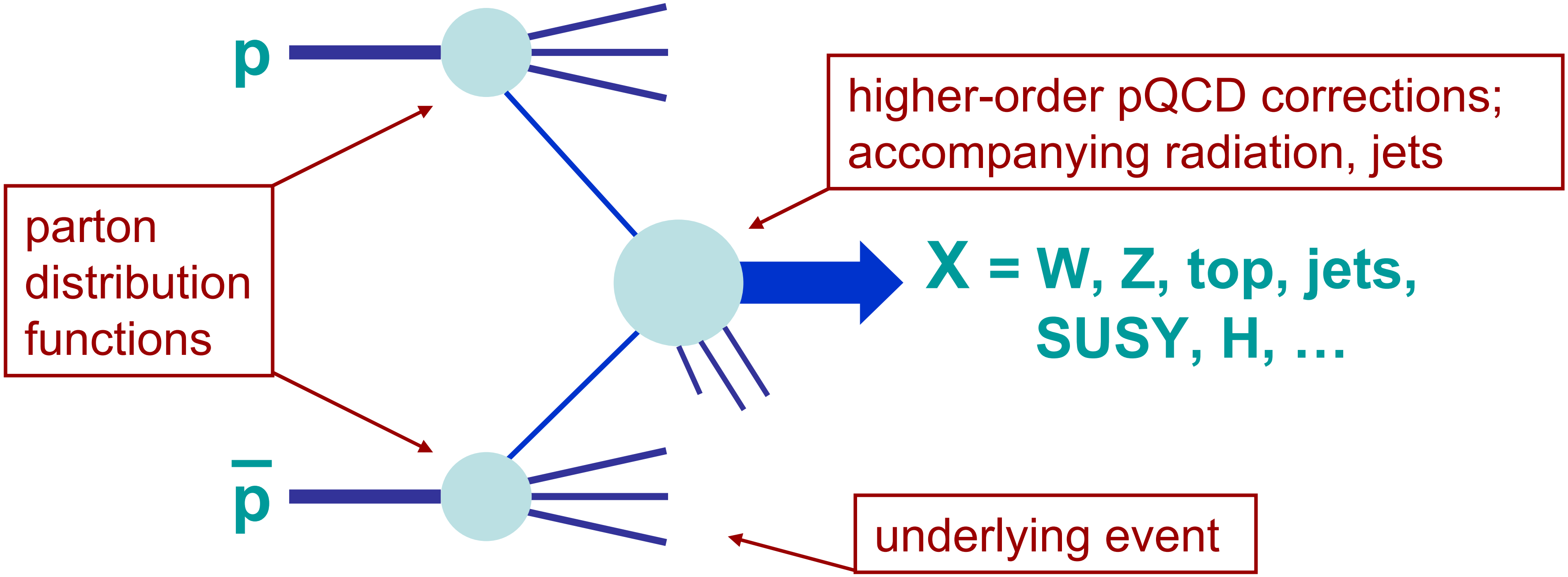
Quantum Chromodynamics (QCD) is the underlying theory for **all** such processes, but the approach (and the level of understanding) is very different for the two cases

For **HARD** processes, e.g. W or high-E_T jet production, the rates and event properties can be predicted with some precision using perturbation theory

For **SOFT** processes, e.g. the total cross section or diffractive processes, the rates and properties are dominated by non-perturbative QCD effects, which are much less well understood



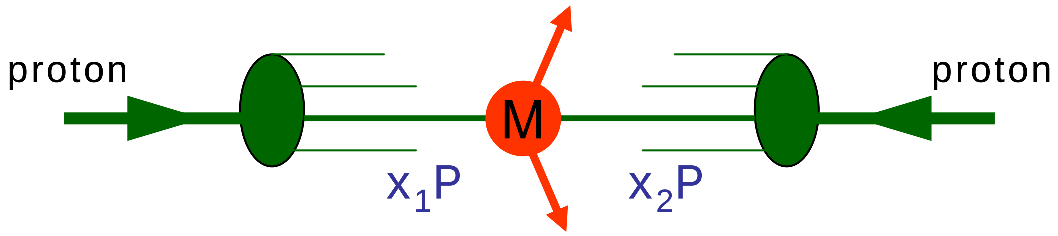
hard scattering in hadron-hadron collisions



for inclusive production, the basic calculational framework is provided by the QCD FACTORISATION THEOREM:

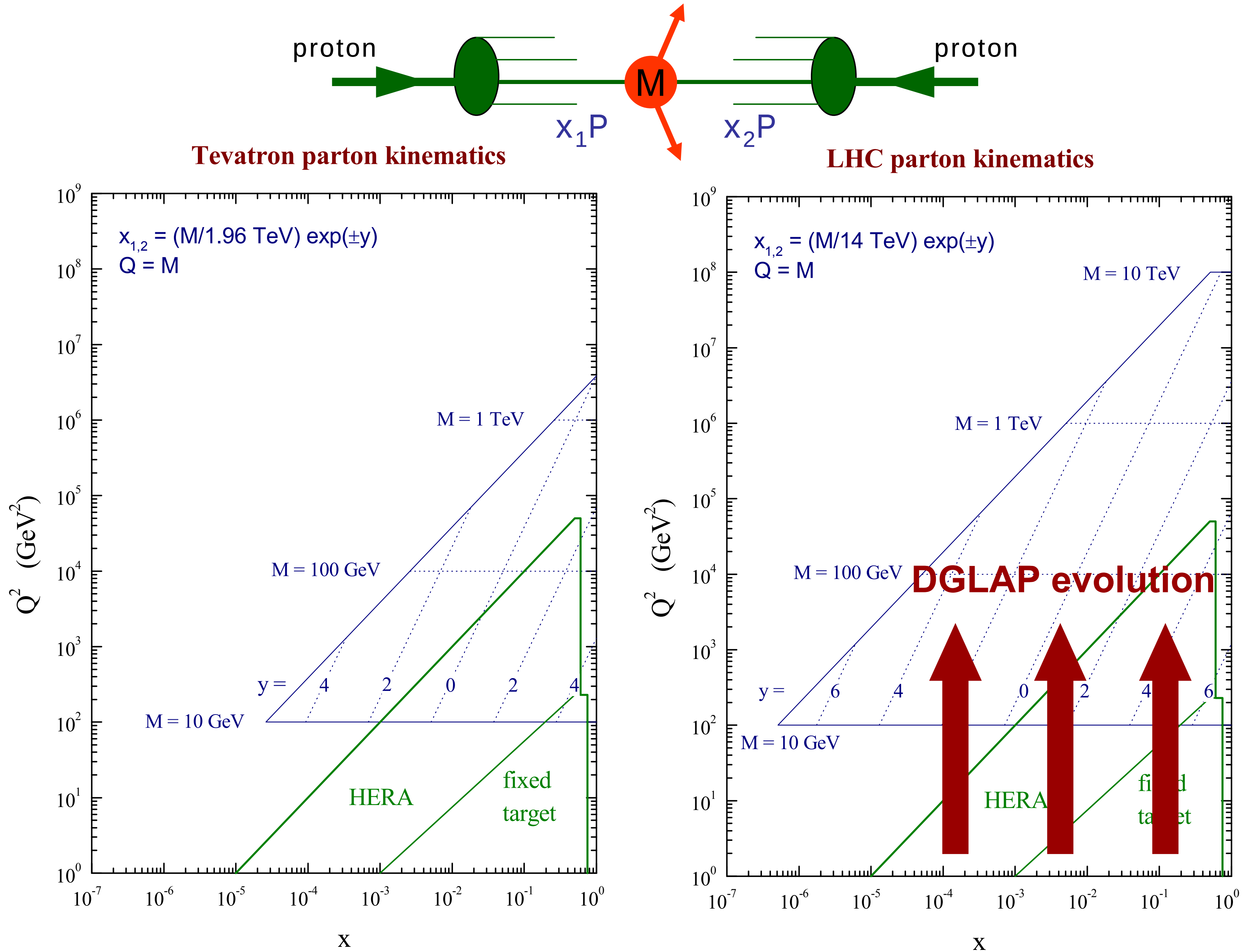
$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2 \, f_a(x_1, \mu_F^2) \, f_b(x_2, \mu_F^2) \times \hat{\sigma}_{ab\to X}\left(x_1, x_2, \{p_i^\mu\}; \alpha_S(\mu_R^2), \alpha(\mu_R^2), \frac{Q^2}{\mu_R^2}, \frac{Q^2}{\mu_F^2}\right)$$

kinematics



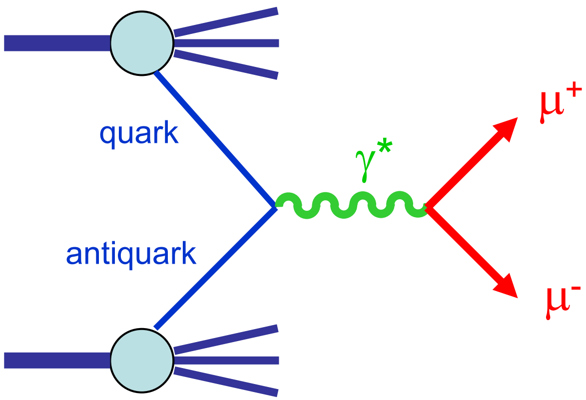
- collision energy: $\sqrt{s}$
- parton momenta:
$$p_1^\mu = x_1 \sqrt{s}/2\,(1,0,0,1)$$
$$p_2^\mu = x_2 \sqrt{s}/2\,(1,0,0,-1)$$
- invariant mass: $M^2 = (p_1+p_2)^2 \equiv \hat{s} = x_1 x_2 s$
- rapidity: $y = \frac{1}{2}\log\frac{E+p_z}{E-p_z} = \frac{1}{2}\log\frac{x_1}{x_2} \Rightarrow \frac{x_1}{x_2} = \mathrm{e}^{2y}$

$$x_1 = \frac{M}{\sqrt{s}}\,\mathrm{e}^{y}\ , \quad x_2 = \frac{M}{\sqrt{s}}\,\mathrm{e}^{-y}$$



proton
M
proton
x1P
x2P
Tevatron parton kinematics
LHC parton kinematics
x1,2 = (M/1.96 TeV) exp(±y)
Q = M
M = 1 TeV
M = 100 GeV
M = 10 GeV
y = 4 2 0 2 4
HERA
fixed target
Q2 (GeV2)
x
x1,2 = (M/14 TeV) exp(±y)
Q = M
M = 10 TeV
M = 1 TeV
M = 100 GeV
M = 10 GeV
y = 6 4 0 2 4 6
DGLAP evolution
HERA

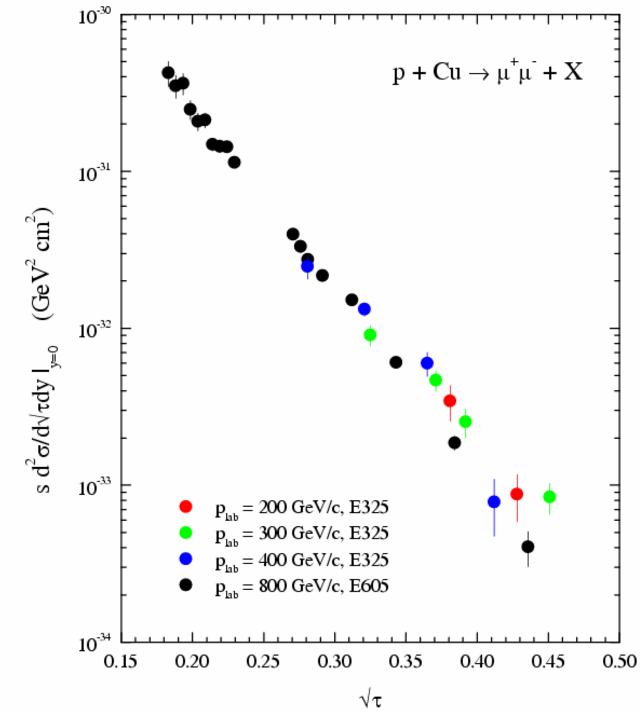
early history: the Drell-Yan process



$\tau = M_{\mu\mu}^2/s$

$$\frac{d^2\sigma}{dM^2} = \frac{4\pi\alpha^2}{3M^4}\int_0^1 dx_1 dx_2 \delta(x_1 x_2 - \tau)\sum_a e_a^2 f_a(x_1) f_{\bar{a}}(x_2)$$

$$= \frac{4\pi\alpha^2}{3M^4}\mathcal{F}(\tau) \qquad \text{(scaling)}$$



"The full range of processes of the type $A + B \to \mu^+\mu^- + X$ with incident p, π, K, γ etc affords the interesting possibility of comparing their parton and antiparton structures" (Drell and Yan, 1970)

(nowadays) … and to study the scattering of quarks and gluons, and how such scattering creates **new particles**

jets! (1981)

Volume 123B, number 1,2 PHYSICS LETTERS

OBSERVATION OF JETS IN HIGH TRANSVERSE ENERGY EVENTS AT THE CERN PROTON ANTIPROTON COLLIDER

UA1 Collaboration, CERN, Geneva, Switzerland

Transverse energy flow of the 5 events with $\sum E_T > 100$ GeV

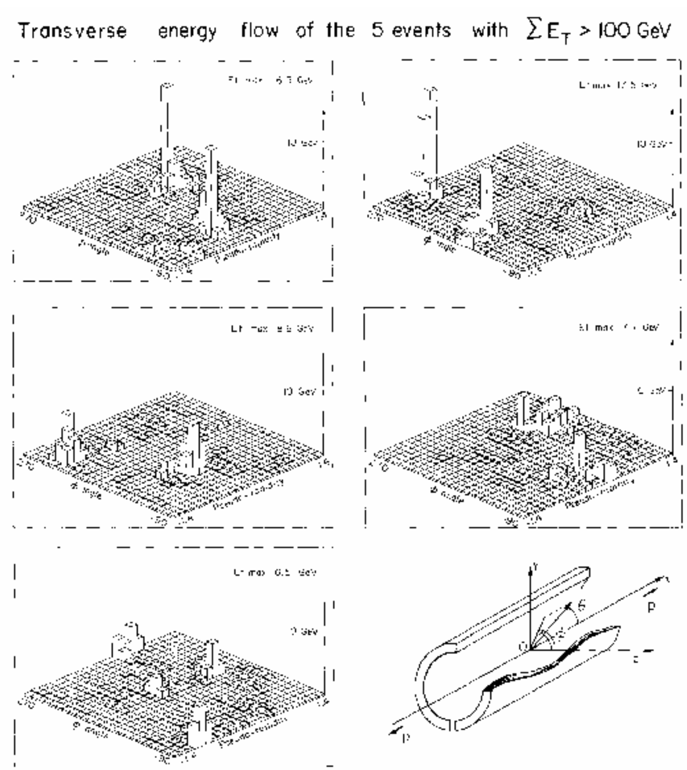
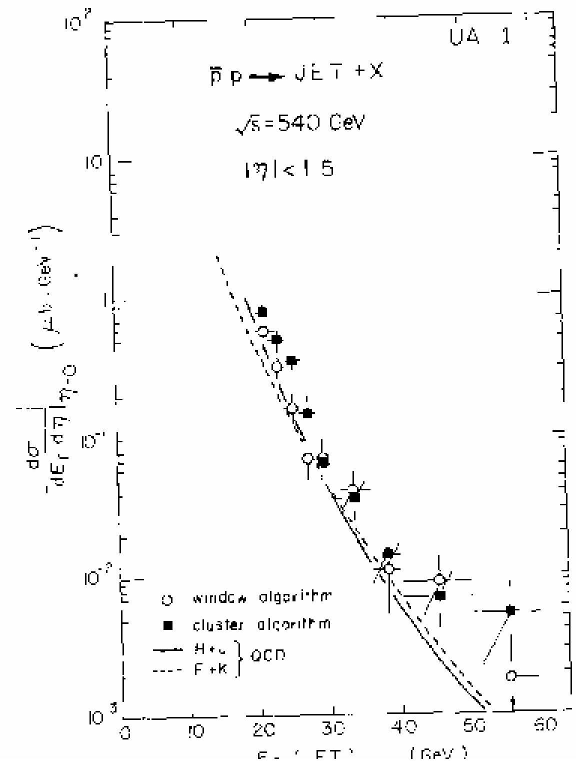
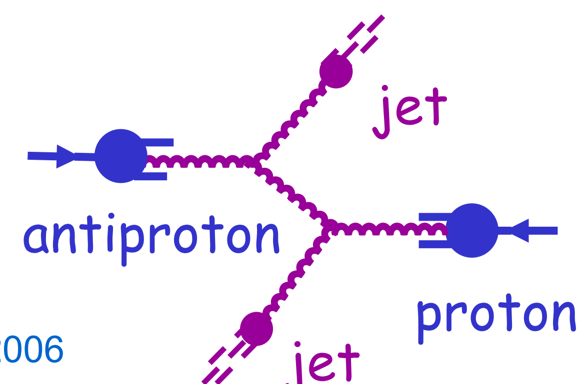


Fig. 3. Distribution of transverse energy versus azimuth ϕ and pseudo-rapidity η, for the five events with the highest ΣE_T

1

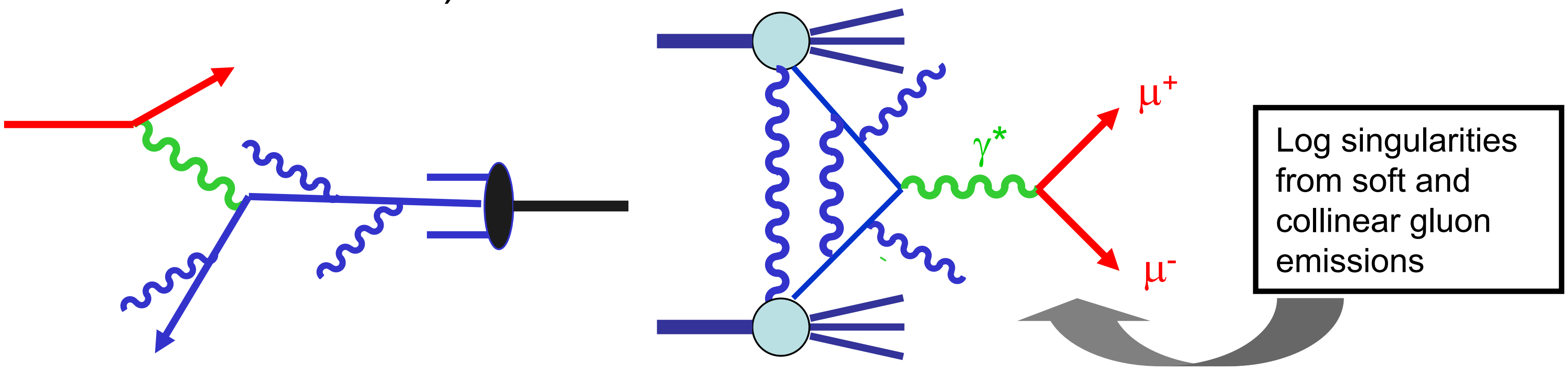




e.g. two gluons scattering at wide angle

factorisation

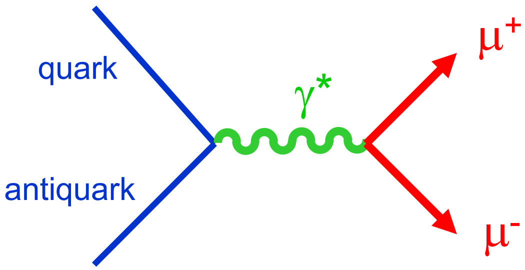
- the factorisation of ‘hard scattering’ cross sections into products of parton distributions was experimentally confirmed and theoretically plausible
- however, it was not at all obvious in QCD (i.e. with quark–gluon interactions included)



- in QCD, for any hard, inclusive process, the soft, nonperturbative structure of the proton can be factored out & confined to universal measurable parton distribution functions $f_a(x,\mu_F^2)$ Collins, Soper, Sterman (1982-5)

and evolution of $f_a(x,\mu_F^2)$ in factorisation scale calculable using the DGLAP equations, as we have seen earlier

Drell-Yan in more detail



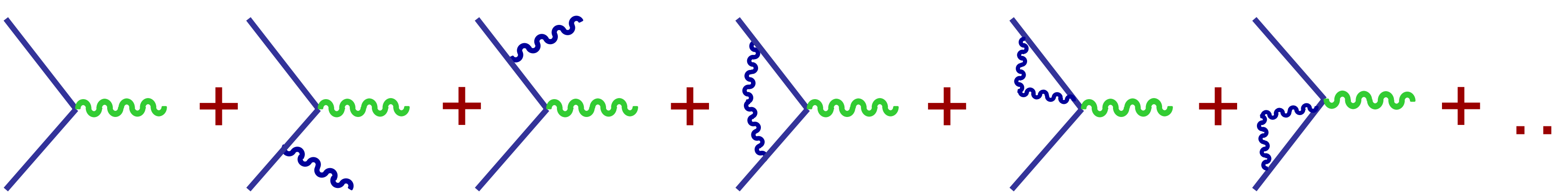
$$\frac{d\hat{\sigma}}{dM^2}^{q\bar{q}\to l^+l^-} = \frac{4\pi\alpha^2}{3N_c M^2}\, e_q^2\, \delta(\hat{s} - M^2)\,, \qquad \hat{s} = x_1 x_2 s$$

$$\frac{d\sigma}{dM^2} = \frac{4\pi\alpha^2}{3N_c M^4}\,\tau \sum_q e_q^2 \int_0^1 dx_1 dx_2\, \delta(x_1 x_2 - \tau)\, q(x_1)\bar{q}(x_2) + (q \leftrightarrow \bar{q})$$

$$\Rightarrow M^4 \frac{d\sigma}{dM^2} = \frac{4\pi\alpha^2}{3N_c}\,\tau\, \mathcal{F}(\tau)$$ scaling!

also $$\frac{d\sigma}{dM^2 dy} = \frac{4\pi\alpha^2}{3N_c M^4}\,\tau \sum_q e_q^2\, q(x_1)\bar{q}(x_2) + (q \leftrightarrow \bar{q})\,, \qquad \tau = M^2/s$$

beyond leading order …



$$d\hat{\sigma} = \hat{\sigma}_0 \left[\delta(x_1 x_2 - \tau) + \frac{\alpha_s}{2\pi} \frac{\theta(x_1 x_2 - \tau)}{x_1 x_2} \left\{ f_q\left(\frac{\tau}{x_1 x_2}\right) + P\left(\frac{\tau}{x_1 x_2}\right) \ln \frac{M^2}{\kappa_1^2} + P\left(\frac{\tau}{x_1 x_2}\right) \ln \frac{M^2}{\kappa_2^2} \right\} \right]$$

Note:

- collinear divergences, with same coefficients of logs as in DIS: *P(x)*
- finite correction: $f_q(x)$
- introduce a factorisation scale, as before:

$$\ln(M^2/\kappa^2) = \ln(M^2/\mu^2) + \ln(\mu^2/\kappa^2)$$

- then fold the parton-level cross section with $q_0(x_1)$ and $q_0(x_2)$, and with the same 'renormalised' distributions as before*, we obtain

$$d\sigma = \int_0^1 dx_1 dx_2 q(x_1,\mu^2)\bar{q}(x_2,\mu^2)\hat{\sigma}_0 \left[\delta(x_1 x_2 - \tau) + \frac{\alpha_s}{2\pi}\frac{1}{x_1 x_2}\left\{ 2P\left(\frac{\tau}{x_1 x_2}\right) \ln \frac{M^2}{\mu^2} + f_q\left(\frac{\tau}{x_1 x_2}\right) - 2\overline{C}\left(\frac{\tau}{x_1 x_2}\right) \right\} + \mathcal{O}(\alpha_s^2) \right]$$

finite

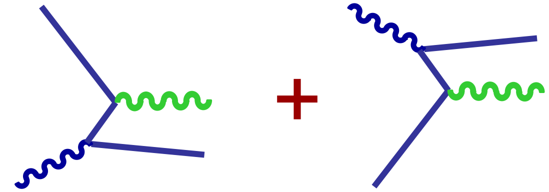
- the standard scale choice is $\mu=M$

Altarelli et al
Kubar et al
1978-80

$$^{*}q(x,\mu^2) = q_0(x) + \frac{\alpha_S}{2\pi}\int_x^1 \frac{dy}{y} q_0(y) \left\{ P(x/y) \ln(\mu^2/\kappa^2) + \overline{C}(x/y) \right\}$$

Note:

- the full calculation at $O(\alpha_S)$ also includes

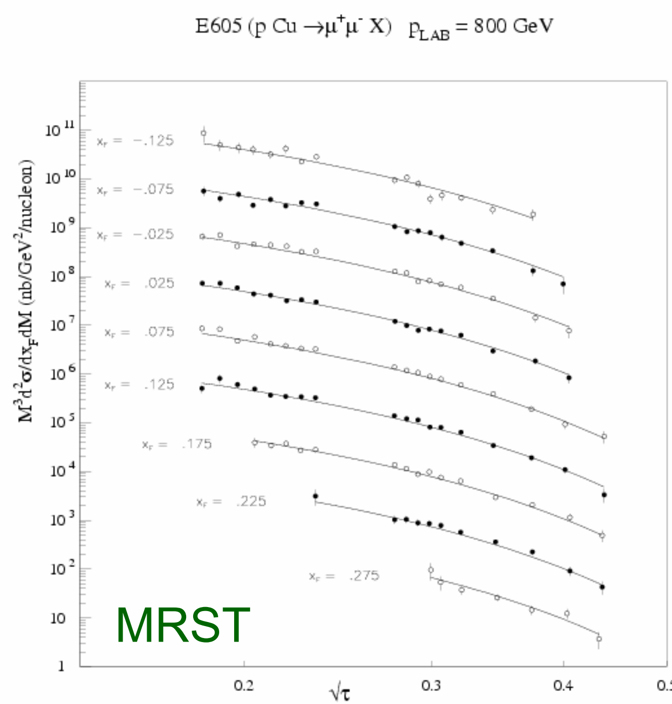


- which gives rise to $\alpha_S\, q * g$ terms in the cross section (see QCD book)

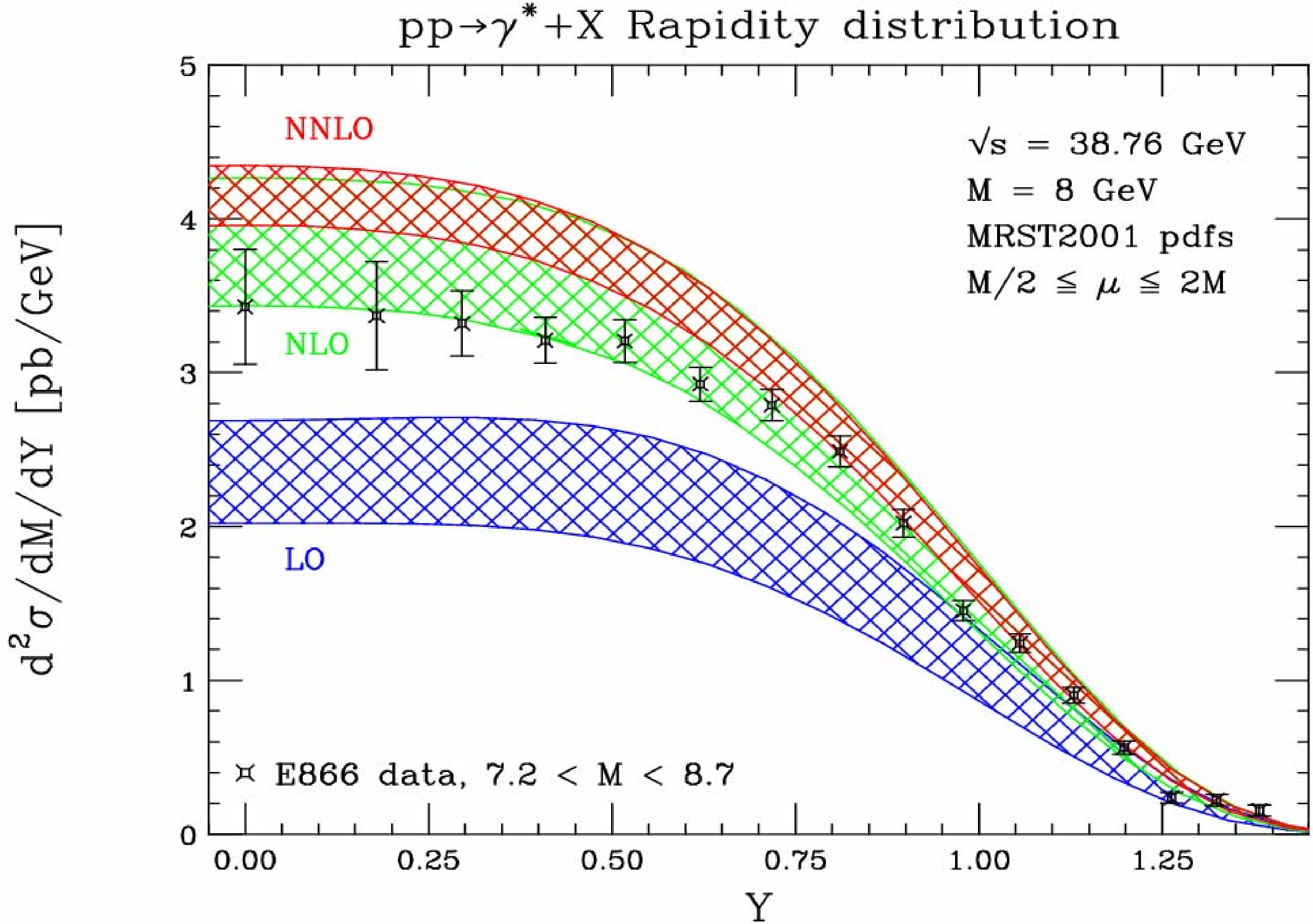
- the (finite) correction is sometimes called the 'K-factor', it is generally large and positive

- … and is factorisation scheme/scale dependent (to compensate the scheme dependence of the pdfs)

using high-precision Drell-Yan data to constrain the sea-quark pdfs



Note: $x_F = \frac{2}{\sqrt{s}}(p_{l+} + p_{l-})_L \approx x_1 - x_2$



Anastasiou, Dixon, Melnikov, Petriello (hep-ph/0306192)

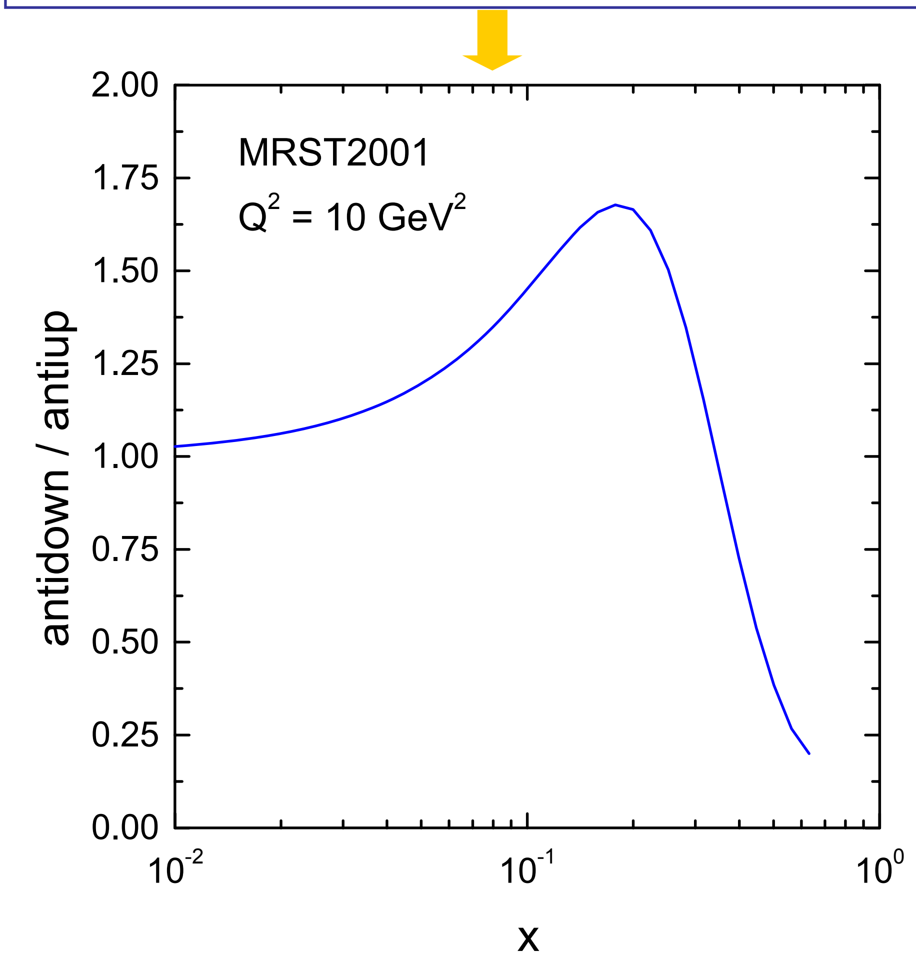
the asymmetric sea

- the sea presumably arises when 'primordial' valence quarks emit gluons which in turn split into quark-antiquark pairs, with suppressed splitting into heavier quark pairs
- so we naively expect

  $\bar{u} \sim \bar{d} > s > c > b$

- why such a big d-u asymmetry? meson cloud, Pauli exclusion, …?
- and is $s(x) = \bar{s}(x)$?

The ratio of Drell-Yan cross sections for $pp, pn \rightarrow \mu^+\mu^- + X$ provides a measure of the difference between the *u* and *d* sea quark distributions



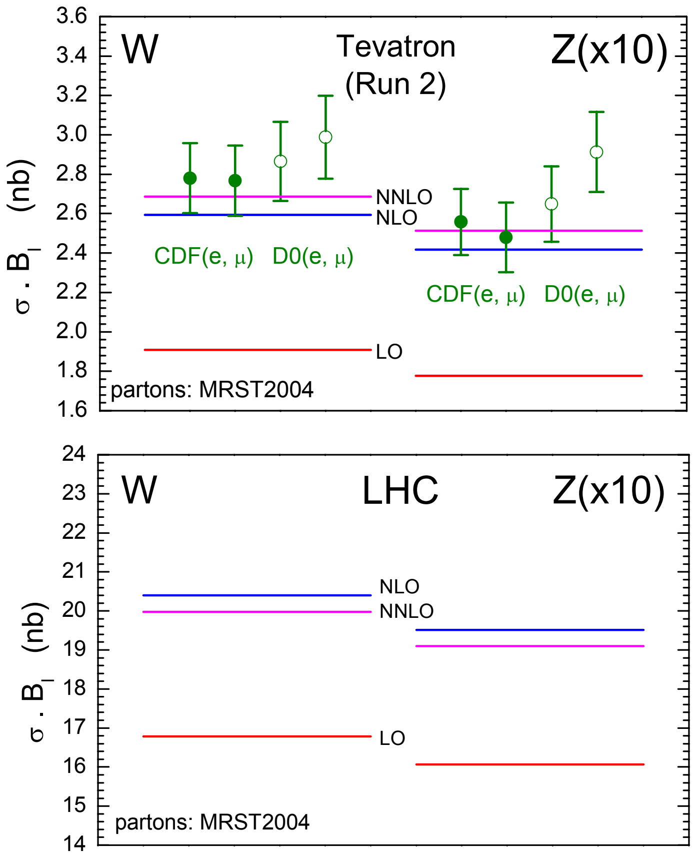
W, Z cross sections: Tevatron and LHC

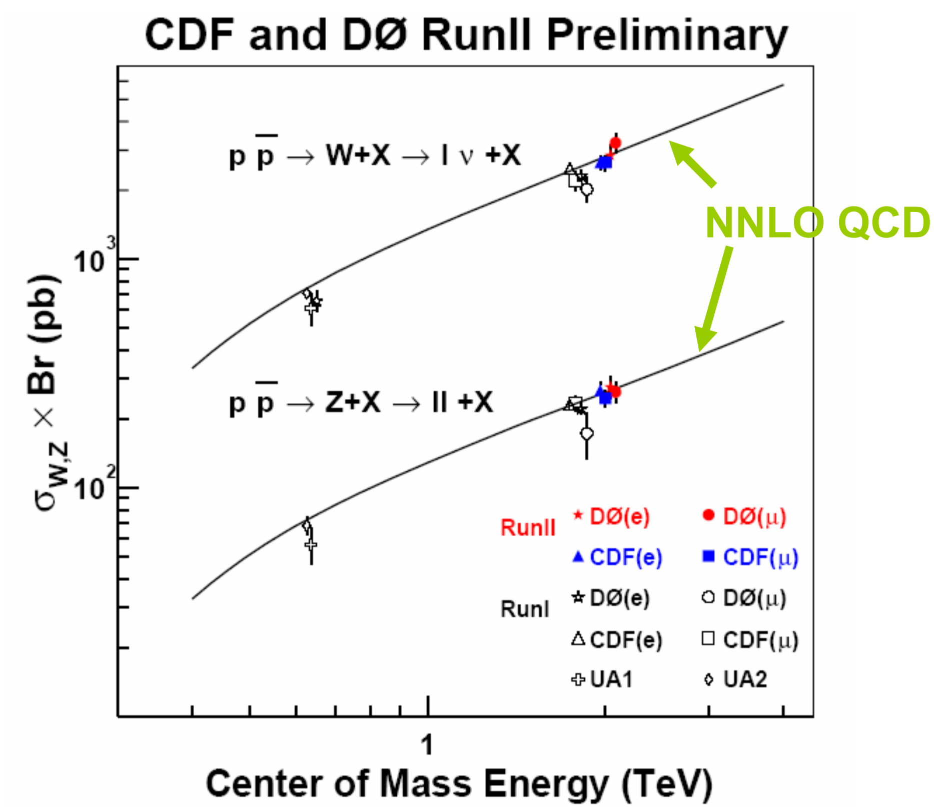
parton level cross sections (narrow width approximation)

$$\hat{\sigma}^{q\bar{q}\to Z} = \frac{4\pi\alpha}{3}\frac{v_q^2 + a_q^2}{4\sin^2\theta_W\cos^2\theta_W}\,\delta(\hat{s}-M_Z^2)$$

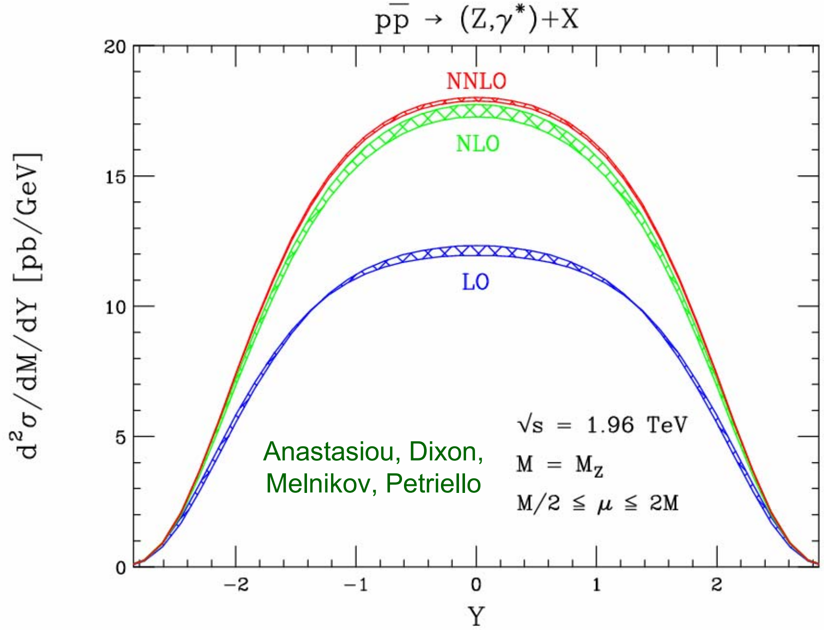
$$\hat{\sigma}^{q\bar{q}'\to W} = \frac{4\pi\alpha}{3}\frac{1}{4\sin^2\theta_W}\,\delta(\hat{s}-M_W^2)$$

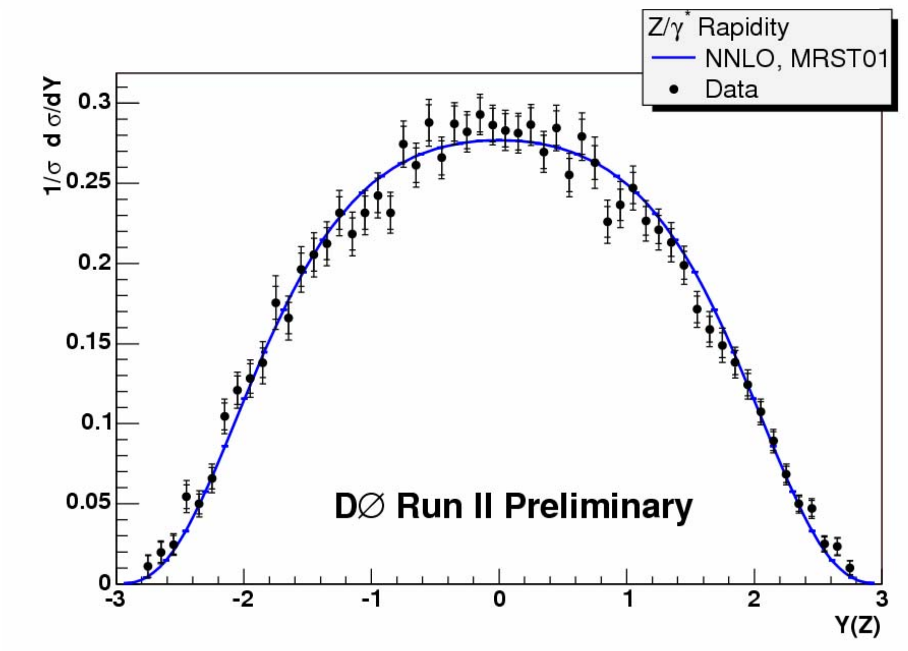
+ pQCD corrections to NNLO, EW to NLO





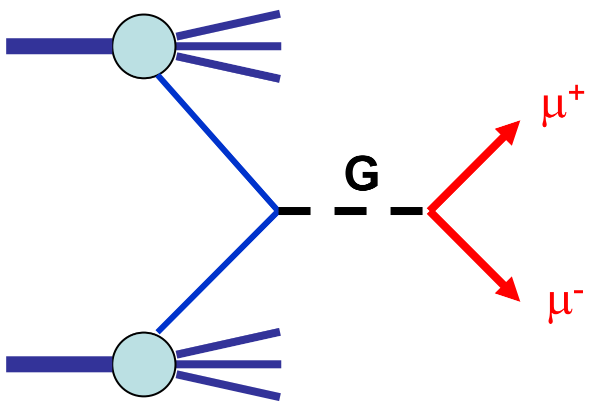
Z rapidity distribution at the Tevatron

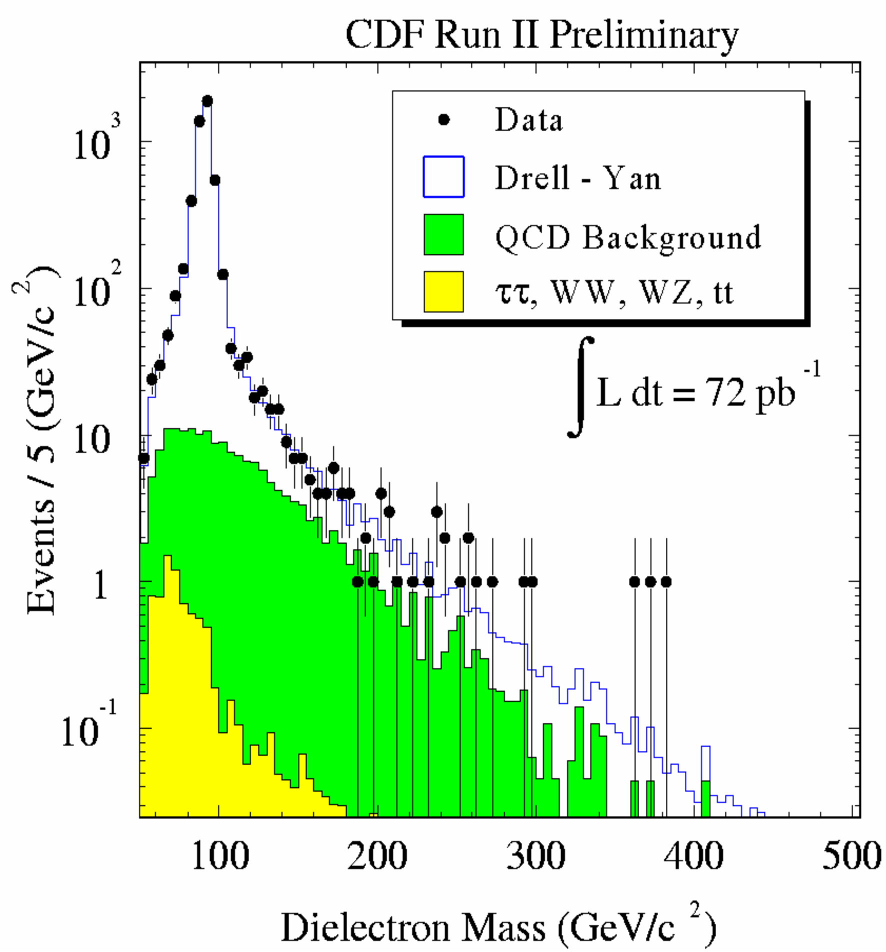




Drell-Yan as a probe of new physics

Large Extra Dimension models have new resonances which could contribute to Drell-Yan





$\Rightarrow$ need to understand the SM contribution to high precision!

Summary: the QCD **factorization theorem** for hard-scattering (short-distance) inclusive processes

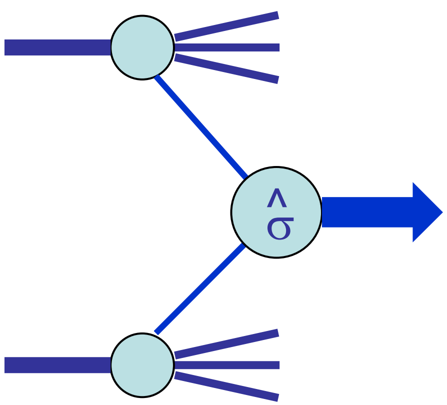
$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2\, f_a(x_1, \mu_F^2)\, f_b(x_2, \mu_F^2)$$
$$\times\ \hat{\sigma}_{ab\to X}\left(x_1, x_2, \{p_i^\mu\}; \alpha_S(\mu_R^2), \alpha(\mu_R^2), \frac{Q^2}{\mu_R^2}, \frac{Q^2}{\mu_F^2}\right)$$

where X=W, Z, H, high-E_T jets, SUSY sparticles, black hole, …, and Q is the 'hard scale' (e.g. = M_X), usually $\mu_F = \mu_R = Q$, and $\hat{\sigma}$ is known …

- to some fixed order in pQCD, e.g. high-E_T jets

$$\hat{\sigma} = A\alpha_S^2 + B\alpha_s^3$$

- or in some leading logarithm approximation (LL, NLL, …) to all orders via resummation

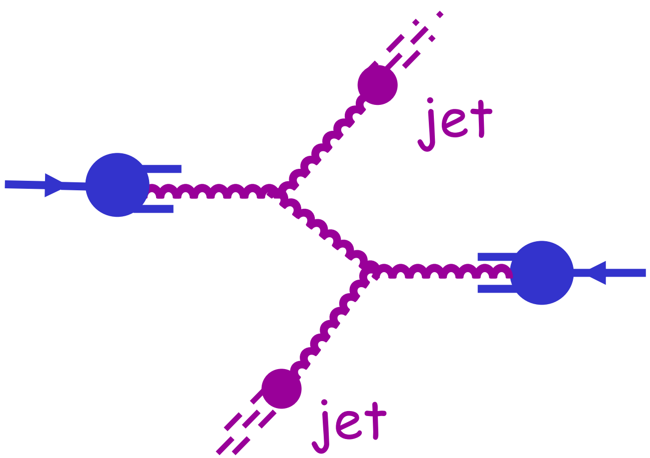


High-E_T jet production

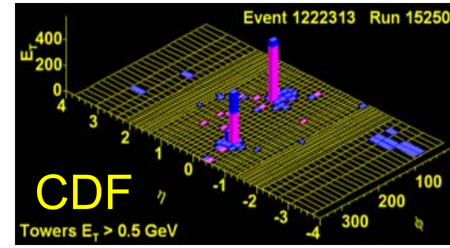
$$E_J \frac{d\sigma}{d^3 p_J} = \sum_{a,b,c,d=q,g} \int_0^1 dx_a dx_b \; f_{a/A}(x_a, Q^2) f_{b/B}(x_b, Q^2) \times \delta(\hat{s} + \hat{t} + \hat{u}) \frac{1}{16\pi^2 \hat{s}} |\overline{M}^{ab\to cd}|^2$$

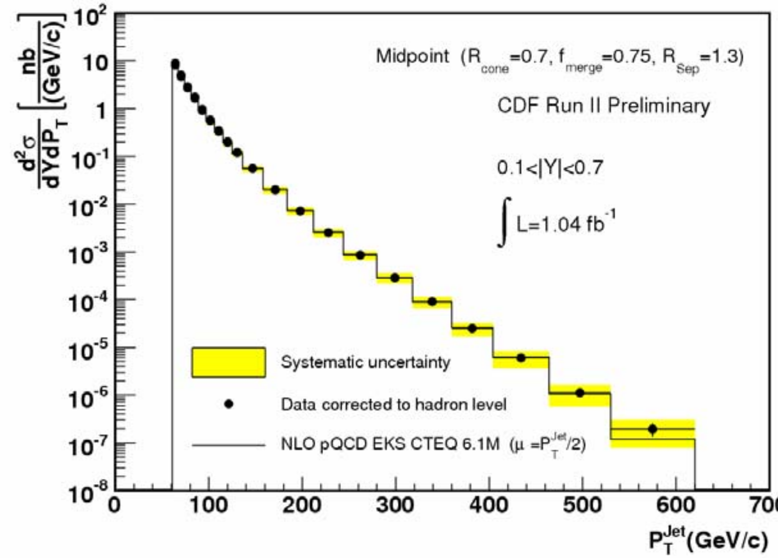
see QCD book

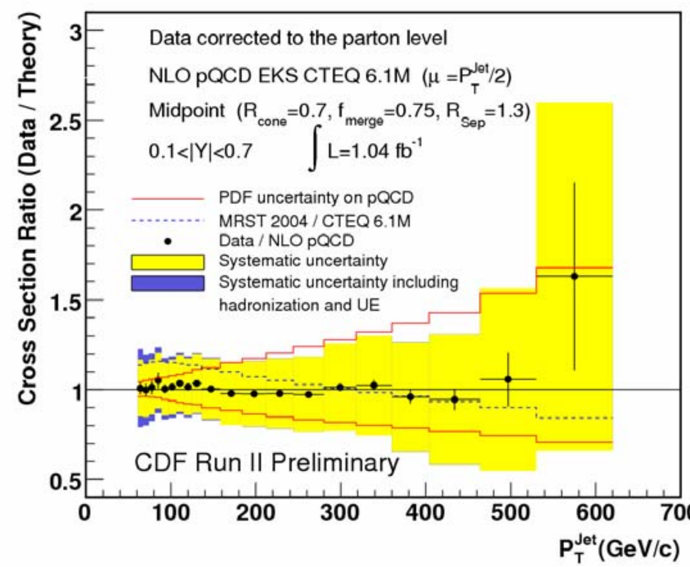
- where *ab→cd* represents all quark & gluon 2→2 scattering processes



- NLO pQCD corrections also known





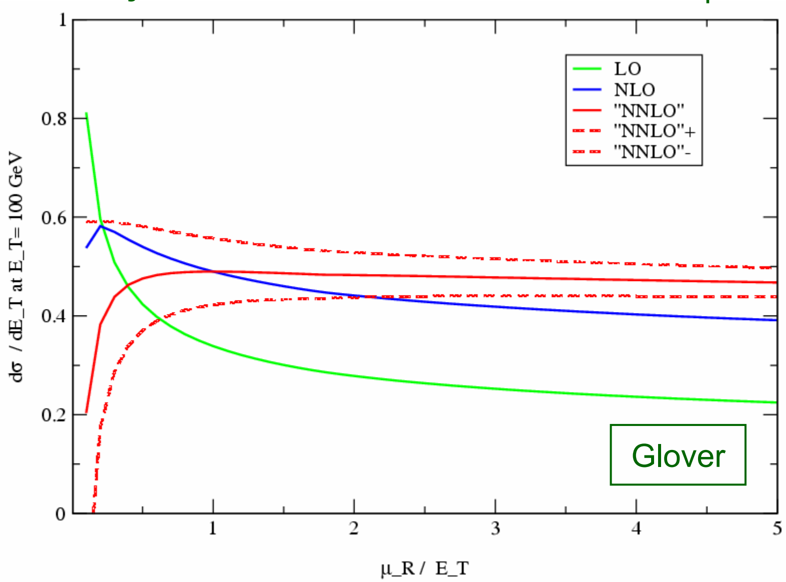


jets at NNLO

$$\frac{d\sigma^{\rm jet}}{dE_T} = \alpha_s^2(\mu_R) A$$
$$+ \alpha_s^3(\mu_R)\left(B + 2b_0 L A\right)$$
$$+ \alpha_s^4(\mu_R)\left(C + 3b_0 L B + (3b_0^2 L^2 + 2b_1 L) A\right)$$

$L = \ln(\mu_R/E_T)$

Tevatron jet inclusive cross section at E_T = 100 GeV



The NNLO coefficient C is not yet known, the curves show guesses $C=0$ (solid), $C=\pm B^2/A$ (dashed) → the scale dependence and hence $\delta\sigma_{th}$ is significantly reduced

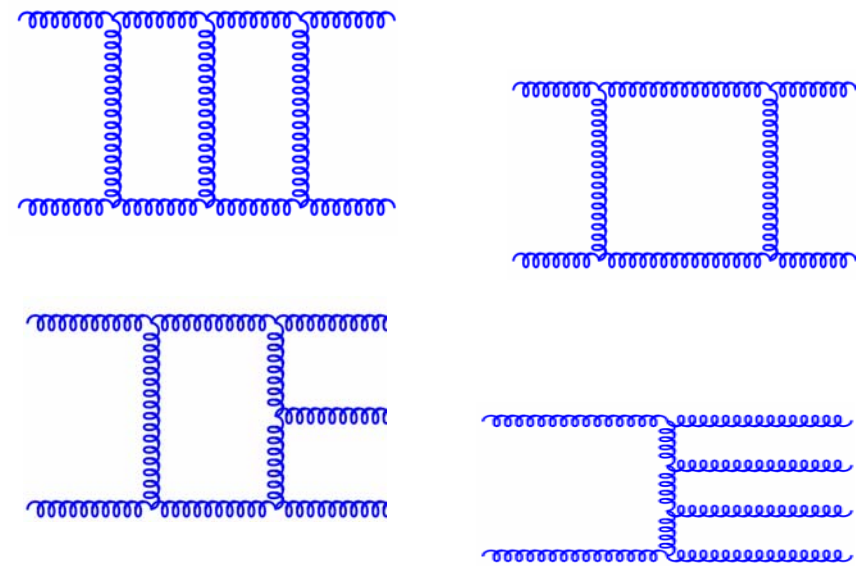
Other advantages of NNLO:

- better matching of partons ⇔hadrons
- reduced power corrections
- better description of final state kinematics (e.g. transverse momentum)

jets at NNLO contd.

- 2 loop, 2 parton final state
- | 1 loop $|^{2}$, 2 parton final state
- 1 loop, 3 parton final states
  or 2 +1 final state

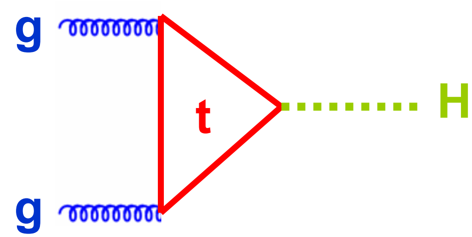
  soft, collinear
- tree, 4 parton final states
  or 3 + 1 parton final states
  or 2 + 2 parton final state



⇒ rapid progress in recent years [many authors]

- many 2→2 scattering processes with up to one off-shell leg now calculated at two loops
- … to be combined with the tree-level 2→4, the one-loop 2→3 and the self-interference of the one-loop 2→2 to yield physical NNLO cross sections
- complete results expected ‘soon’

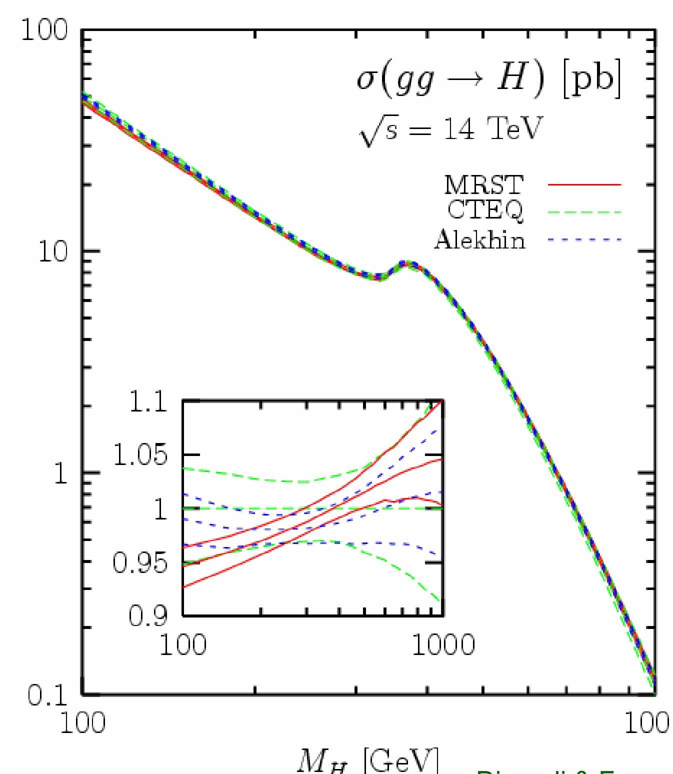
Higgs production



$$\hat{\sigma}^{gg\to H} = \frac{\alpha\alpha_S^2 M_H^2}{576\sin^2\theta_W M_W^2}\left|I\left(\frac{m_t^2}{M_H^2}\right)\right|^2$$

$$\begin{aligned} I(x) &= 3x[2+(4x-1)F(x)] \\ F(x) &= \theta(1-4x)\frac{1}{2}\left[\log\left(\frac{1+\sqrt{1-4x}}{1-\sqrt{1-4x}}\right)-i\pi\right]^2 \\ &\quad -\theta(4x-1)2\left[\sin^{-1}(1/2\sqrt{x})\right]^2 \end{aligned}$$

- the HO pQCD corrections to $\sigma(gg\to H)$ are large (more diagrams, more colour)



Djouadi & Ferrag

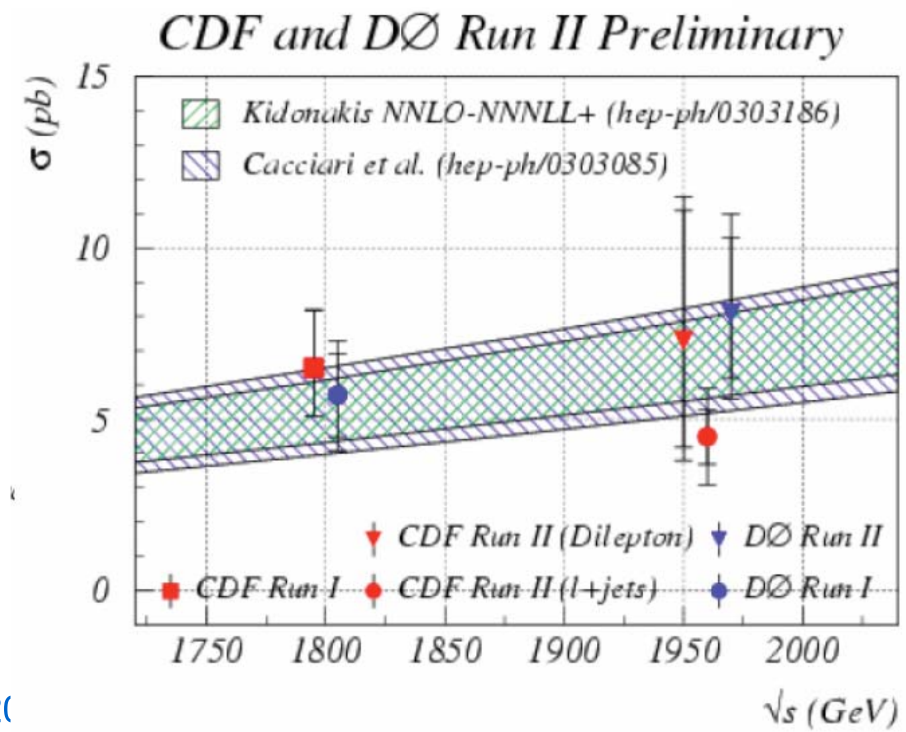
top quark production

$$\hat{\sigma}^{q\bar{q}\to Q\bar{Q}} = \frac{\pi\alpha_S^2\beta\rho}{27M_Q^2}(2+\rho)$$

$$\hat{\sigma}^{gg\to Q\bar{Q}} = \frac{\pi\alpha_S^2\beta\rho}{192M_Q^2}[\frac{1}{\beta}(\rho^2+16\rho+16)\log\frac{1+\beta}{1-\beta}-28-31\rho],$$

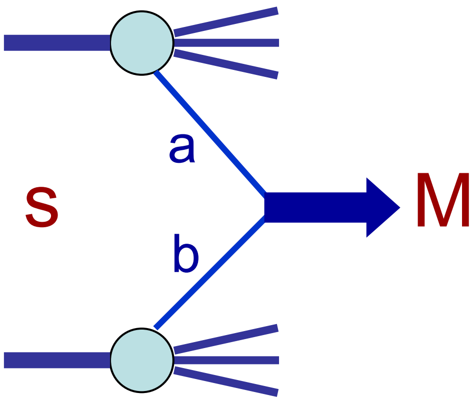
where $\rho = 4M_Q^2/\hat{s},\ \beta=\sqrt{1-\rho}$.

NLO known, but awaits full NNLO pQCD calculation; NNLO & N^nLL "soft+virtual" approximations exist



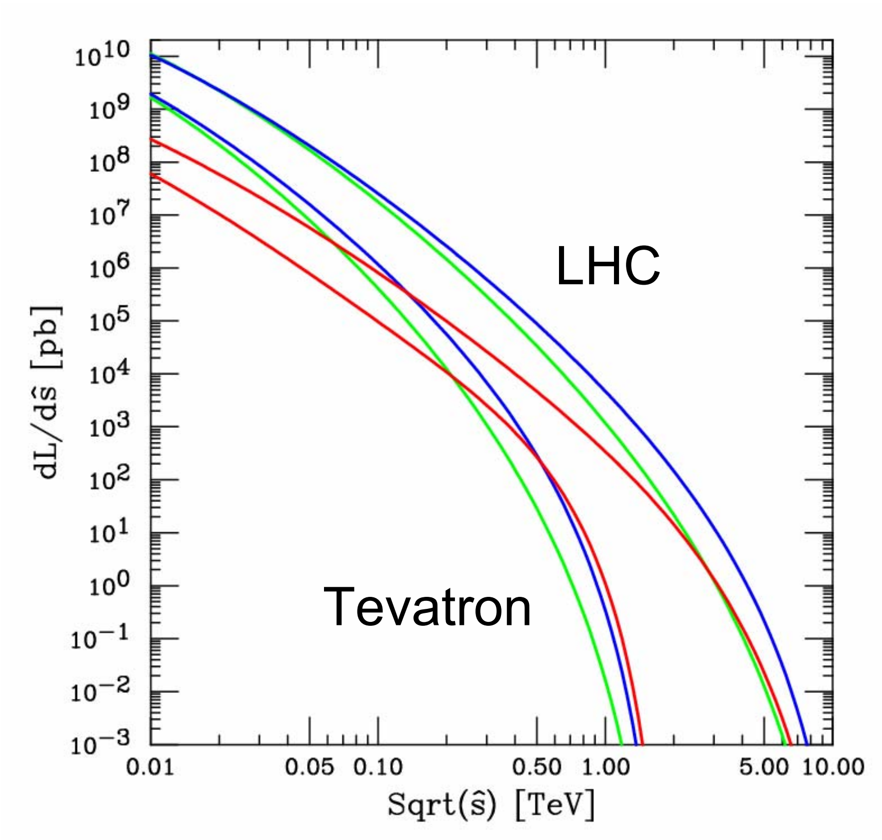
parton luminosity functions

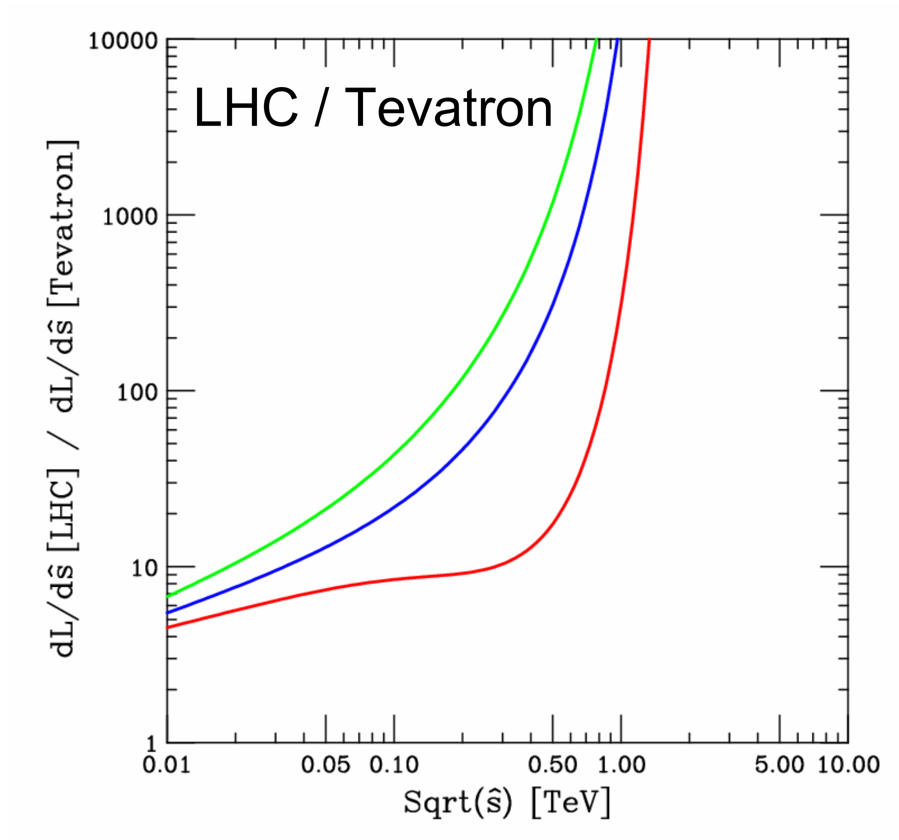
- a quick and easy way to assess the mass and collider energy dependence of production cross sections



$$
\begin{aligned}
\hat{\sigma}_{ab\to X} &= C_X \delta(\hat{s} - M^2) \\
\sigma_X &= \int_0^1 dx_a dx_b \, f_a(x_a, M^2) f_b(x_b, M^2) \, C_X \, \delta(x_a x_b - \tau) \\
&\equiv C_X \left[\frac{1}{s} \frac{\partial \mathcal{L}_{ab}}{\partial \tau} \right] \qquad\qquad (\tau = M^2/s) \\
\frac{\partial \mathcal{L}_{ab}}{\partial \tau} &= \int_0^1 dx_a dx_b \, f_a(x_a, M^2) f_b(x_b, M^2) \, \delta(x_a x_b - \tau)
\end{aligned}
$$

- i.e. all the mass and energy dependence is contained in the X-independent parton luminosity function in [ ]
- useful combinations are $ab = gg, \sum_q q\bar{q}, \ldots$
- and also useful for assessing the uncertainty on cross sections due to uncertainties in the pdfs (see later)





$$= \; gg$$

$$= \sum_i (gq_i + g\bar{q}_i + q_i g + \bar{q}_i g)$$

$$= \sum_i (q_i\bar{q}_i + \bar{q}_i q_i)$$

see CHS for more

future hadron colliders: energy *vs* luminosity?

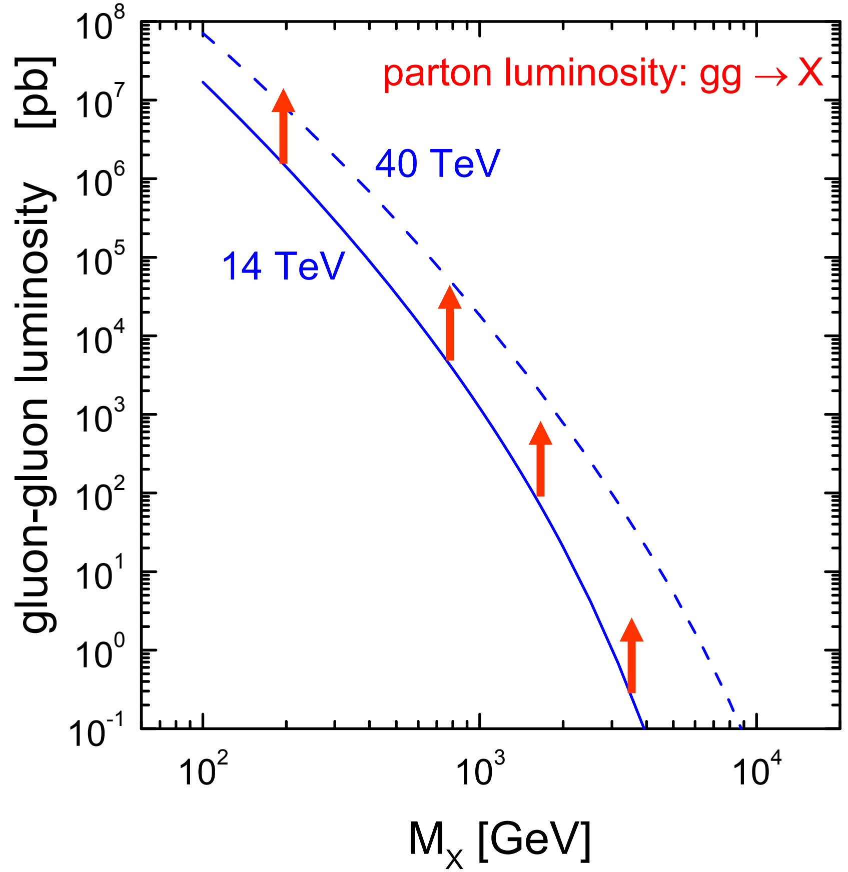
recall parton-parton luminosity:

$$\frac{\partial \mathcal{L}_{ab}}{\partial \tau} = \int_\tau^1 \frac{dx}{x}\, f_a(x, Q^2) f_b(\tau/x, Q^2)$$

so that

$$\sigma_X \ \propto \ \frac{1}{s} \frac{\partial \mathcal{L}_{ab}}{\partial \tau}$$

with $\tau = M_X^2/s$



for M_X > O(1 TeV), energy × 3 is better than luminosity × 10 (everything else assumed equal!)

what limits the precision of the predictions?

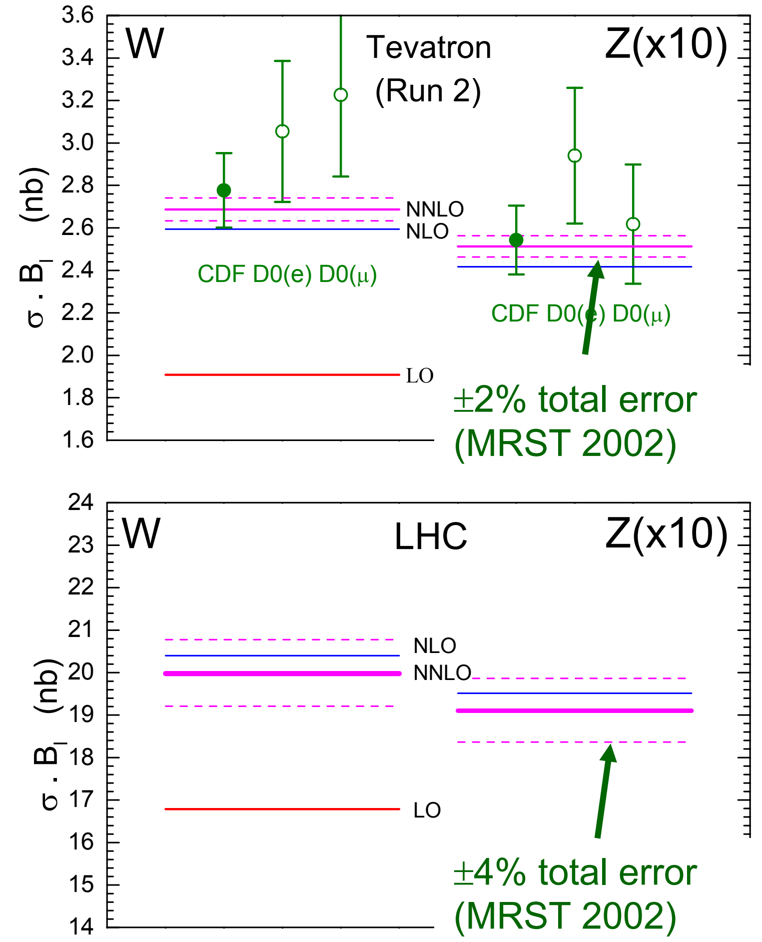
- the order of the perturbative expansion
- the uncertainty in the input parton distribution functions
- example: σ(Z) @ LHC

$$\delta\sigma_{pdf} \approx \pm 3\%, \quad \delta\sigma_{pt} \approx \pm 2\%$$

$$\rightarrow \quad \delta\sigma_{theory} \approx \pm 4\%$$

whereas for gg→H :

$$\delta\sigma_{pdf} << \delta\sigma_{pt}$$

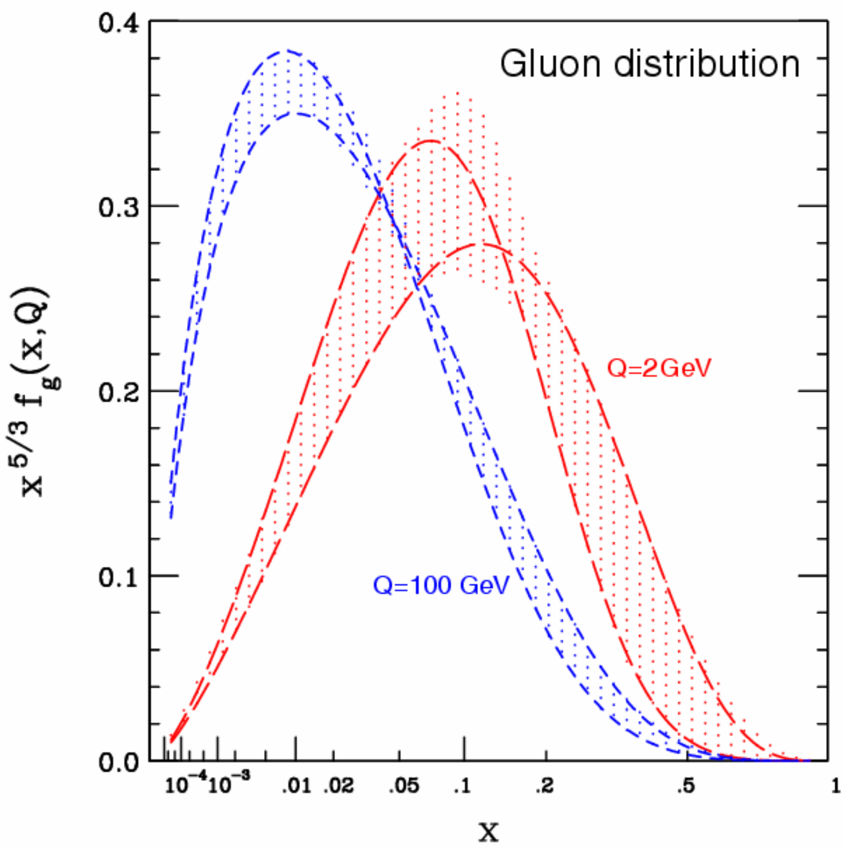


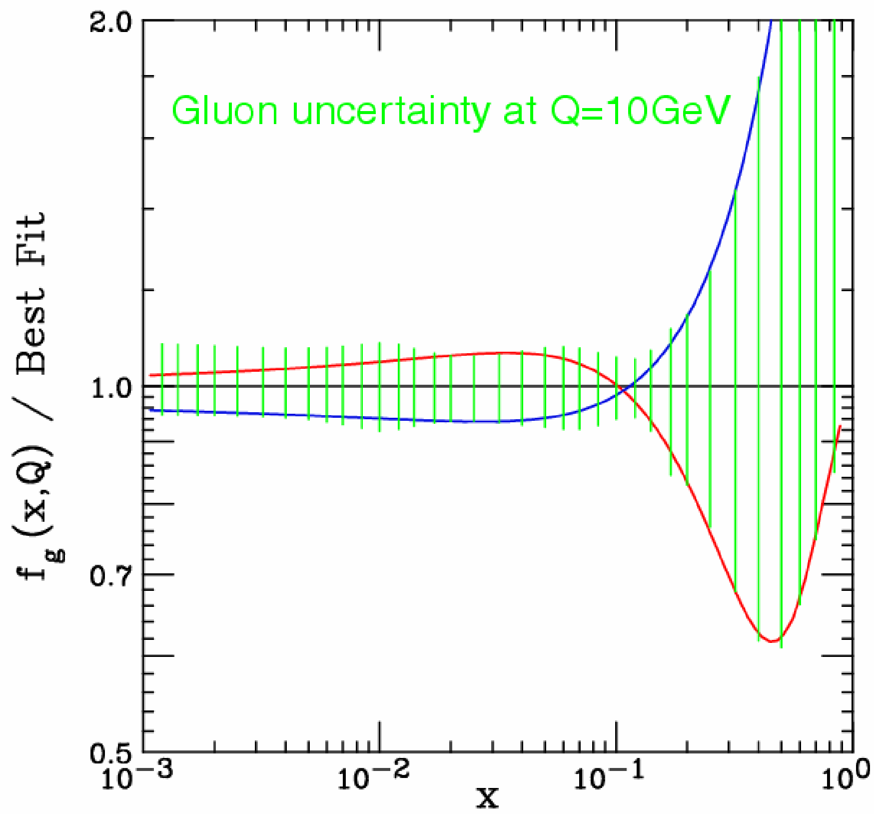
partons: MRST2002
NNLO evolution: van Neerven, Vogt approximation to Vermaseren et al. moments
NNLO W,Z corrections: van Neerven et al. with Harlander, Kilgore corrections

pdf uncertainties

- MRST, CTEQ, Alekhin, … also produce ‘pdfs with errors’
- typically, 30-40 ‘error’ sets based on a ‘best fit’ set to reflect $\pm 1\sigma$ variation of all the parameters $\{A_i, a_i, \ldots, \alpha_S\}$ inherent in the fit
- these reflect the uncertainties on the data used in the global fit (e.g. $\delta F_2 \approx \pm 3\% \rightarrow \delta u \approx \pm 3\%$)
- however, there are also systematic pdf uncertainties reflecting theoretical assumptions/prejudices in the way the global fit is set up and performed

uncertainty in gluon distribution (CTEQ)



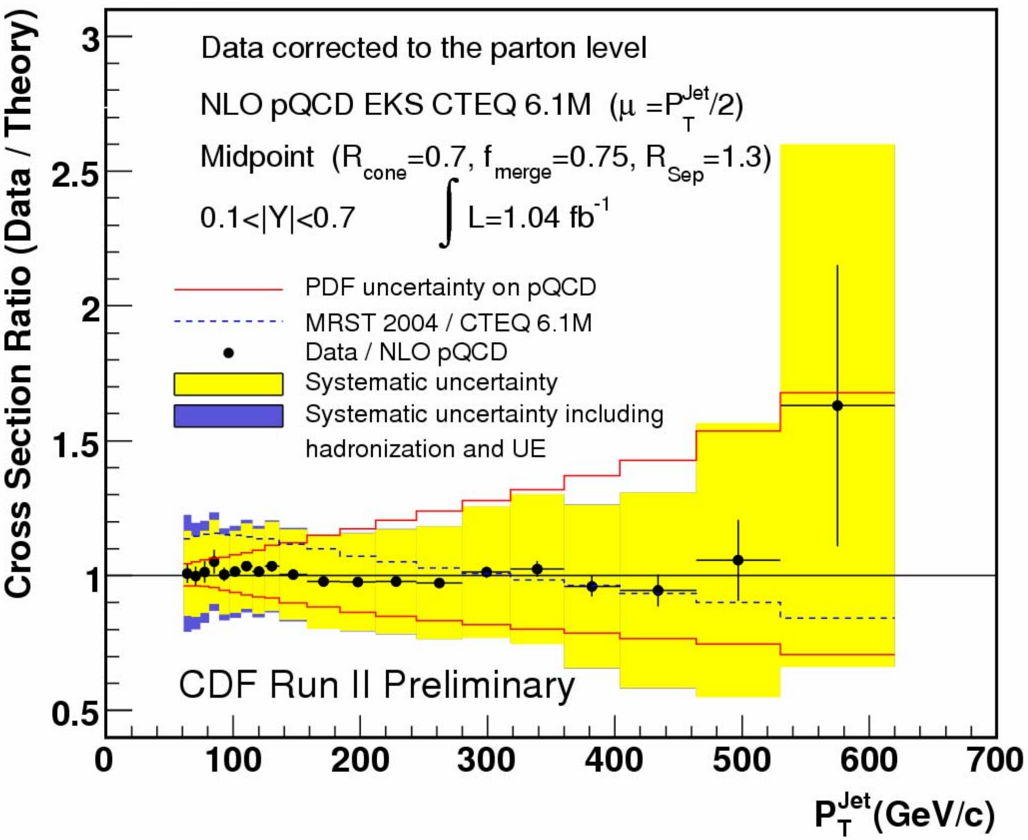


CTEQ6.1E:	1 + 40 error sets
MRST2001E:	1 + 30 error sets

high-x gluon from high E_T jets data

- both MRST and CTEQ use Tevatron jets data to determine the gluon pdf at large x

- the errors on the gluon therefore reflect the measured cross section uncertainties

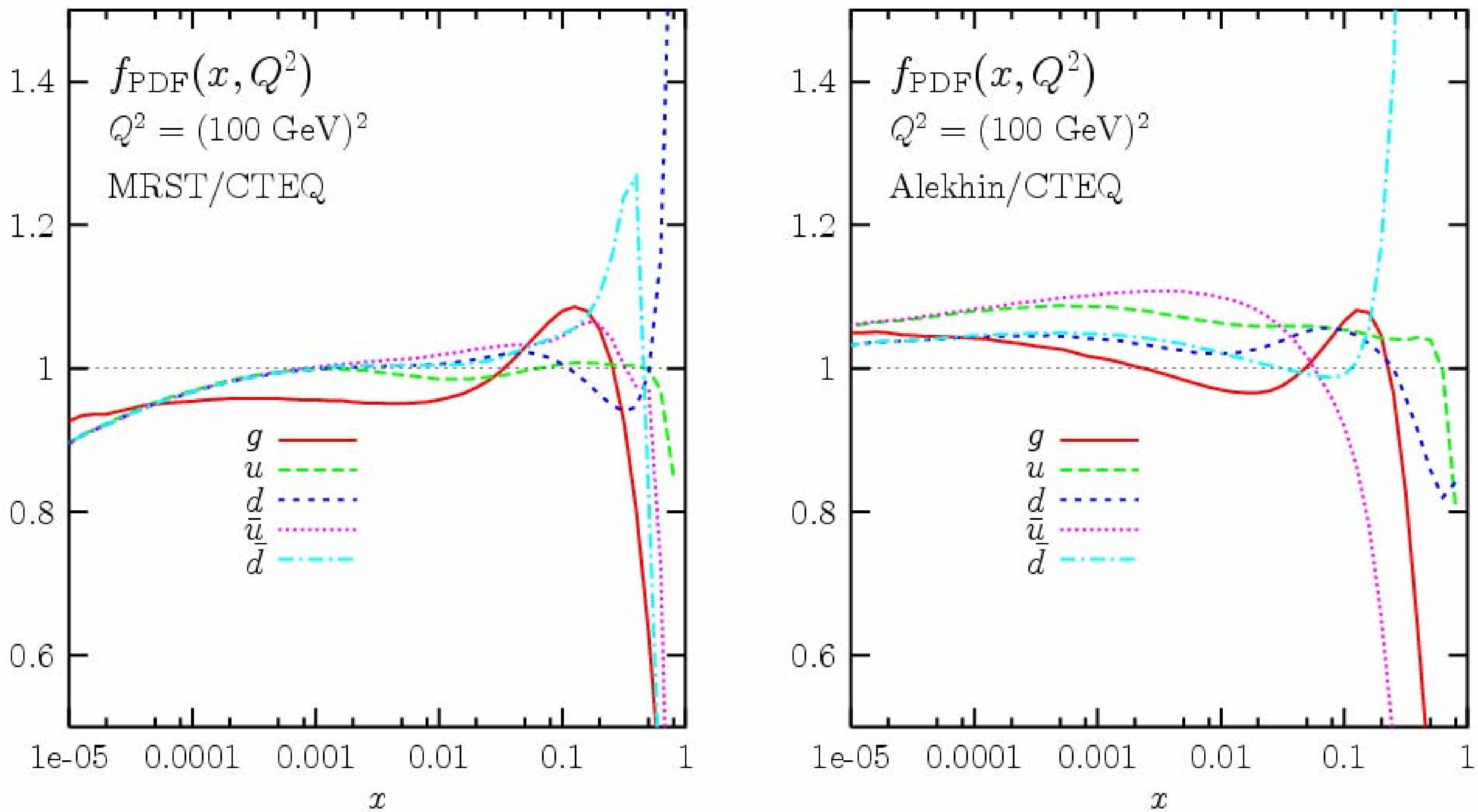


why do ‘best fit’ pdfs and errors differ?

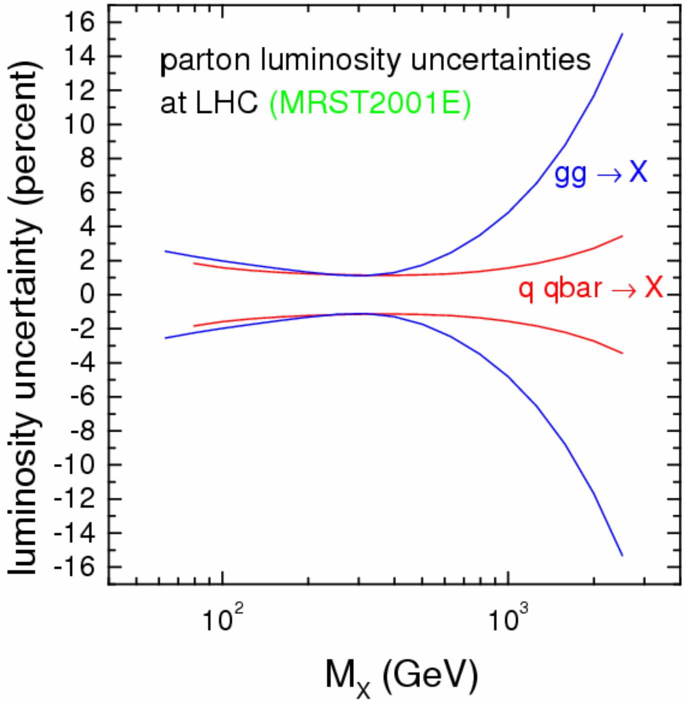
- different data sets in fit
  - different subselection of data
  - different treatment of exp. sys. errors
- different choice of
  - tolerance to define $\pm\delta f_i$
    (MRST: $\Delta\chi^2=50$, CTEQ: $\Delta\chi^2=100$, Alekhin: $\Delta\chi^2=1$)
  - factorisation/renormalisation scheme/scale
  - Q_0^2
  - parametric form $Ax^a(1-x)^b[..]$ etc
  - α_S
  - treatment of heavy flavours
  - theoretical assumptions about $x\to0,1$ behaviour
  - theoretical assumptions about sea flavour symmetry
  - evolution and cross section codes (removable differences!)

LHC	$\sigma_{NLO}(W)$ (nb)
MRST2002	204 ± 4 (expt)
CTEQ6	205 ± 8 (expt)
Alekhin02	215 ± 6 (tot)

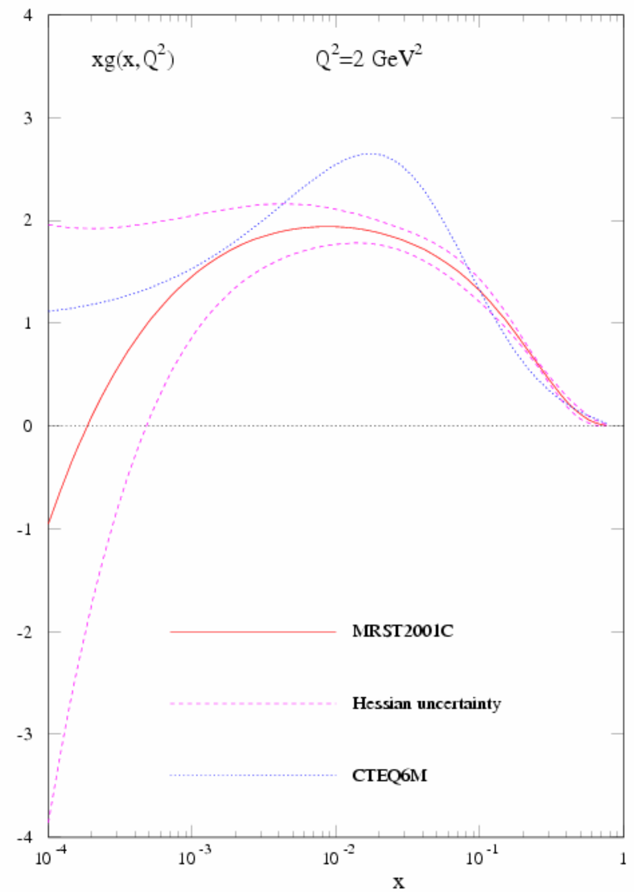
similar partons
different partons
different $\Delta\chi^2$



Djouadi & Ferrag, hep-ph/0310209



Note: CTEQ gluon 'more or less' consistent with MRST gluon

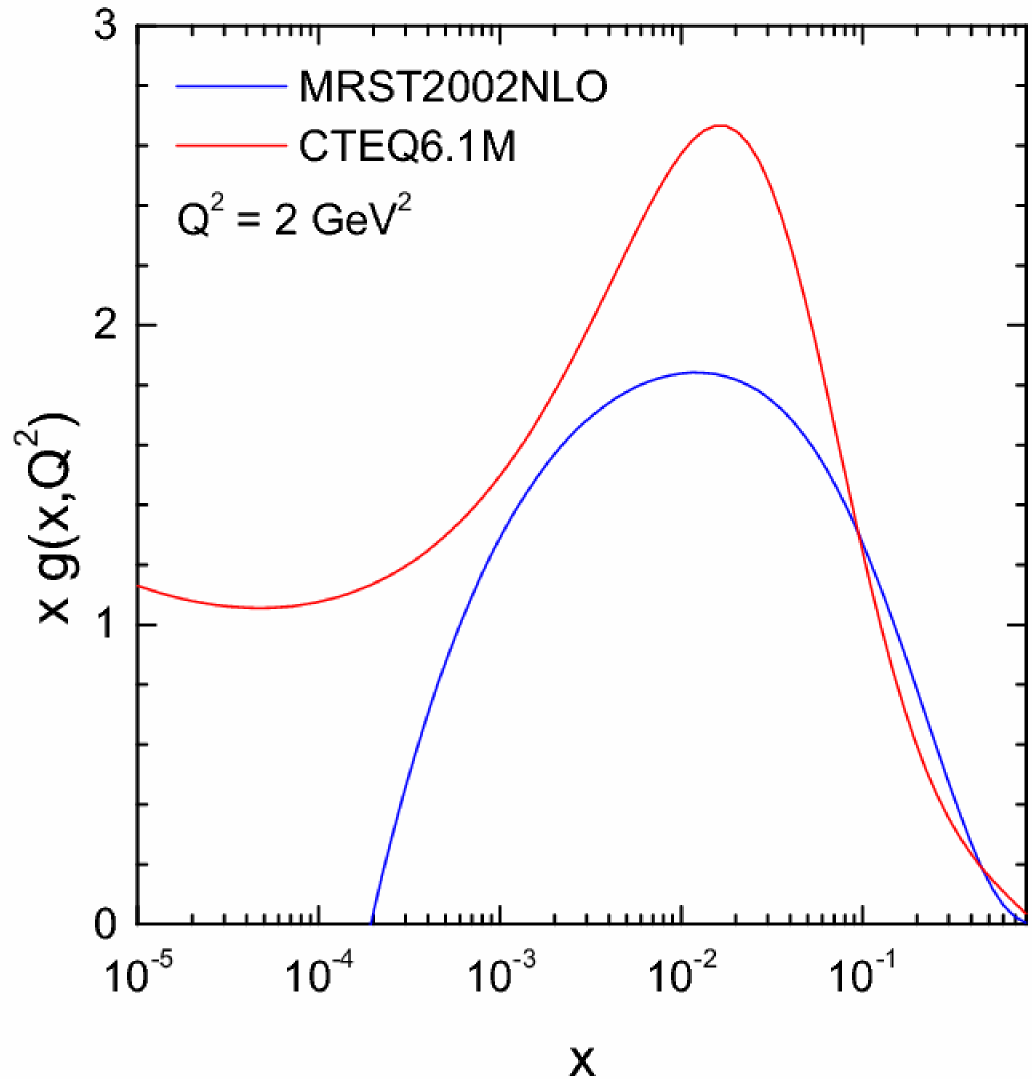


- MRST: Q_0^2 = 1 GeV^2, Q_{cut}^2 = 2 GeV^2

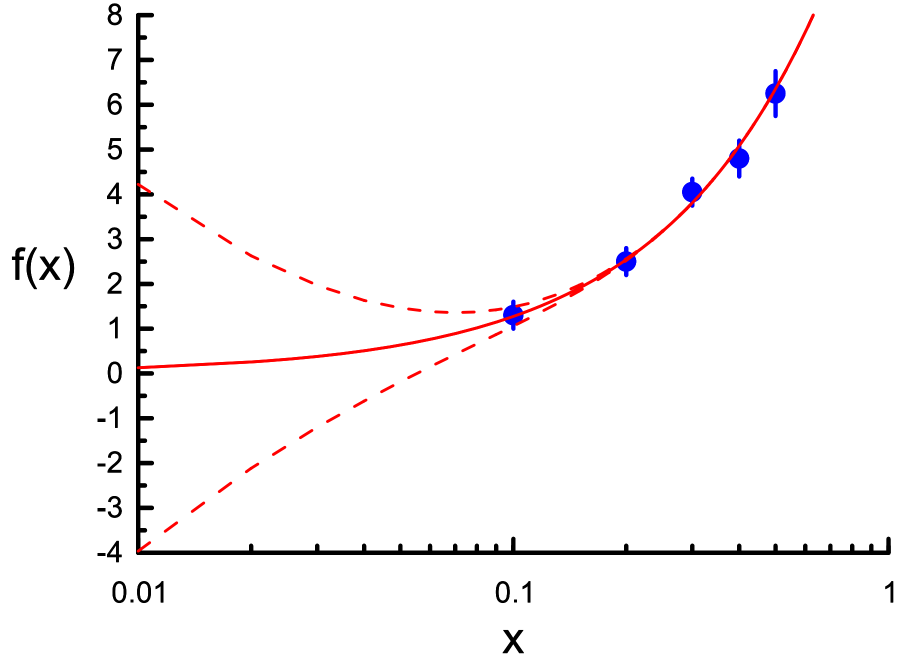
$$xg = Ax^a(1-x)^b(1+Cx^{0.5}+Dx) - Ex^c(1-x)^d$$

- CTEQ6: Q_0^2 = 1.69 GeV^2, Q_{cut}^2 = 4 GeV^2

$$xg = Ax^a(1-x)^b e^{cx}(1+Cx)^d$$



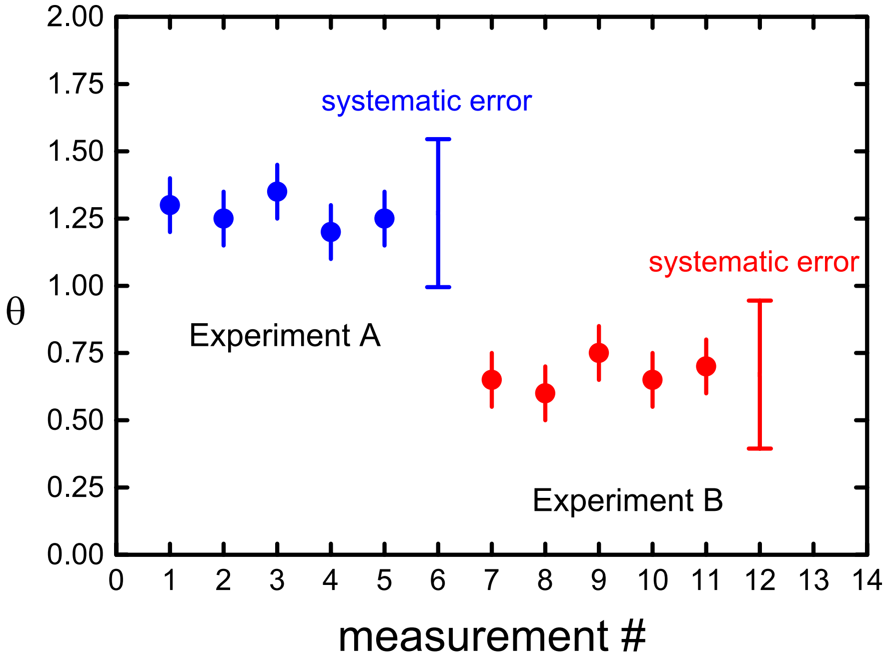
extrapolation errors

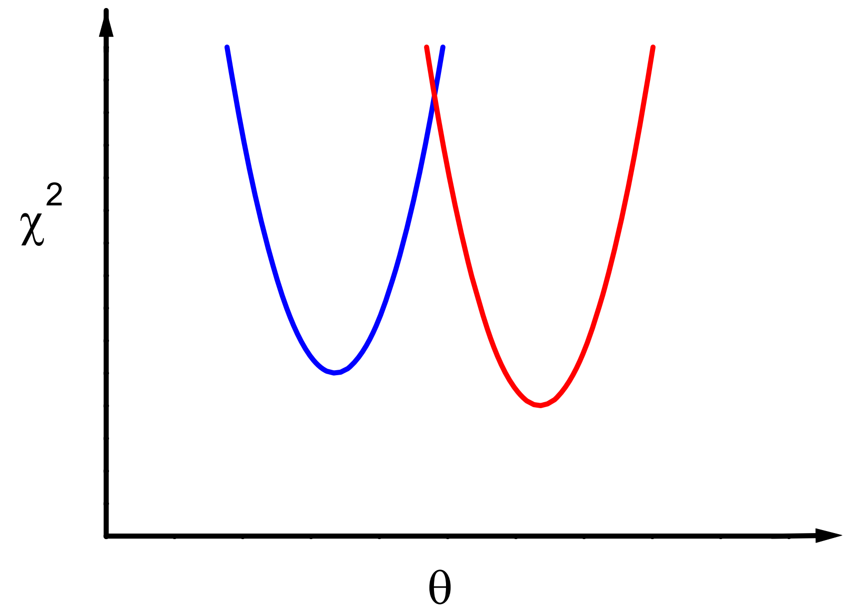


theoretical insight/guess: $f \sim A\,x$ as $x \to 0$

theoretical insight/guess: $f \sim \pm A\,x^{-0.5}$ as $x \to 0$

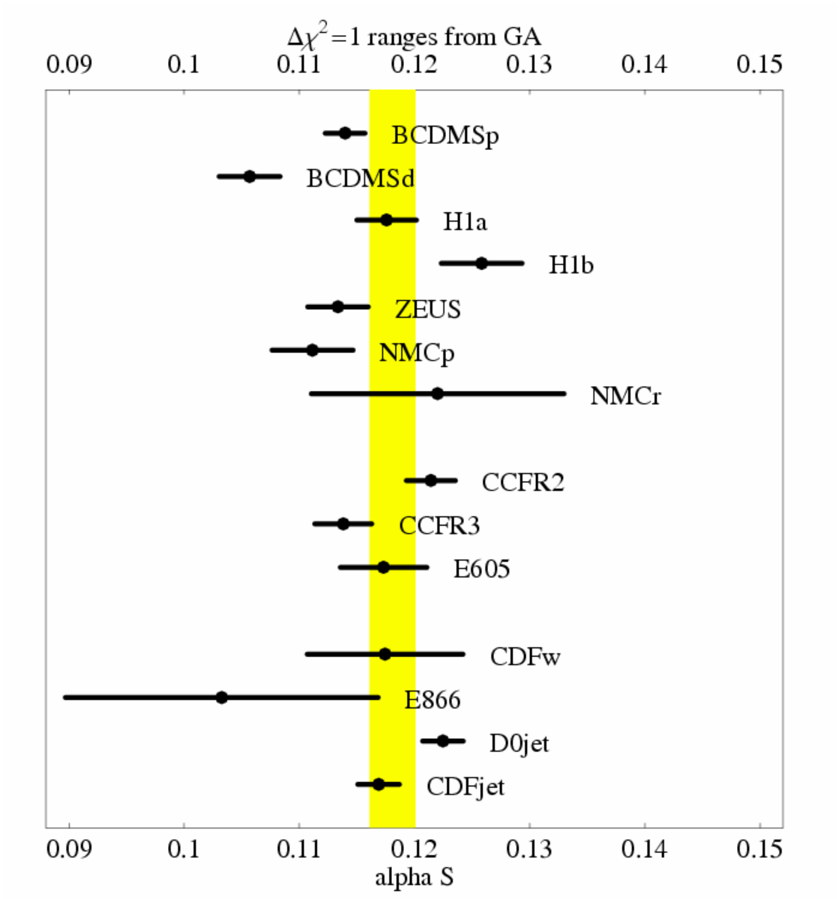
tensions within the global fit

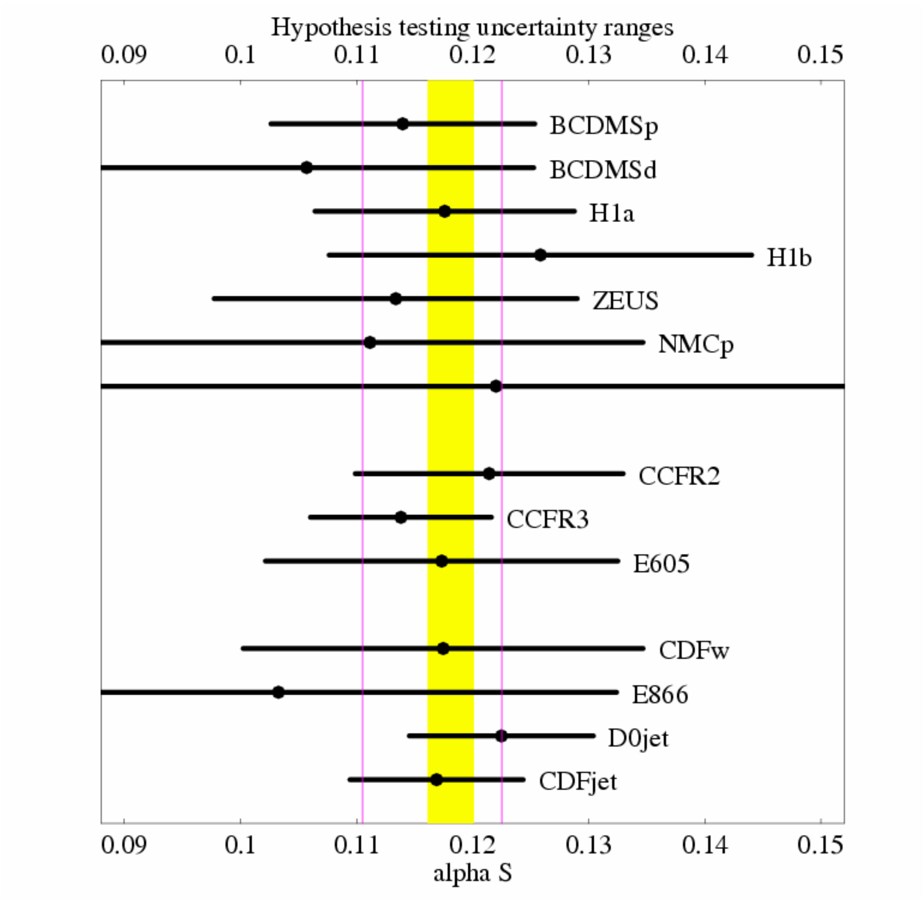




- with dataset A in fit, $\Delta\chi^2=1$; with A and B in fit, $\Delta\chi^2=?$
- 'tensions' between data sets arise, for example,
  - between DIS data sets (e.g. **μH** and **νN** data)
  - when jet and Drell-Yan data are combined with DIS data

CTEQ $\alpha_S(M_Z)$ values from global analysis with $\Delta\chi^2 = 1, 100$





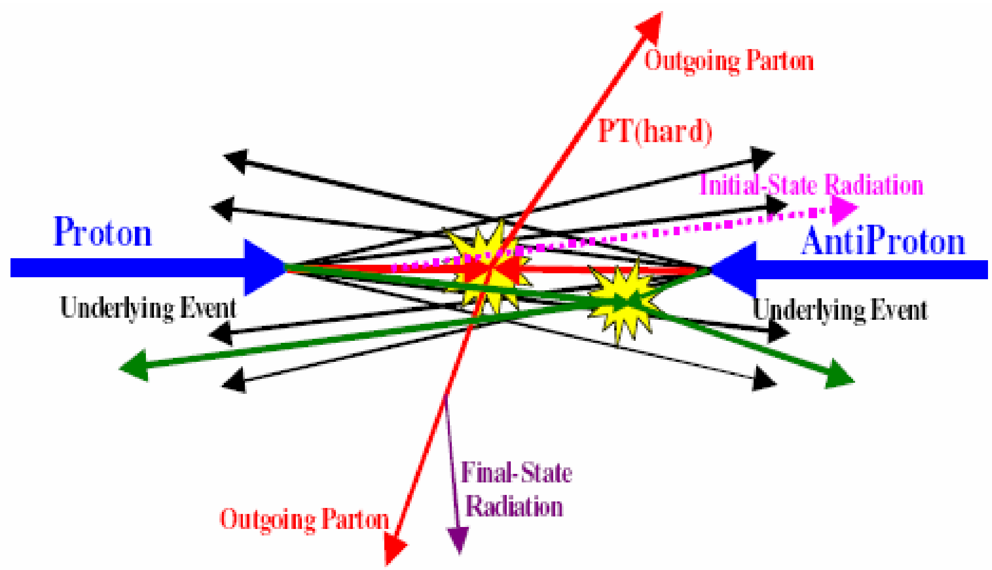
beyond perturbation theory

non-perturbative effects arise in many different ways

- emission of gluons with $k_T < Q_0$ off 'active' partons
- soft exchanges between partons of the same or different beam particles

manifestations include…

- hard scattering occurs at net non-zero transverse momentum
- 'underlying event' additional hadronic energy

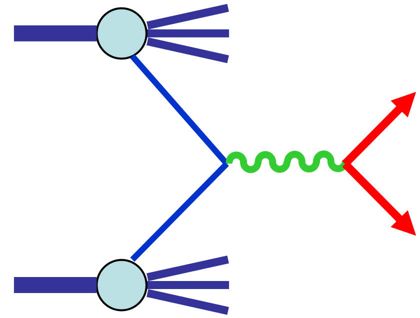


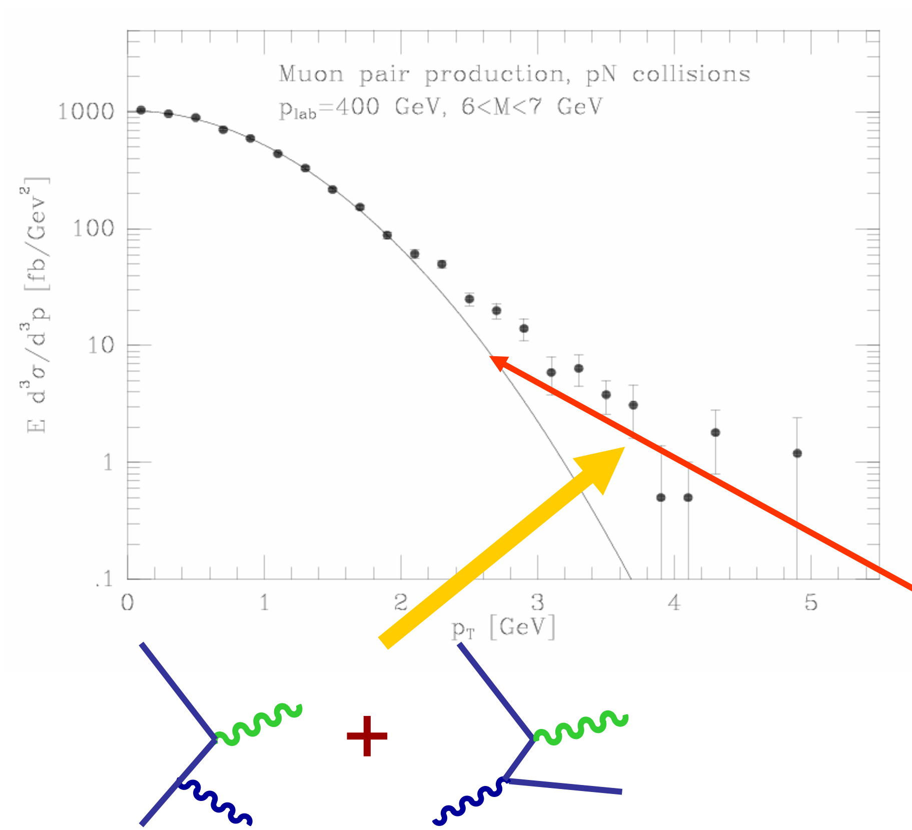
precision phenomenology requires a quantitative understanding of these effects!

‘intrinsic’ transverse momentum

simple parton model assumes partons have zero transverse momentum

… but data shows that the DY lepton pair is produced with non-zero $\langle p_T \rangle$





Generalise

$$d\xi \; f(\xi) \longrightarrow d^2k_T d\xi \; P(\vec{k}_T \xi)\;,$$

with $\int d^2k_T P(\vec{k}_T, \xi) = f(\xi)$.
If assume

$$P(\vec{k}_T, \xi) = h(\vec{k}_T)\; f(\xi)$$

then

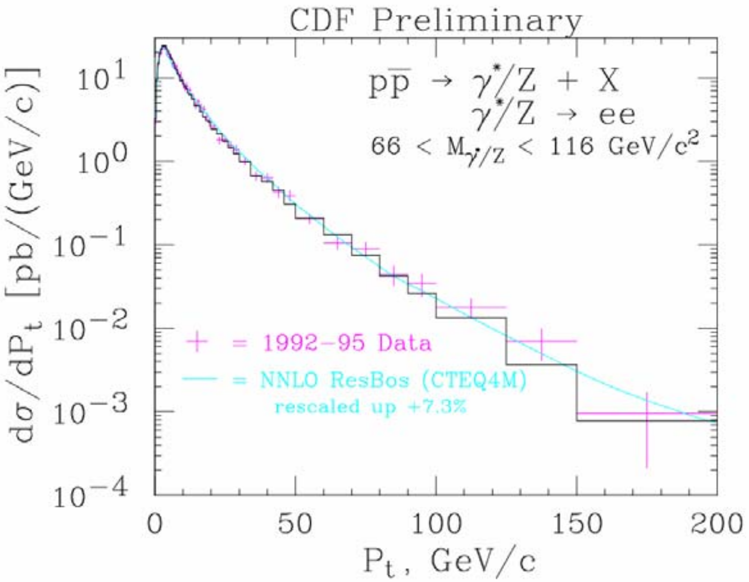
$$\frac{1}{\sigma}\frac{d^2\sigma}{d^2p_T} = \int d^2k_{T1} d^2k_{T2}\; \delta^{(2)}(\vec{k}_{T1} + \vec{k}_{T2} - \vec{p}_T)\; h(\vec{k}_{T1}) h(\vec{k}_{T2})$$

A fit to data gives

$$h(\vec{k}_T) = \frac{b}{\pi}\;\exp(-bk_T^2)$$

with $\langle k_T \rangle = (\pi/4b)^{1/2} = 760$ MeV.

the perturbative tail is even more apparent in *W, Z* production at the Tevatron, and can be well accounted for by the 2→2 scattering processes:



$$\overline{\sum}|\mathcal{M}^{q\bar{q}'\to Wg}|^2 = \pi\alpha_S\sqrt{2}G_F M^2|V_{qq'}|^2\,\frac{8}{9}\,\frac{t^2+u^2+2M^2s}{tu}$$

$$\overline{\sum}|\mathcal{M}^{gq\to Wq'}|^2 = \pi\alpha_S\sqrt{2}G_F M^2|V_{qq'}|^2\,\frac{1}{3}\,\frac{s^2+u^2+2tM^2}{-su}$$

… with known NLO pQCD corrections. Note that the p_T distribution diverges as $p_T\to 0$ due to soft gluon emission:

$$\frac{d\sigma^R}{dp_T^2} = \alpha_S\left(A\,\frac{\ln(M^2/p_T^2)}{p_T^2} + B\,\frac{1}{p_T^2} + C(p_T^2)\right)$$

the $O(\alpha_S)$ virtual gluon correction contributes at $p_T=0$, in such a way as to make the integrated distribution finite

$$\frac{d\sigma^{R+V}}{dp_T^2} = \alpha_S\left(A\left[\frac{\ln(M^2/p_T^2)}{p_T^2}\right]_+ + B\left[\frac{1}{p_T^2}\right]_+ + \overline{C}(p_T^2)\right)$$

intrinsic k_T can also be included, by convoluting with the pQCD contribution

resummation

- when $p_T \ll M$, the pQCD series contains large logarithms $\ln(M^2/p_T^2)$ at each order:

$$\frac{1}{\sigma}\frac{d\sigma}{dp_T^2} \simeq \frac{1}{p_T^2}\left[A_1\alpha_S \ln\frac{M^2}{p_T^2} + A_2\alpha_S^2\ln^3\frac{M^2}{p_T^2} + \ldots + A_n\alpha_S^n\ln^{2n-1}\frac{M^2}{p_T^2} + \ldots\right]$$

which spoils the convergence of the series when $\alpha_S \ln^2\frac{M^2}{p_T^2} \sim 1$

- fortunately, these logarithms can be *resummed* to all orders in pQCD, to generate a Sudakov form factor:

$$\frac{1}{\sigma}\frac{d\sigma}{dp_T^2} \simeq \frac{d}{dp_T^2}\exp\left(-\frac{\alpha_S}{2\pi}C_F\ln^2\frac{M^2}{p_T^2}\right) = \frac{\alpha_S C_F}{\pi}\frac{\ln(M^2/p_T^2)}{p_T^2}\exp\left(-\frac{\alpha_S}{2\pi}C_F\ln^2\frac{M^2}{p_T^2}\right)$$

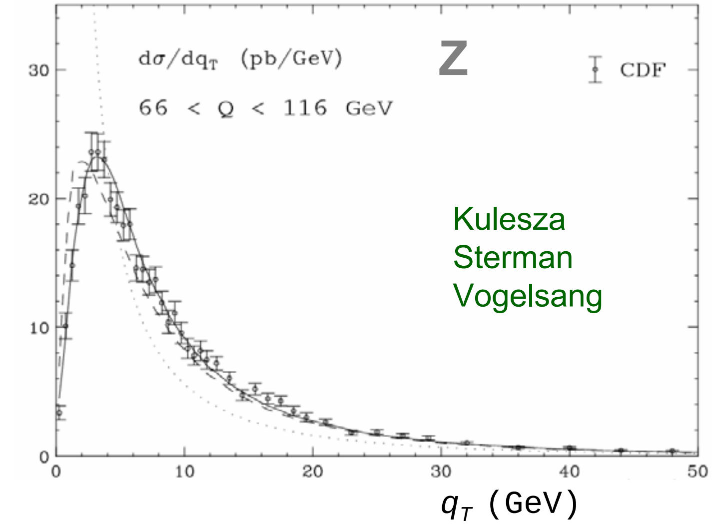
… which regulates the LO singularity at $p_T = 0$

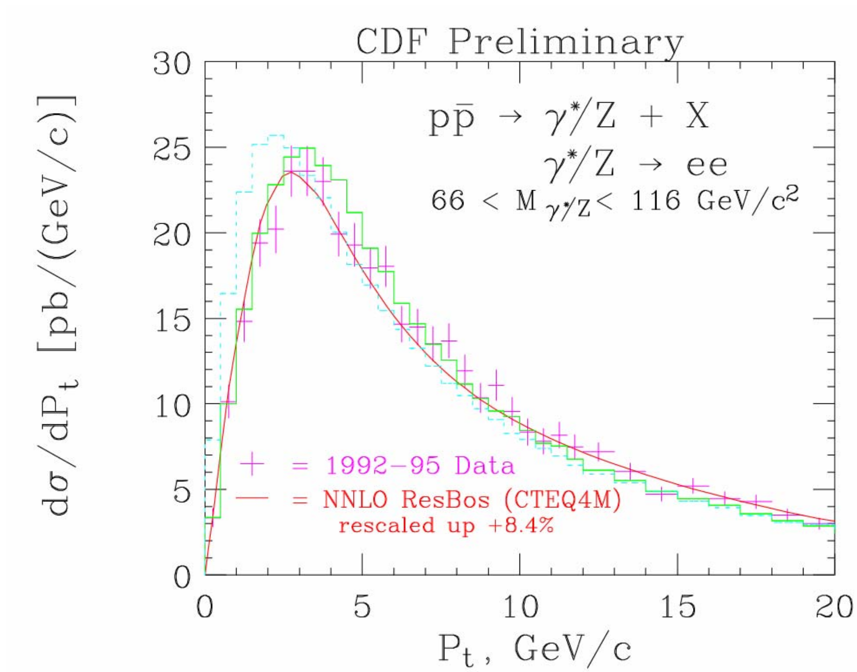
- the effect of the form factor is (just about) visible in the (Tevatron) data

resummation contd.

- theoretical refinements include the addition of sub-leading logarithms (e.g. NNLL) and nonperturbative contributions, and merging the resummed contributions with the fixed order (e.g. NLO) contributions appropriate for large p_T

- the resummation formalism is also valid for Higgs production at LHC via gg→H



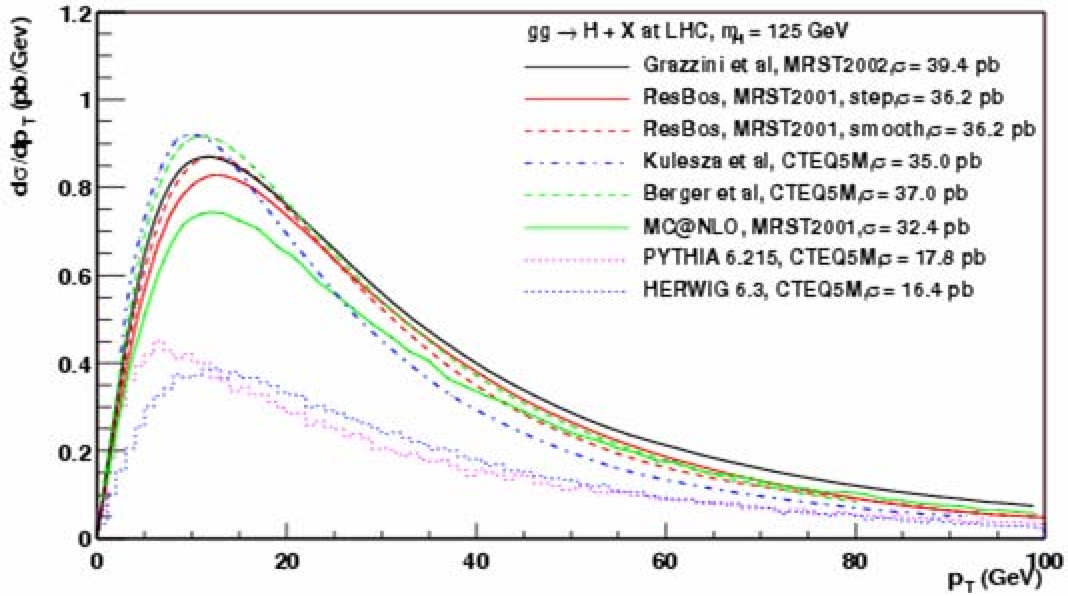


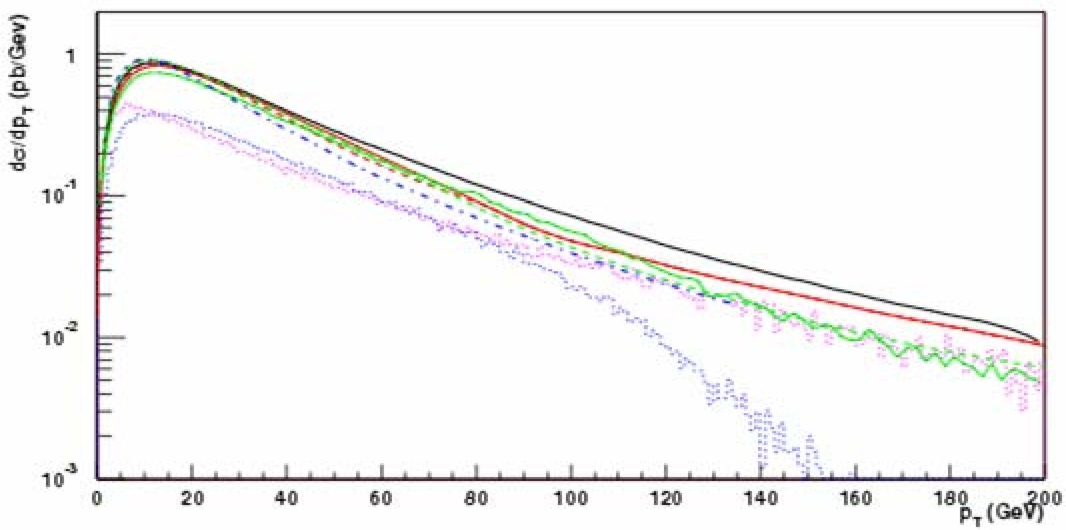
• comparison of resummed / fixed-order calculations for Higgs (M_H = 125 GeV) p_T distribution at LHC

Balazs et al, hep-ph/0403052

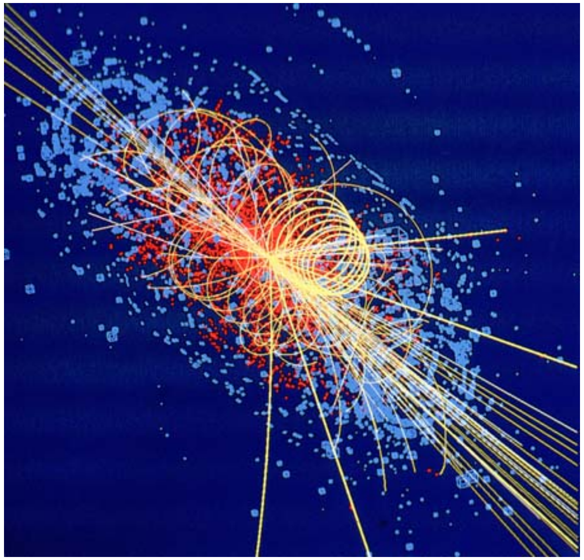
• differences due mainly to different N^nLO and N^nLL contributions included

• Tevatron $d\sigma(Z)/dp_T$ provides good test of calculations

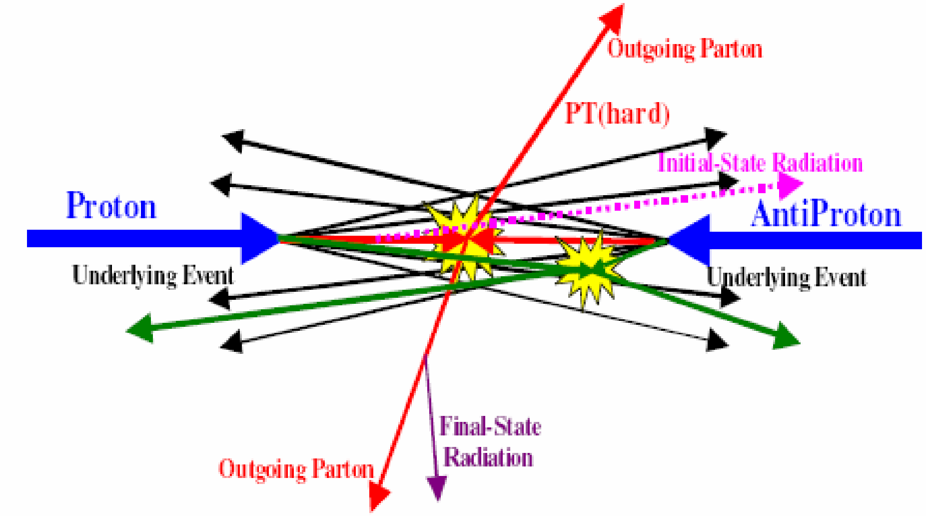




full event simulation at hadron colliders



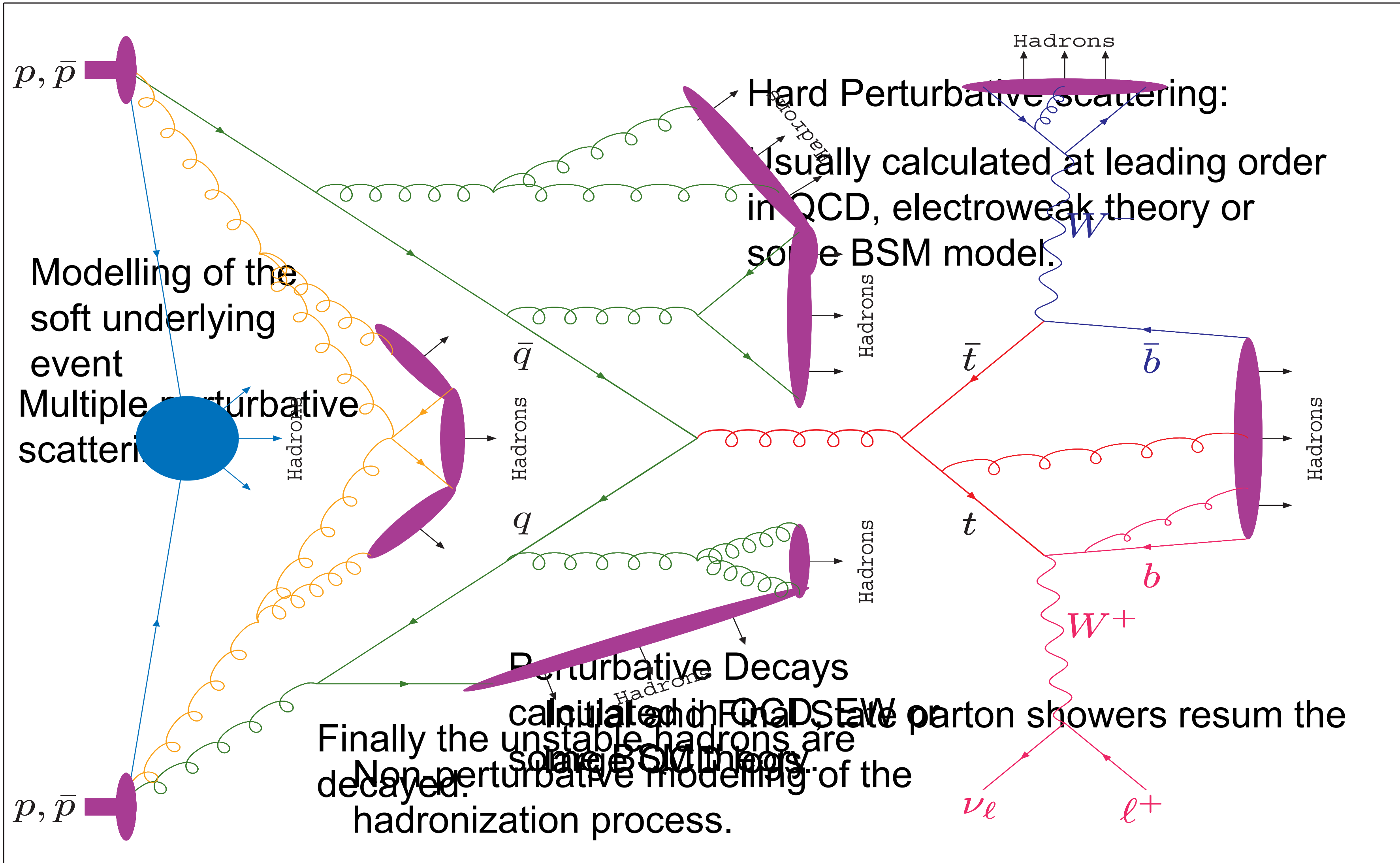
- it is important (designing detectors, interpreting events, etc.) to have a good understanding of all features of the collisions – not just the 'hard scattering' part
- this is very difficult because our understanding of the non-perturbative part of QCD is still quite primitive
- at present, therefore, we have to resort to models (PYTHIA, HERWIG, …) …



Monte Carlo Event Generators

- programs that simulates particle physics events with the same probability as they occur in nature
- widely used for signal and background estimates
- the main programs in current use are PYTHIA and HERWIG
- the simulation comprises different phases:
  - start by simulating a hard scattering process – the fundamental interaction (usually a 2→2 process but could be more complicated for particular signal/background processes)
  - this is followed by the simulation of (soft and collinear) QCD radiation using a parton shower algorithm
  - non-perturbative models are then used to simulate the hadronization of the quarks and gluons into the observed hadrons and the underlying event

a Monte Carlo event



from Peter Richardson (HERWIG)

(hadron collider) processes in HERWIG

1300	$q\bar{q} \to Z^0/\gamma \to q'\bar{q}'$ (all flavours)
1300+IQ	$q\bar{q} \to Z^0/\gamma \to q'\bar{q}'$ (IQ $= 1,2,3,4,5,6$ for $q = d,u,s,c,b,t$)
1350	$q\bar{q} \to Z^0/\gamma \to \ell\bar{\ell}$ (all lepton species)
1350+IL	$q\bar{q} \to Z^0/\gamma \to \ell\bar{\ell}$ (IL $= 1-6$ for $\ell = e, \nu_e, \mu, \nu_\mu$, etc.)
1399	$q\bar{q} \to Z^0/\gamma \to$ anything
1400	$q\bar{q} \to W^\pm \to q'\bar{q}''$ (all flavours)
1400+IQ	$q\bar{q} \to W^\pm \to q'\bar{q}''$ (q' or q'' as above)
1450	$q\bar{q} \to W^\pm \to \ell\nu_\ell$ (all lepton species)
1450+IL	$q\bar{q} \to W^\pm \to \ell\nu_\ell$ (IL $= 1,2,3$ for $\ell = e,\mu,\tau$)
1499	$q\bar{q} \to W^\pm \to$ anything
1500	QCD $2 \to 2$ hard parton scattering
	After generation, IHPRO is subprocess (see sect. 4.6.2)
1600+ID	$gg/q\bar{q} \to H^0_{\rm SM}$ (ID as in IPROC $= 300 +$ ID)
1700+IQ	QCD heavy quark production (IQ as above)
	After generation, IHPRO is subprocess (see sect. 4.6.2)
1800	QCD direct photon + jet production
IPROC	Process
	After generation, IHPRO is subprocess (see sect. 4.6.5)
1900+ID	$q\bar{q} \to q'\bar{q}'W^+W^-/Z^0Z^0 \to q'\bar{q}'H^0_{\rm SM}$ (ID as in IPROC $= 300 +$ ID)
2000	t production via $W^\pm$ exchange (sum of 2001–2008)
2001–4	$\bar{u}b \to \bar{d}t$, $db \to ut$, $\bar{d}b \to \bar{u}t$, $ub \to dt$
2005–8	$\bar{c}b \to \bar{s}t$, $sb \to ct$, $\bar{s}b \to \bar{c}t$, $cb \to st$
2100	$W^\pm$ + jet production
2110	$W^\pm$ + jet production (Compton only: $gq \to Wq$)
2120	$W^\pm$ + jet production (annihilation only: $q\bar{q} \to Wg$)
2150	Z^0 + jet production
2160	Z^0 + jet production (Compton only: $gq \to Zq$)
2170	Z^0 + jet production (annihilation only: $q\bar{q} \to Zg$)
2200	QCD direct photon pair production
	After generation, IHPRO is subprocess (see sect. 4.6.5)
2300+ID	QCD SM Higgs + jet production (ID as in IPROC=300+ID)
	After generation, IHPRO is subprocess (see sect. 4.6.10)
2400	Mueller-Tang colour singlet exchange
2450	Quark scattering via photon exchange
2500+ID	$gg/q\bar{q} \to t\bar{t}H^0_{\rm SM}$ (ID as in IPROC=300+ID)
2600+ID	$q\bar{q}' \to W^\pm H^0_{\rm SM}$ (ID as in IPROC=300+ID)
2700+ID	$q\bar{q} \to Z^0 H^0_{\rm SM}$ (ID as in IPROC=300+ID)

2800	W^+W^- production in hadron-hadron collisions
2810	Z^0Z^0 production in hadron-hadron collisions (including photon terms)
2815	Z^0Z^0 production in hadron-hadron collisions (Z^0 only)
2820	$W^\pm Z^0$ production in hadron-hadron collisions (including photon terms)
2825	$W^\pm Z^0$ production in hadron-hadron collisions (Z^0 only)
2850	hadron-hadron $\to W^+W^-X$ using MC@NLO
2860	hadron-hadron $\to Z^0Z^0X$ using MC@NLO
2870	hadron-hadron $\to W^+Z^0X$ using MC@NLO
2880	hadron-hadron $\to W^-Z^0X$ using MC@NLO
2900+IQ	$gg + q\bar{q} \to Q\bar{Q}Z^0$ for massless Q and $\bar{Q}$ (IQ=1…6 for $Q = d \ldots t$)
2910+IQ	$gg + q\bar{q} \to Q\bar{Q}Z^0$, for massive Q and $\bar{Q}$ (IQ=1…6 for $Q = d \ldots t$)
3000-3999	Minimal Supersymmetric Standard Model (MSSM) processes
3000	2-parton → 2-sparticle processes (sum of those below)
3010	2-parton → 2-sparton processes
3020	2-parton → 2-gaugino processes
3030	2-parton → 2-slepton processes
3100+ISQ	$gg/q\bar{q} \to \tilde{q}\tilde{q}'^* H^\pm$ (ISQ=IPROC−3100 as from table 15)
3200+ISQ	$gg/q\bar{q} \to \tilde{q}\tilde{q}'^* h, H, A$ (ISQ=IPROC−3200 as from table 16)

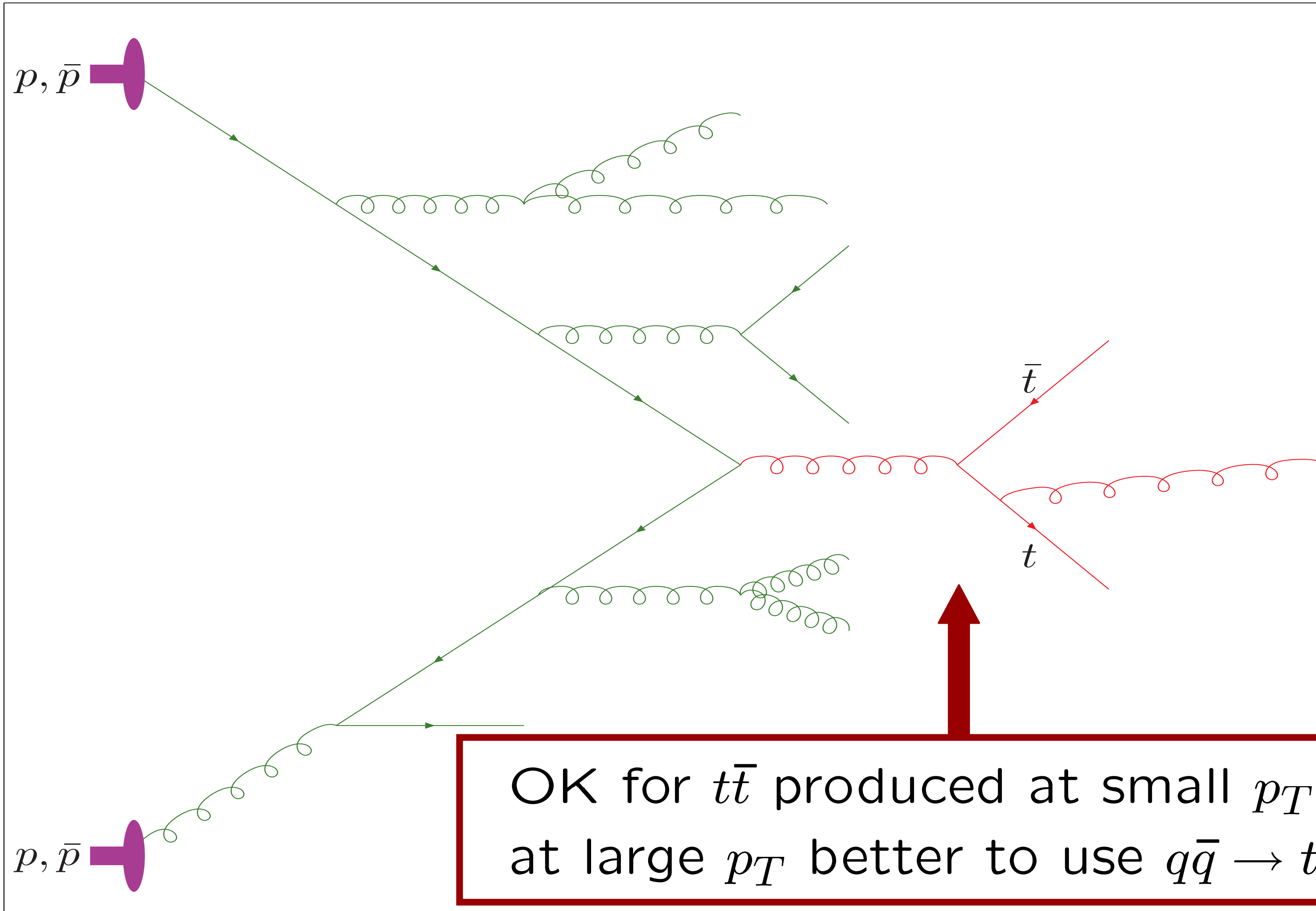
IPROC	Process
3310,3315	$q\bar{q}' \to W^\pm h^0, H^\pm h^0$(all q, q' flavours – gauge bosons mediated only)
3320,3325	$q\bar{q}' \to W^\pm H^0, H^\pm H^0$ (")
3335	$q\bar{q}' \to H^\pm A^0$ (")
3350	$q\bar{q} \to W^\pm H^\mp$ (Higgstrahlung and Higgs mediated)
3355	$q\bar{q} \to H^\pm H^\mp$ (all q flavours — gauge boson mediated only)
3360,3365	$q\bar{q} \to Z^0h^0, A^0h^0$ (")
3370,3375	$q\bar{q} \to Z^0H^0, A^0H^0$ (")
3410	$bg \to b\, h^0$ + ch. conj.
3420	$bg \to b\, H^0$ + ch. conj.
3430	$bg \to b\, A^0$ + ch. conj.
3450	$bg \to t\, H^-$ + ch. conj.
3500	$bq \to bq'H^\pm$ + ch. conj.
3610	$q\bar{q}/gg \to h^0$ (light scalar Higgs)
3620	$q\bar{q}/gg \to H^0$ (heavy scalar Higgs)
3630	$q\bar{q}/gg \to A^0$ (pseudoscalar Higgs)
3710	$q\bar{q} \to q'\bar{q}'W^+W^-/Z^0Z^0 \to q'\bar{q}'h^0$
3720	$q\bar{q} \to q'\bar{q}'W^+W^-/Z^0Z^0 \to q'\bar{q}'H^0$
3810+IQ	$gg + q\bar{q} \to Q\bar{Q}h^0$ (all q flavours in s-channel, IQ as usual for Q flavour)
3820+IQ	$gg + q\bar{q} \to Q\bar{Q}H^0$ (")
3830+IQ	$gg + q\bar{q} \to Q\bar{Q}A^0$ (")
3839	$gg + q\bar{q} \to b\bar{t}H^+$ + ch. conjg. (all q flavours in s-channel)
3840+IQ	$gg \to Q\bar{Q}h^0$ (IQ as above)
3850+IQ	$gg \to Q\bar{Q}H^0$ (")
3860+IQ	$gg \to Q\bar{Q}A^0$ (")
3869	$gg \to b\bar{t}H^+$ + ch. conjg.
3870+IQ	$q\bar{q} \to Q\bar{Q}h^0$ (all q flavours in s-channel, IQ as above)
3880+IQ	$q\bar{q} \to Q\bar{Q}H^0$ (")
3890+IQ	$q\bar{q} \to Q\bar{Q}A^0$ (")
3899	$q\bar{q} \to b\bar{t}H^+$ + ch. conjg. (all q flavours in s-channel)
3900–99	Reserved for other hadron-hadron MSSM processes
4000–99	R-parity violating supersymmetric processes via LQD
4000	single sparticle production, sum of 4010–4050
4010	$\bar{u}_j d_k \to \tilde{\chi}^0 l_i^-$, $\bar{d}_j d_k \to \tilde{\chi}^0 \nu_i$ (all neutralinos)
4010+IN	$\bar{u}_j d_k \to \tilde{\chi}^0_{\rm IN} l_i^-$, $\bar{d}_j d_k \to \tilde{\chi}^0_{\rm IN} \nu_i$ (IN=neutralino mass state)
4020	$\bar{u}_j d_k \to \tilde{\chi}^- \nu_i$, $\bar{d}_j d_k \to \tilde{\chi}^- e_i^+$ (all charginos)
4020+IC	$\bar{u}_j d_k \to \tilde{\chi}^-_{\rm IC} \nu_i$, $\bar{d}_j d_k \to \tilde{\chi}^-_{\rm IC} e_i^+$ (IC=chargino mass state)
4040	$u_j\bar{d}_k \to \tilde{\tau}_i^+ Z^0$, $u_j\bar{d}_k \to \tilde{\nu}_i W^+$ and $d_j\bar{d}_k \to \tilde{\ell}_i^+ W^-$
4050	$u_j\bar{d}_k \to \tilde{\ell}_i^+ h^0/H^0/A^0$, $u_j\bar{d}_k \to \tilde{\nu}_i H^+$ and $d_j\bar{d}_k \to \tilde{\ell}_i^+ H^-$

IPROC	Process
4060	Sum of 4070 and 4080
4070	$\bar{u}_j d_k \to \bar{u}_l d_m$ and $\bar{d}_j d_k \to \bar{d}_l d_m$, via LQD only
4080	$\bar{u}_j d_k \to \nu_j l_k^-$ and $\bar{d}_j d_k \to l_j^+ l_k^-$, via LQD and LLE
4100-99	R-parity violating supersymmetric processes via UDD
4100	single sparticle production, sum of 4110–4150
4110	$u_i d_j \to \tilde{\chi}^0 \bar{d}_k$, $d_j d_k \to \tilde{\chi}^0 \bar{u}_i$ (all neutralinos)
4110 +IN	$u_i d_j \to \tilde{\chi}^0_{\rm IN} \bar{d}_k$, $d_j d_k \to \tilde{\chi}^0_{\rm IN} \bar{u}_i$(IN as above)
4120	$u_i d_j \to \tilde{\chi}^+ \bar{u}_k$, $d_j d_k \to \tilde{\chi}^- \bar{d}_i$ (all charginos)
4120 +IC	$u_i d_j \to \tilde{\chi}^+_{\rm IC} \bar{u}_k$, $d_j d_k \to \tilde{\chi}^-_{\rm IC} \bar{d}_i$ (IC as above)
4130	$u_i d_j \to \tilde{g}\bar{d}_k$, $d_j d_k \to \tilde{g}\bar{u}_i$
4140	$u_i d_j \to \tilde{b}_1^* Z^0$, $d_j d_k \to \tilde{t}_1^* Z^0$, $u_i d_j \to \tilde{t}_i^* W^+$ and $d_j d_k \to \tilde{b}_i^* W^-$
4150	$u_i d_j \to \tilde{d}_{k1}^* h^0/H^0/A^0, d_j d_k \to \tilde{u}_{i1}^* h^0/H^0/A^0, u_i d_j \to \tilde{u}_{k\alpha}^* H^+,$ $d_j d_k \to \tilde{d}_{i\alpha}^* H^-$
4160	$u_i d_j \to u_l d_m$, $d_j d_k \to d_l d_m$ via UDD.
4200-99	Graviton resonance production
4200	Sum of 4210, 4250 and 4270
4210	$gg/q\bar{q} \to G \to gg/q\bar{q}$ (all partons)
4210+IQ	$gg/q\bar{q} \to G \to q\bar{q}$ (IQ as above)
4220	$gg/q\bar{q} \to G \to gg$
4250	$gg/q\bar{q} \to G \to \ell\bar{\ell}$ (all leptons)
4250+IL	$gg/q\bar{q} \to G \to \ell\bar{\ell}$ (IL $= 1-6$ for $\ell = e, \nu_e, \mu, \nu_\mu$, etc.)
4260	$gg/q\bar{q} \to G \to \gamma\gamma$
4270	$gg/q\bar{q} \to G \to W^+W^-/Z^0Z^0/H^0_{\rm SM}H^0_{\rm SM}$
4271	$gg/q\bar{q} \to G \to W^+W^-$
4272	$gg/q\bar{q} \to G \to Z^0Z^0$
4273	$gg/q\bar{q} \to G \to H^0_{\rm SM}H^0_{\rm SM}$

from Peter Richardson

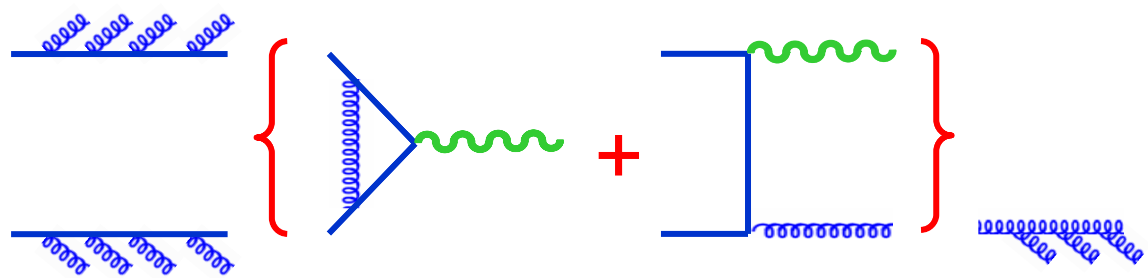
however …

- the tuning of the nonperturbative parts of the models is performed a single collider energy (or limited range of energies) – can we trust the extrapolation to LHC?!
- in general the event generators only use leading order matrix elements and therefore the normalisation is uncertain
- this can be overcome by renormalising to known NLO etc results or by incorporating next-to-leading order matrix elements (real + virtual emissions) – see below
- only soft and collinear emission is accounted for in the parton shower, therefore the emission of additional hard, high E_T jets is generally significantly underestimated
- for this reason, it is possible that many of the previous LHC studies of new physics signals have significantly underestimated the Standard Model backgrounds



OK for $t\bar{t}$ produced at small p_T;
at large p_T better to use $q\bar{q} \to t\bar{t}g$ ME

interfacing N^nLO and parton showers

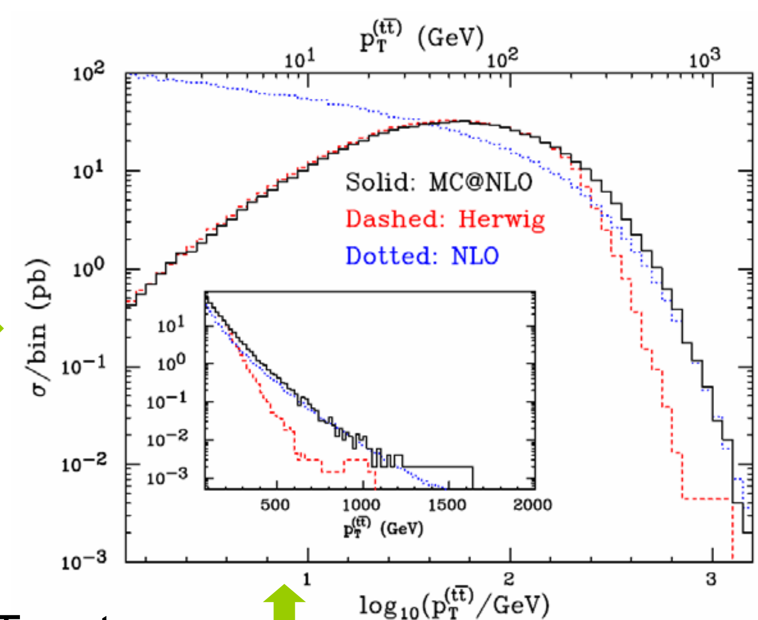


Benefits of both:

N^nLO correct overall rate, hard scattering kinematics, reduced scale dep.
PS complete event picture, correct treatment of collinear logs to all orders

Example: MC@NLO
Frixione, Webber, Nason,
www.hep.phy.cam.ac.uk/theory/webber/MCatNLO/

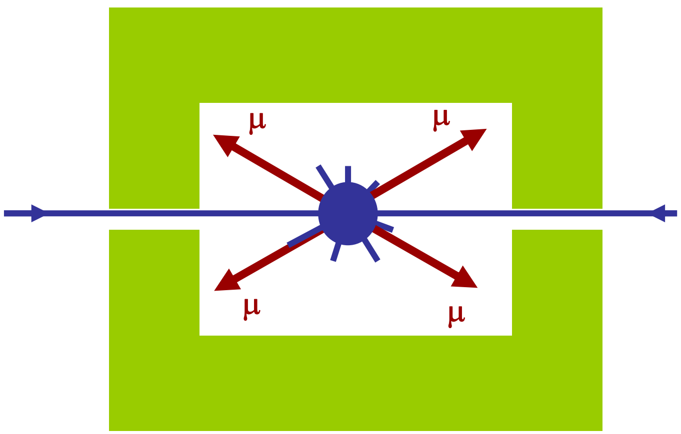
processes included so far …
$pp \to WW, WZ, ZZ, bb, tt, H^0, W, Z/\gamma$



p_T distribution of tt at Tevatron

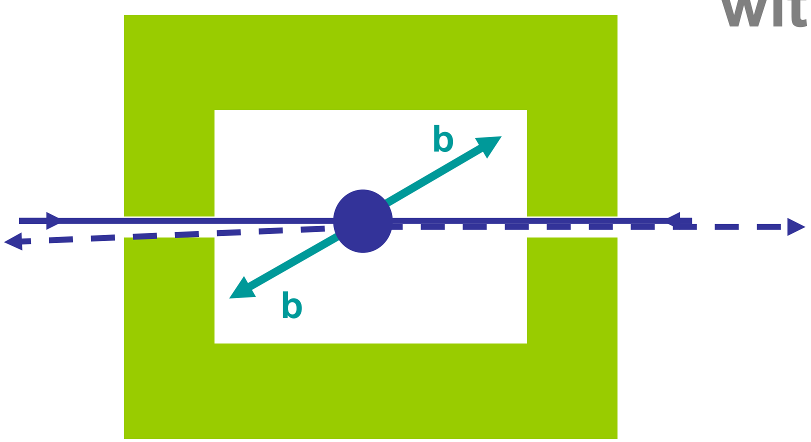
and finally …

central exclusive diffractive physics



compare …

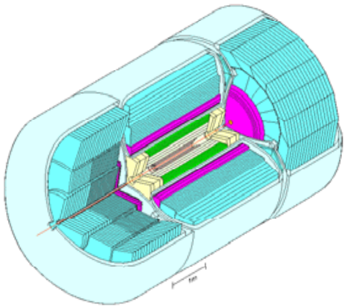
- $p + p \rightarrow H + X$
  - the rate (σ_{parton}, pdfs, α_S)
  - the kinematic distribtns. ($d\sigma/dydp_T$)
  - the environment (jets, underlying event, backgrounds, …)

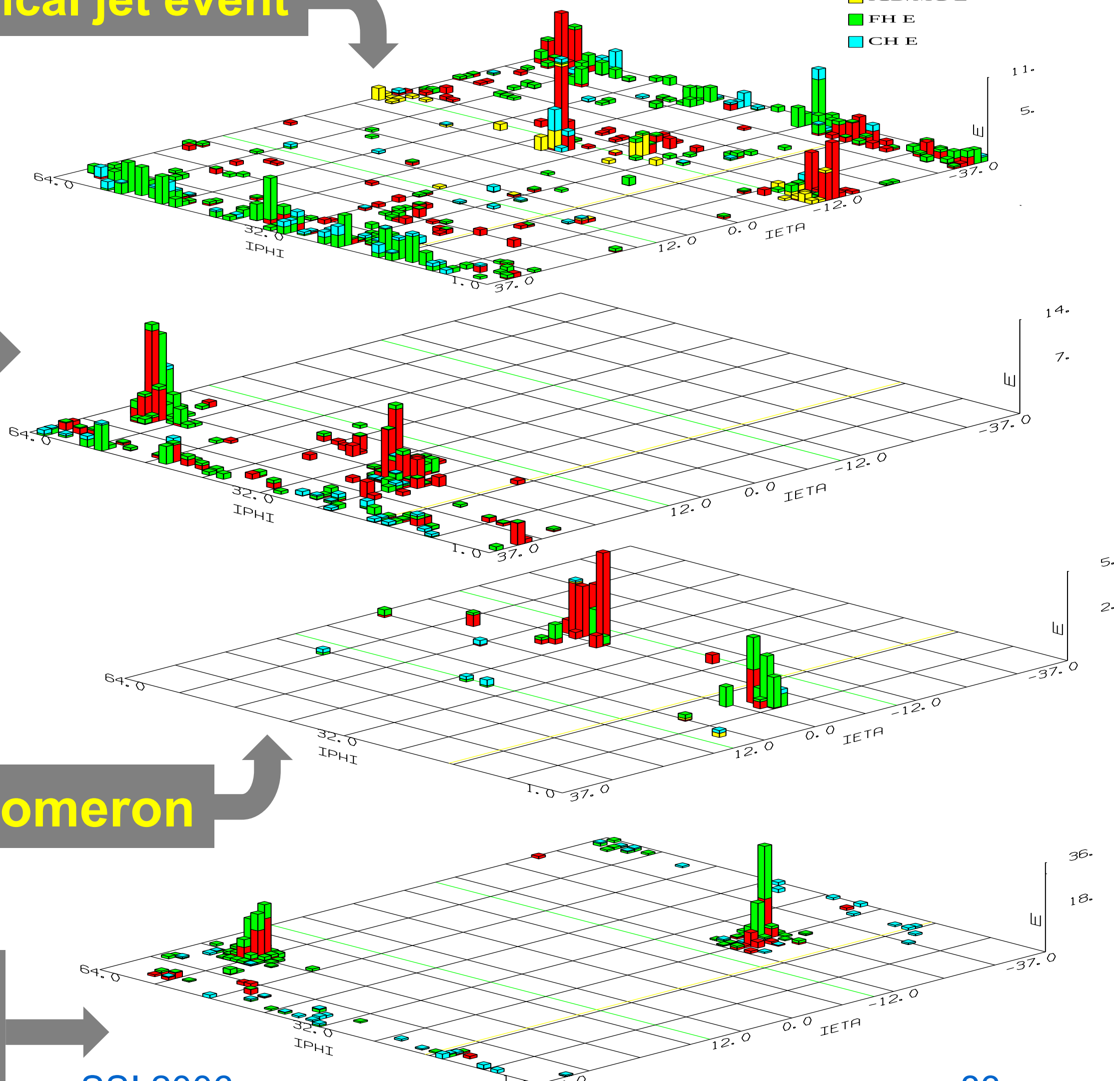


with …

- $p + p \rightarrow p + H + p$
  - a real challenge for theory (pQCD + npQCD) and experiment (tagging forward protons, triggering, …)

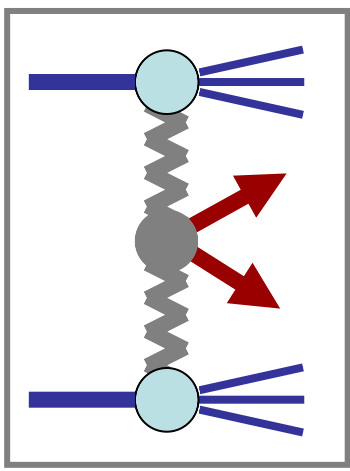
‘rapidity gap’ collision events





typical jet event

hard single diffraction



hard double pomeron

hard color singlet exchange

forward proton tagging at LHC: the physics case

$p + p \to p \oplus X \oplus p$

- all objects produced this way must be in a 0^{++} state → spin-parity filter/analyser
- with a mass resolution of ~O(1 GeV) from the proton tagging, the Standard Model $H \to bb$ decay mode opens up, with S/B > 1
- $H \to WW^{(*)}$ also looks very promising
- in certain regions of MSSM parameter space, S/B > 20, and double proton tagging is THE discovery channel

e.g. SM Higgs → bb
ΔM = 1 GeV, L = 30 fb^{-1}

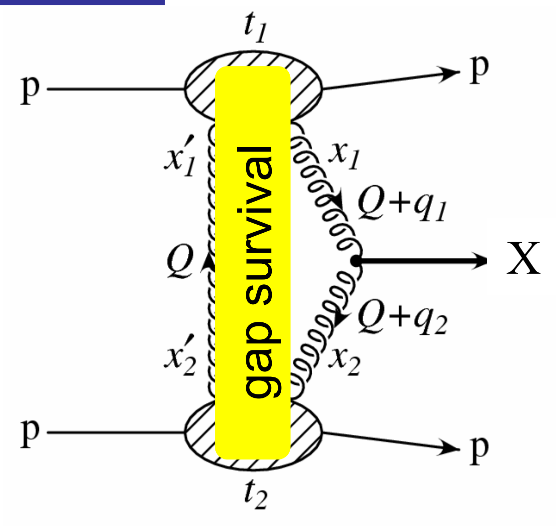
	S	B
m_h = 120 GeV	11	4
m_H = 135.5 GeV	124	2
m_A = 130 GeV	1	2

- in other regions of MSSM parameter space, explicit CP violation in the Higgs sector shows up as an azimuthal asymmetry in the tagged protons → direct probe of CP structure of Higgs sector at LHC
- any exotic 0^{++} state, which couples strongly to glue, is a real possibility: radions, gluinoballs, …

Khoze Martin Ryskin

the challenges …

theory



need to calculate production amplitude and gap Survival Factor: mix of pQCD and npQCD ⇒ significant uncertainties

Khoze
Martin
Ryskin
Kaidalov
WJS
de Roeck

Cox
Forshaw
Monk
Pilkington

Helsinki group

Saclay group

…

experiment

many! (forward proton/antiproton tagging, pile-up, low event rate, triggering, …)

important checks from Tevatron for X= dijets, $\gamma\gamma$, quarkonia, …

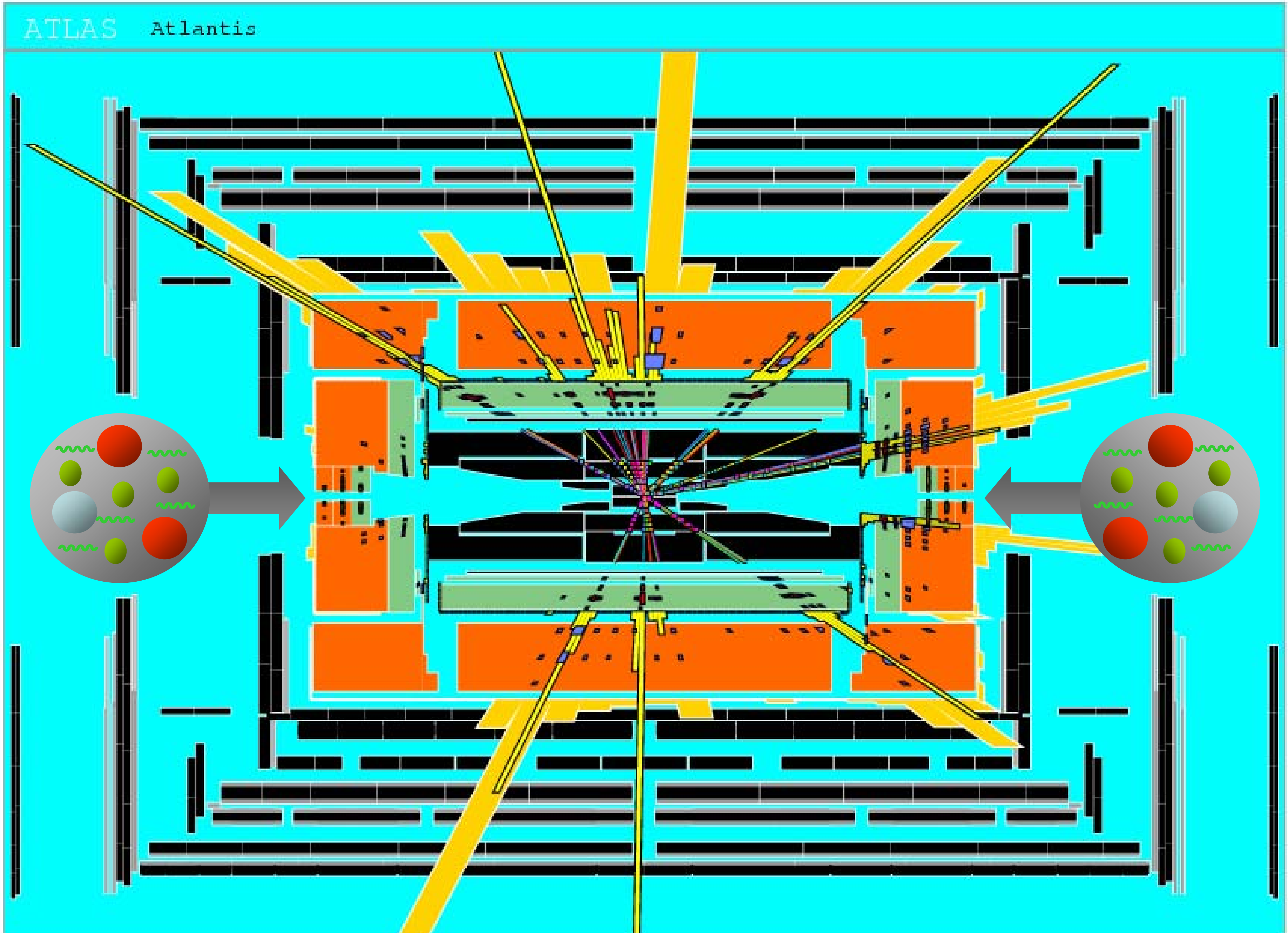
summary

- thanks to > 30 years theoretical studies, supported by experimental measurements, we now know how to calculate (an important class of) proton-proton collider event rates reliably and with a high precision

- the key ingredients are the factorisation theorem and the universal parton distribution functions

- such calculations underpin searches (at the Tevatron and the LHC) for Higgs, SUSY, etc

- …but much work still needs to be done, in particular
  - calculating more and more NNLO pQCD corrections (and some missing NLO ones too) – see Lance Dixon's lectures
  - further refining the pdfs, and understanding their uncertainties
  - understanding the detailed event structure, which is outside the domain of pQCD and is currently simply modelled
  - extending the calculations to new types of New Physics production processes, e.g. exclusive/diffractive production


ATLAS
Atlantis