Physics of Particle Detection

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Basic idea

Every effect of particles or radiation can be used as a working principle for a particle detector.

Outline of the Lectures

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- *methods of particle identification*:
  - Measure the bending radius ρ in a magnetic field B ($\vec{p} \perp \vec{B}$):

$$\frac{mv^2}{\rho} = z \cdot e \cdot v \cdot B \quad \Rightarrow \quad \rho = \frac{p}{zeB} \propto \frac{\gamma m_0 \beta c}{z}$$

with $\quad p$: momentum; $\quad \beta = \frac{v}{c}; \quad \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}.$

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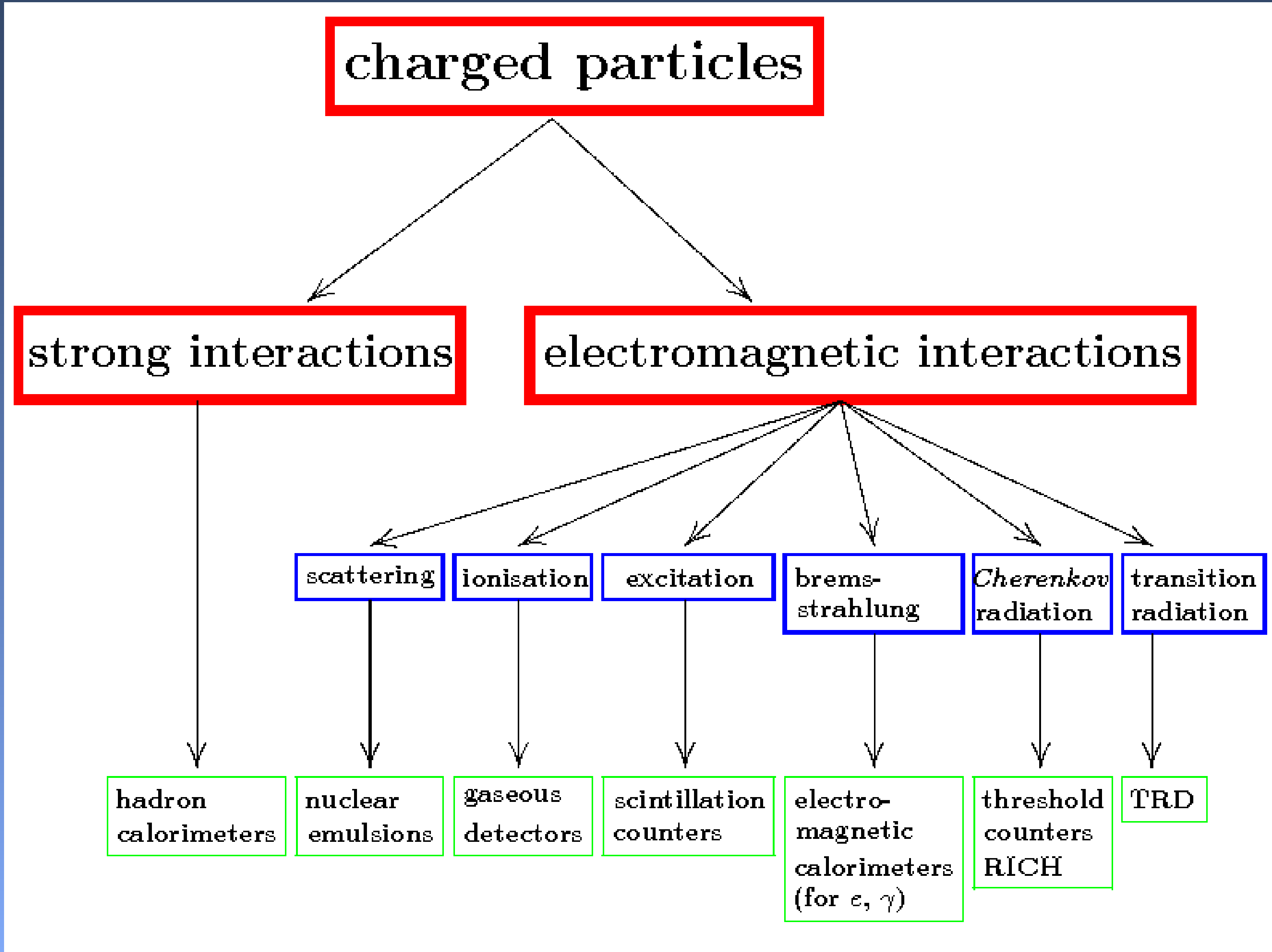
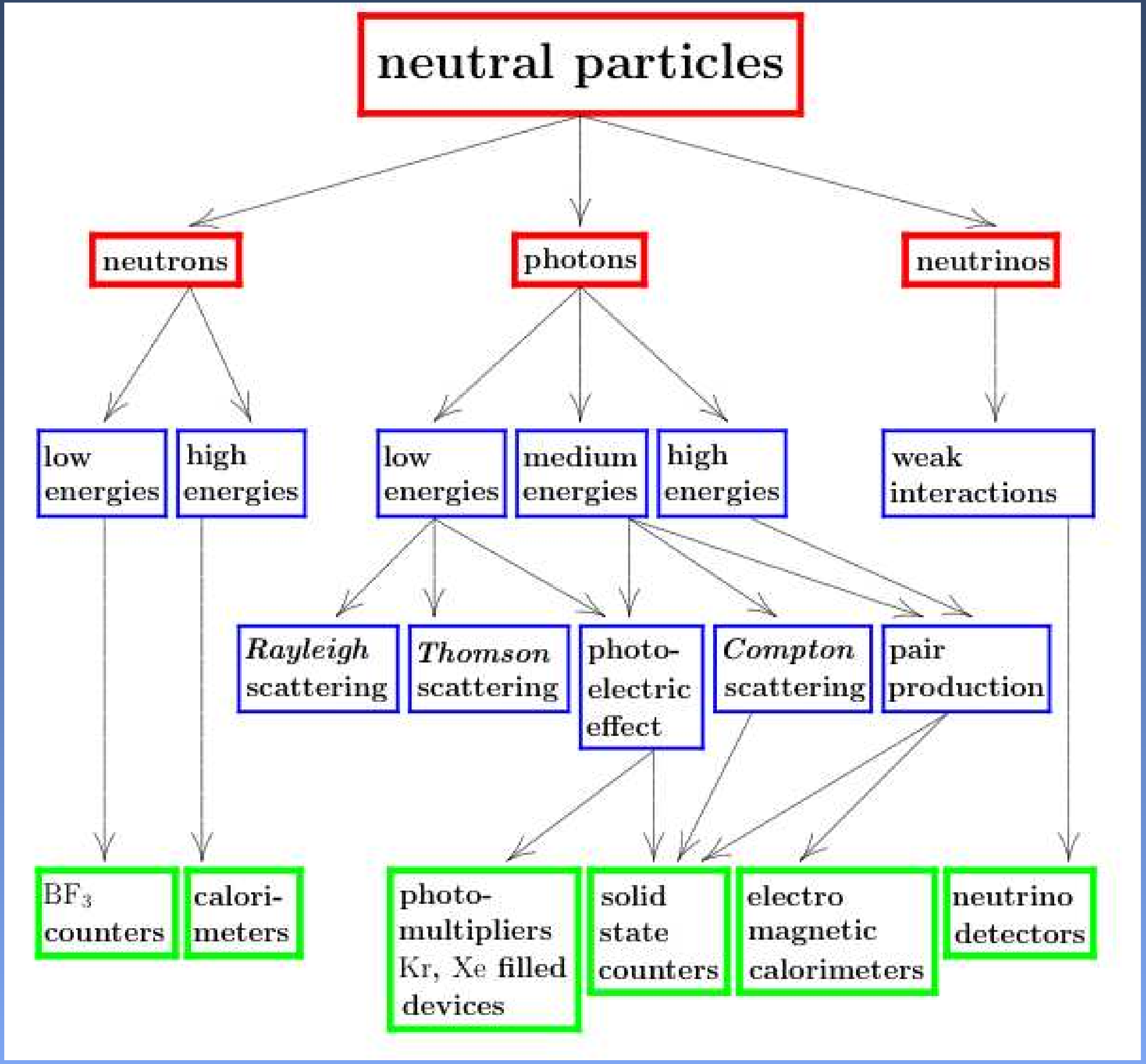
- Measure the energy loss due to transition radiation:
  $\left(\frac{\mathrm{d}E}{\mathrm{d}x}\right)_{\mathrm{transition}} \propto z^2\gamma$.

Charged Particles



Neutral Particles



Interaction of Charged Paricles

Kinematics: a particle of mass m_0 and velocity $v = \beta c$ collides with an electron; maximum transferable energy:

$$E_{\max}^{\text{kin}} = \frac{2m_e c^2 \beta^2 \gamma^2}{1 + 2\gamma \frac{m_e}{m_0} + \left(\frac{m_e}{m_0}\right)^2} = \frac{2m_e p^2}{m_0^2 + m_e^2 + 2m_e E/c^2}$$

with E: total energy of the particle, $\gamma = \frac{E}{m_0 c^2}$.

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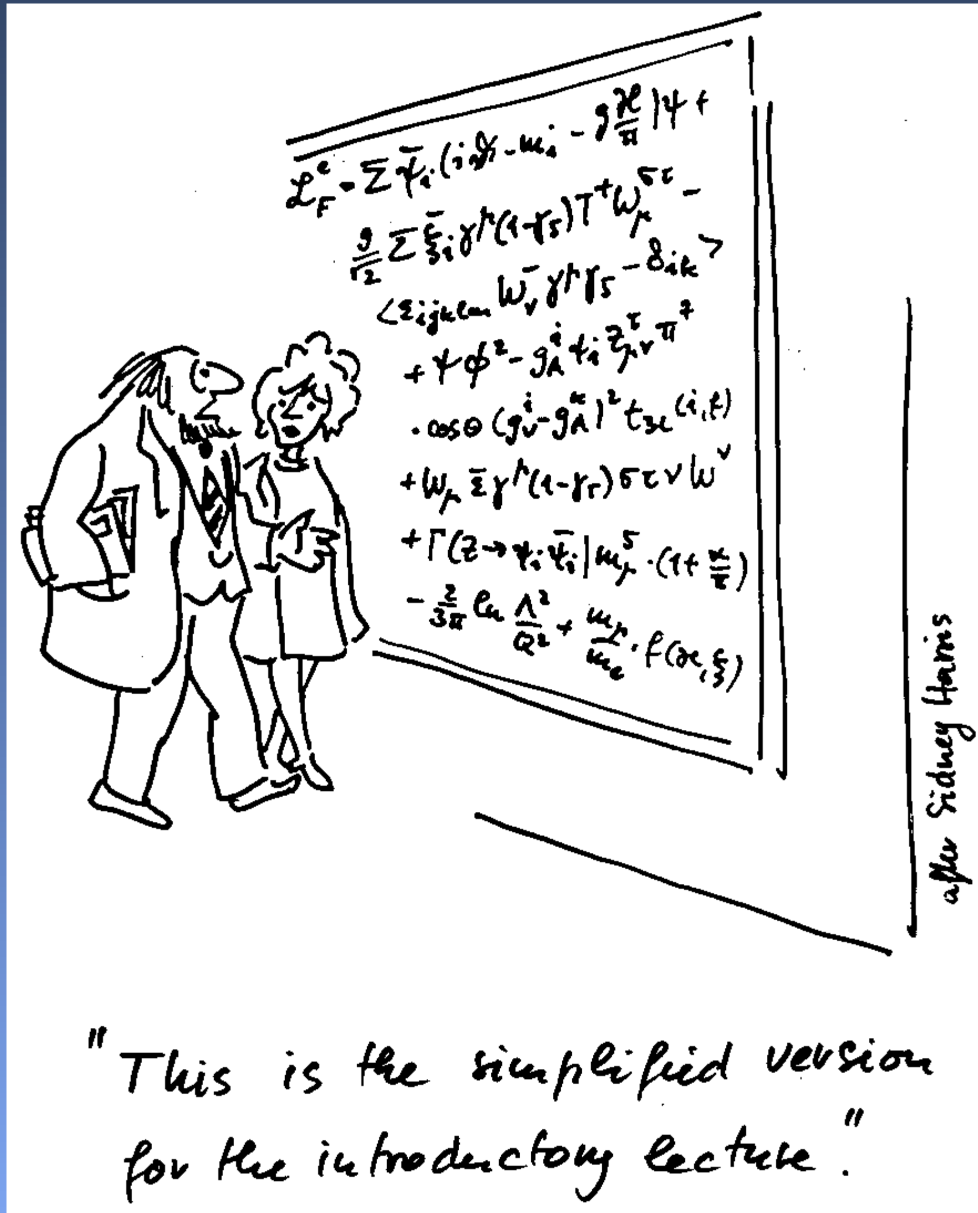
Examples:

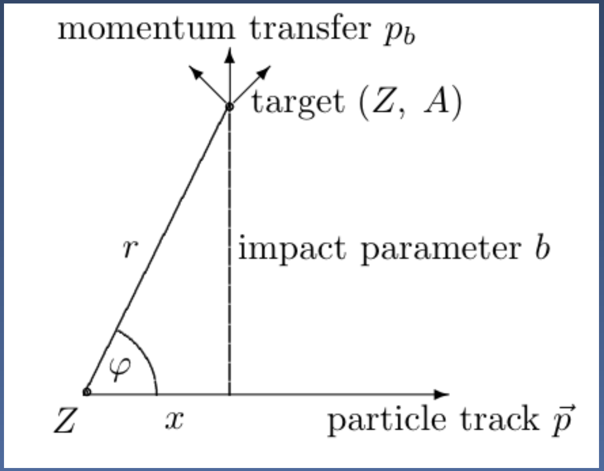
(a) $\mu - e$ - collision: $E_{\max} = \frac{E^2}{E+11}$ (E in GeV)

(b) if $m_0 = m_e$: $E_{\max}^{\mathrm{kin}} = \frac{p^2}{m_e + E/c^2} = \frac{E^2 - m_e^2 c^4}{E + m_e c^2} = E - m_e c^2$

Simplified Version



Rutherford Scattering



$$\vec{F} = \frac{ze \cdot Ze}{r^2} \cdot \frac{\vec{r}}{r},$$

$$p_b = \int_{-\infty}^{\infty} F_b \mathrm{d}t = \int_{-\infty}^{\infty} \frac{zZe^2}{r^2} \cdot \frac{b}{r} \cdot \frac{\mathrm{d}x}{\beta c},$$

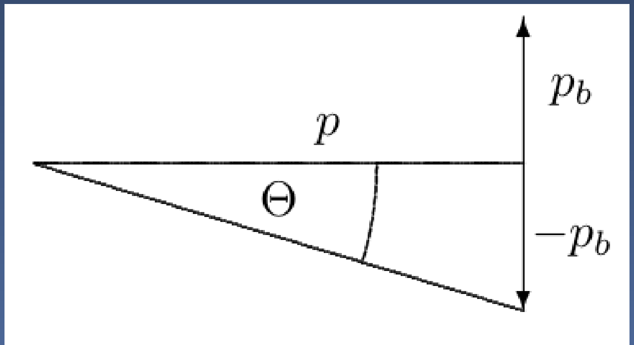
$$p_b = \frac{zZe^2}{\beta c} \int_{-\infty}^{\infty} \frac{b\ \mathrm{d}x}{(\sqrt{x^2 + b^2})^3}$$

$$= \frac{zZe^2}{\beta c b} \int_{-\infty}^{\infty} \frac{\mathrm{d}(x/b)}{(\sqrt{1 + (x/b)^2})^3} = \frac{2zZe^2}{\beta c b},$$

$$\rightsquigarrow p_b = \frac{2 r_e m_e c}{b \beta} zZ \quad \text{with} \quad r_e = \frac{e^2}{m_e c^2}.$$

Scattering Angle

$$\Theta = \frac{p_b}{p} = \frac{2zZe^2}{bc\beta} \cdot \frac{1}{p}$$



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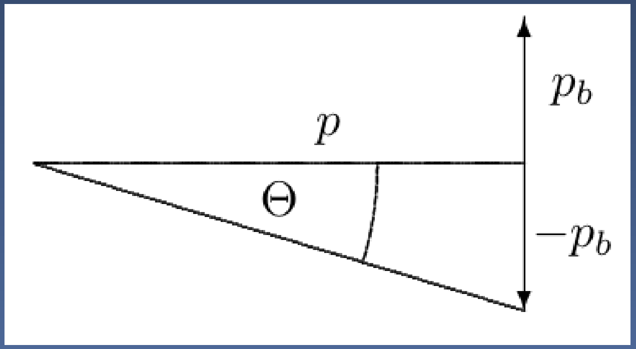

Cross section for scattering into the solid angle:

$$\mathrm{d}\Omega = \sin\Theta \mathrm{d}\Theta\ \mathrm{d}\varphi = -\mathrm{d}\cos\Theta\ \mathrm{d}\varphi$$

with Θ: polar angle, φ azimuthal angle.

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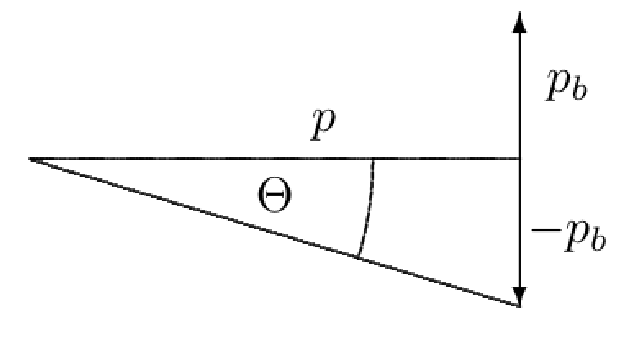

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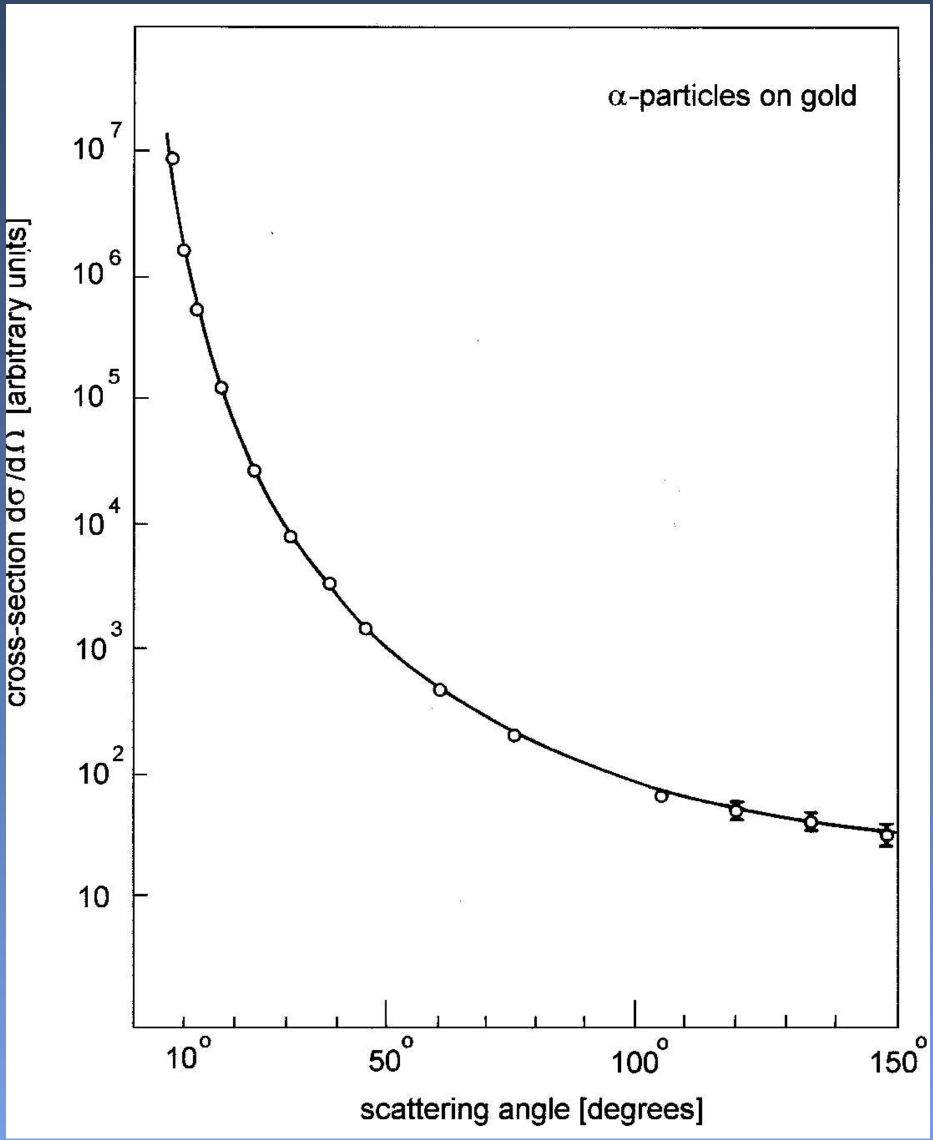
“Rutherford scattering”:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{z^2 Z^2}{4} r_e^2 \left(\frac{m_e c}{\beta p}\right)^2 \cdot \frac{1}{\sin^4 \Theta/2}.$$

Scattering of α-Particles on Gold

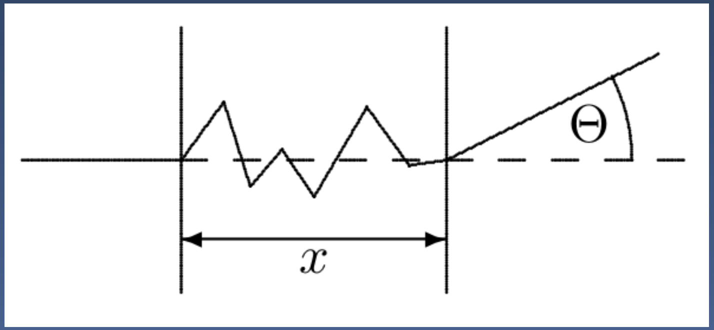
E. Rutherford
Phil. Mag. 21 (1911) 669

H. Geiger, E. Marsden
Phil. Mag. 25 (1913) 604



Multiple Scattering

$\langle \Theta \rangle = 0$
p in MeV/c
X_0: radiation length



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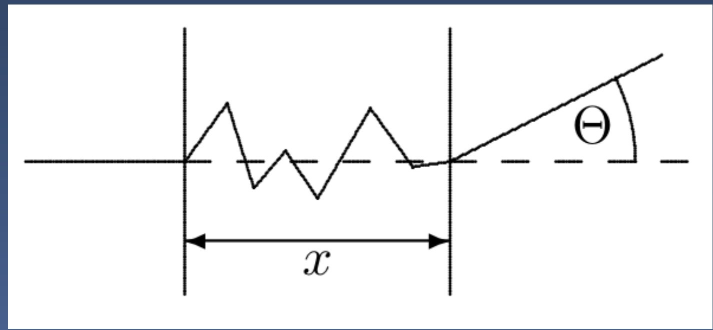

$$\sqrt{\langle \Theta^2 \rangle} = \Theta_{\text{plane}} = \frac{13.6 \text{ MeV}}{\beta \, cp} z \sqrt{\frac{x}{X_0}} \left\{ 1 + 0.038 \ln \left(\frac{x}{X_0} \right) \right\}$$

$$\Theta_{\text{space}} = \sqrt{2} \Theta_{\text{plane}} = \sqrt{2} \Theta_0$$

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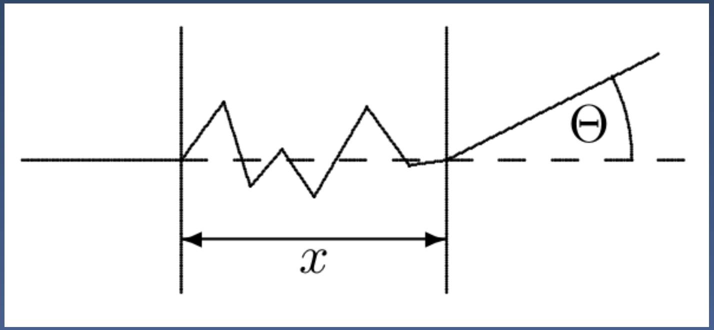

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Projected angular distribution:

$$P(\Theta)\ \text{d}\Theta = \frac{1}{\sqrt{2\pi}\Theta_0} \exp\left\{ -\frac{\Theta^2}{2\Theta_0^2} \right\} \text{d}\Theta$$

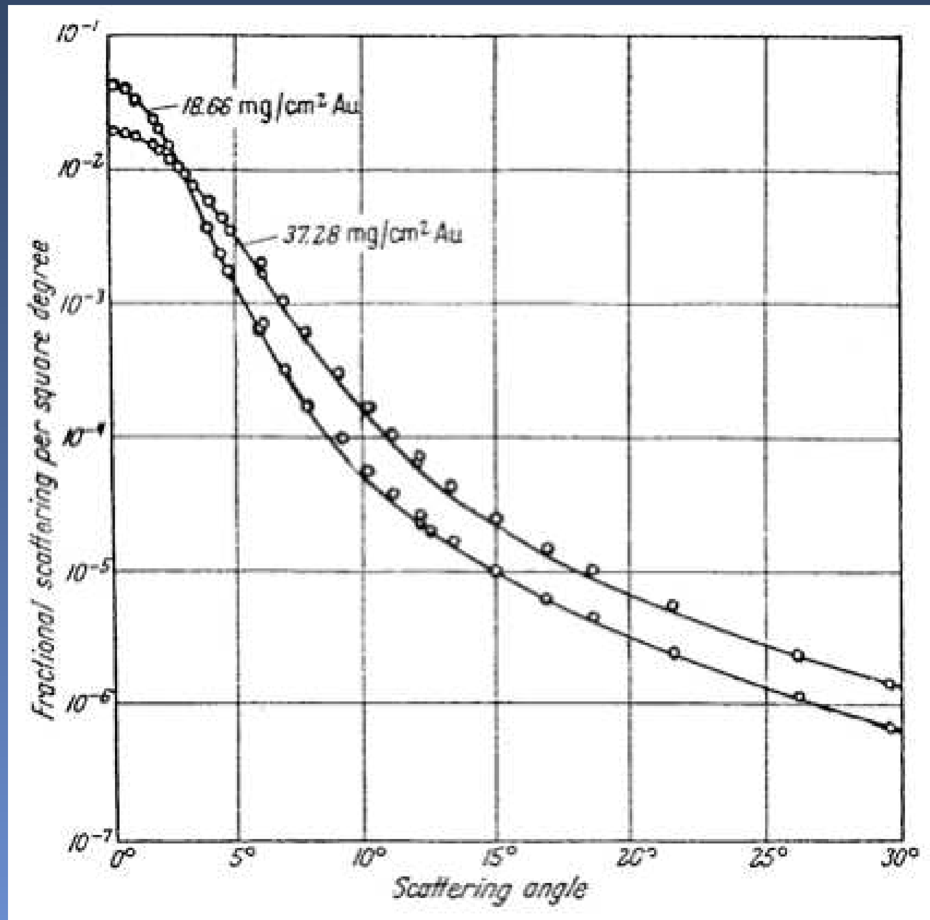
$+$ tail due to single, large angle Coulomb scattering.

Scattering of electrons on Gold

Electrons of $15.7\ \mathrm{MeV}$
on Au-foils

$< 5^\circ$ dominated by multiple scattering

$> 15^\circ$ dominated by single scattering



A.O. Hanson et al., Phys.Rev. 84 (1951) 634
R.O. Birkhoff, Handb.Phys. XXXIV (1958)

Ionisation Energy Loss (Bethe-Bloch formula, 1)

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Interaction rate per $(\mathrm{g}/\mathrm{cm}^2)$, given the atomic cross section:

$$\phi(\mathrm{g}^{-1}\ \mathrm{cm}^2) = \frac{N}{A} \cdot \sigma\ [\mathrm{cm}^2\ /\ \mathrm{atom}]$$

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$$\phi(\varepsilon)\mathrm{d}\varepsilon = \frac{N}{A} \cdot \underbrace{2\pi b\ \mathrm{d}b}_{\text{area of an annulus}} \cdot Z$$

with Z: electrons per target atom.

Ionisation Energy Loss (Bethe-Bloch formula, 2)

$$\varepsilon = f(b) \Rightarrow b^2 = \frac{2r_e^2 m_e c^2}{\beta^2} z^2 \cdot \frac{1}{\varepsilon}.$$

$$\phi(\varepsilon)\mathrm{d}\varepsilon = \frac{N}{A} \cdot Z \cdot 2\pi \cdot \underbrace{\frac{r_e^2 m_e c^2}{\beta^2} z^2 \frac{\mathrm{d}\varepsilon}{\varepsilon^2}}_{b\ \mathrm{d}b} \propto \frac{1}{\varepsilon^2}.$$

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Energy loss:

$$-\mathrm{d}E = \int_0^\infty \phi(\varepsilon) \cdot \varepsilon\ \mathrm{d}\varepsilon\ \mathrm{d}x = \int_0^\infty \frac{N}{A} \cdot 2\pi b\ \mathrm{d}b \cdot Z \cdot \varepsilon\ \mathrm{d}x.$$

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$$\frac{\mathrm{d}E}{\mathrm{d}x} = \frac{2\pi N}{A} \cdot Z \cdot \int_0^\infty \varepsilon b \,\mathrm{d}b = 2\pi \frac{Z \cdot N}{A} \cdot \frac{2r_e^2 m_e c^2}{\beta^2} z^2 \int_0^\infty \frac{\mathrm{d}b}{b}.$$

Problem: the integral is divergent at $b = 0$ *and* $b = \infty \ldots$

Ionisation Energy Loss (Bethe-Bloch formula, 3)

$b = 0$:

Assume $b_{\min} = \frac{h}{2p} = \frac{h}{2\gamma m_e \beta c}$ half the *de Broglie wavelength*.

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$\tau_i = \frac{b_{\max}}{v}\sqrt{1-\beta^2}$

with $\sqrt{1-\beta^2}$: *Lorentz*-contraction of the field at high velocities

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$$\rightsquigarrow -\frac{\mathrm{d}E}{\mathrm{d}x} = \frac{2\pi Z N}{A}\frac{2 r_e^2 m_e c^2}{\beta^2} z^2 \left[\ln \frac{2\gamma^2\beta^2 m_e c^2}{I} - \underbrace{\eta}_{\text{screening effect}} \right]$$

Bethe-Bloch formula: exact treatment

$$\frac{\mathrm{d}E}{\mathrm{d}x} = 2\pi \frac{ZN}{A} \frac{2r_e^2 m_e c^2}{\beta^2} z^2 \left[\frac{1}{2} \ln \left(\frac{2m_e c^2 \gamma^2 \beta^2}{I^2} E_{\mathrm{max}}^{\mathrm{kin}} \right) - \beta^2 - \frac{\delta}{2} \right]$$

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Density correction:

$\frac{\delta}{2} = \ln \left(\frac{\hbar\omega_p}{I} \right) + \ln(\beta\gamma) - \frac{1}{2}$

where $\hbar\omega_p = \sqrt{4\pi N_e r_e^3 \frac{m_e c^2}{\alpha}}$ (plasma energy).

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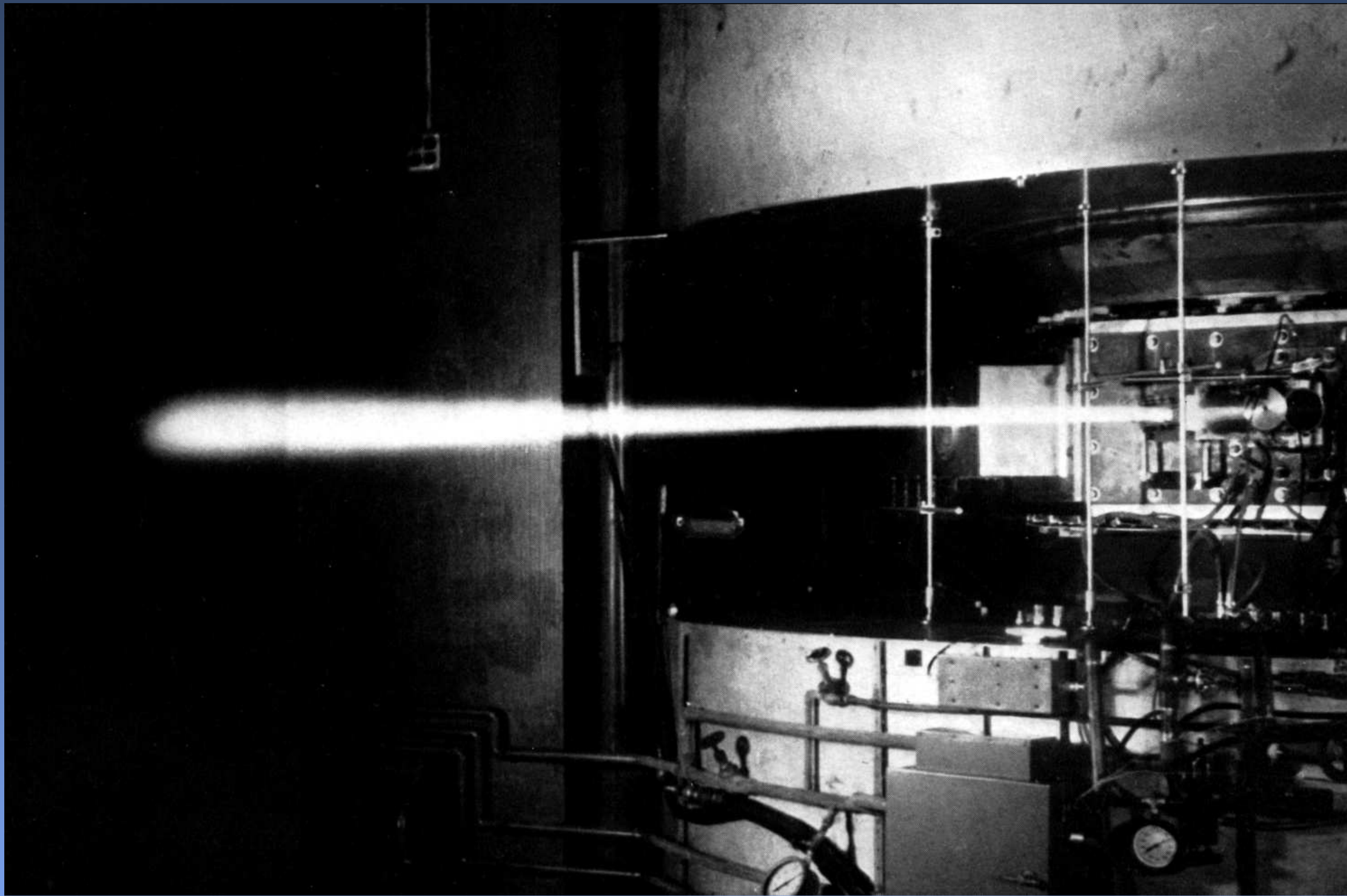
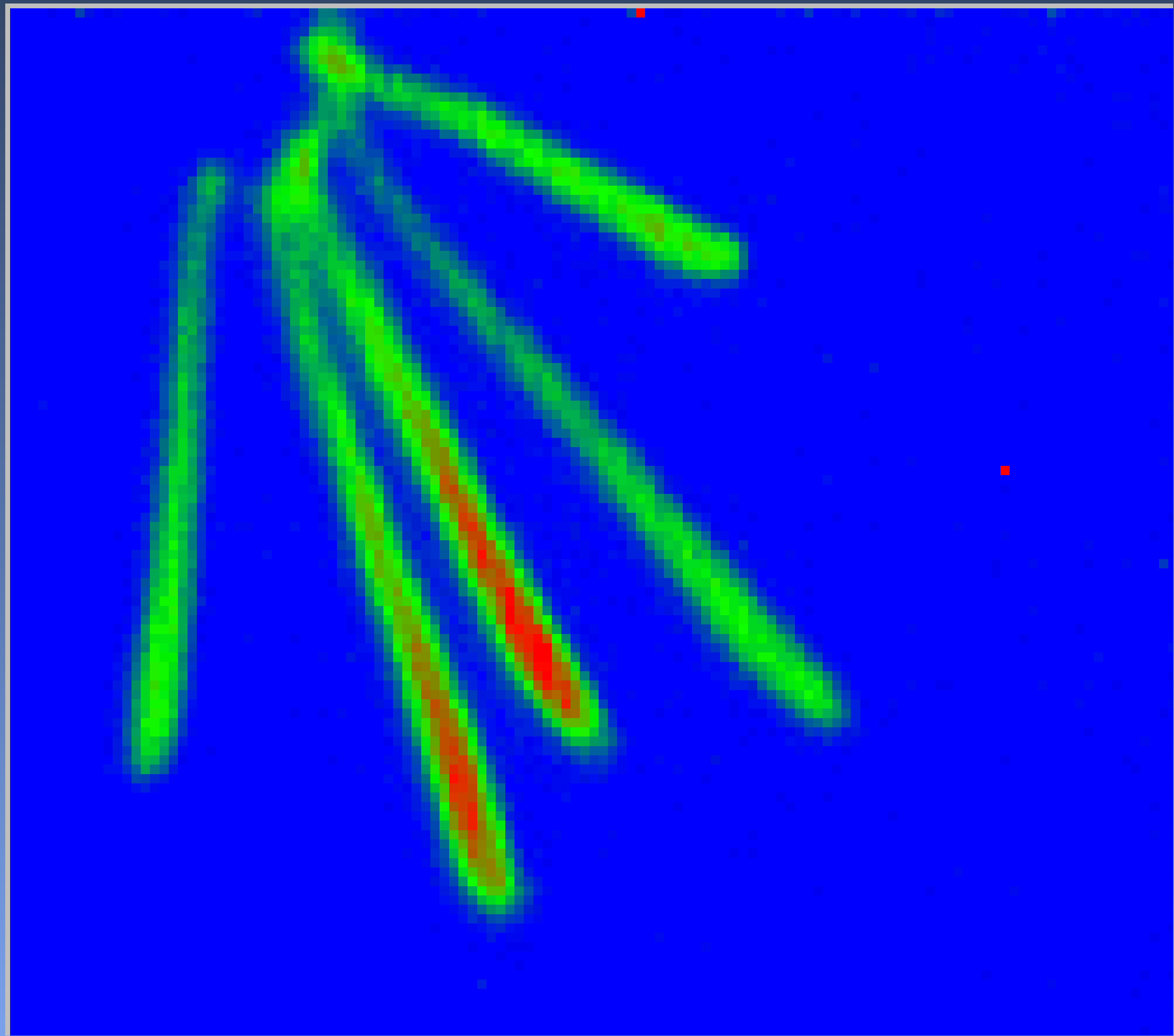
- N_e: electron density of the absorbing material
- α: fine structure constant $= \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{\hbar c}$
- ε_0: permittivity of free space

Speed Limit

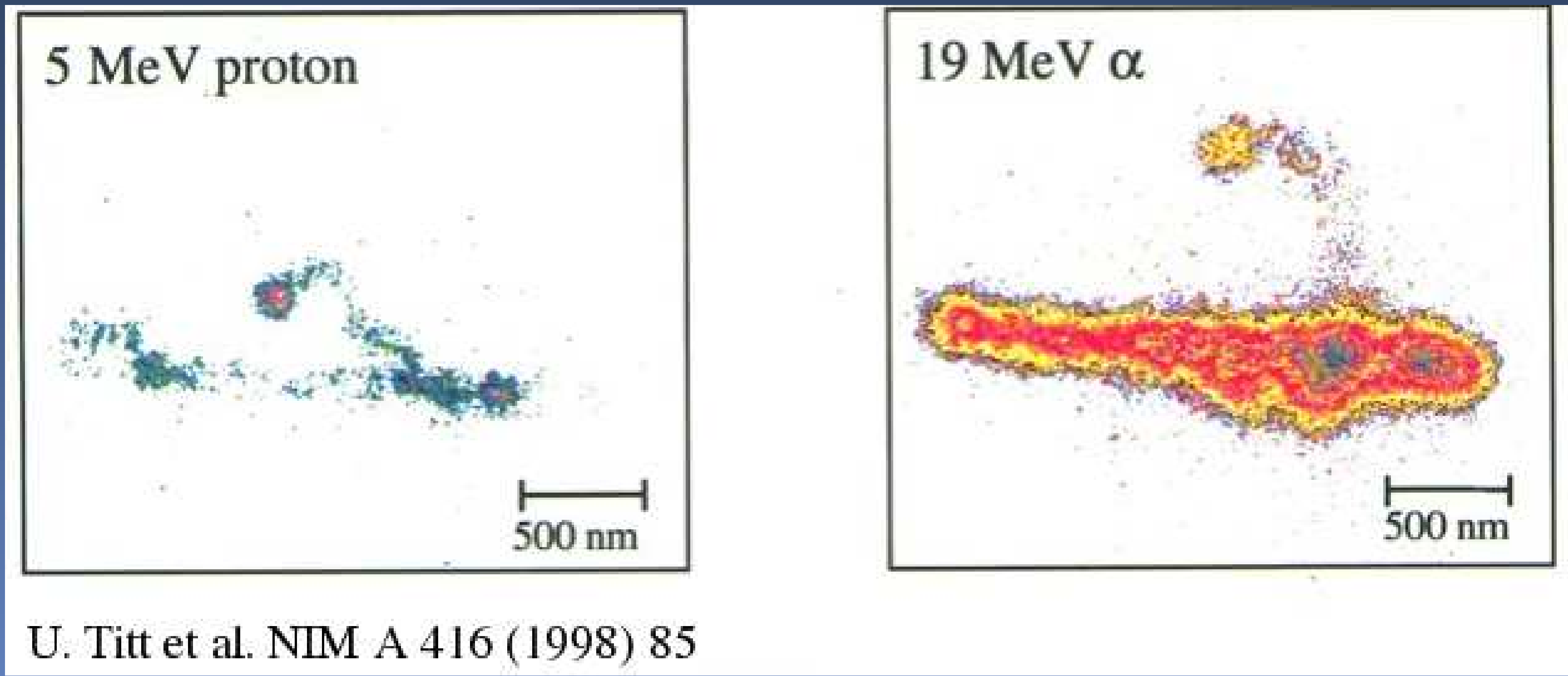
Deuteron Beam Scintillating in Air



α Tracks in a Micro Pattern Chamber



Proton and α Ranging out



Optical avalanche microdosimeter: demonstrates $\left(\frac{\mathrm{d}E}{\mathrm{d}x}\right)_\alpha >> \left(\frac{\mathrm{d}E}{\mathrm{d}x}\right)_\mathrm{p}$ because $\frac{\mathrm{d}E}{\mathrm{d}x} \sim z^2$

$E_\mathrm{p} = 5.0$ MeV with δ-ray

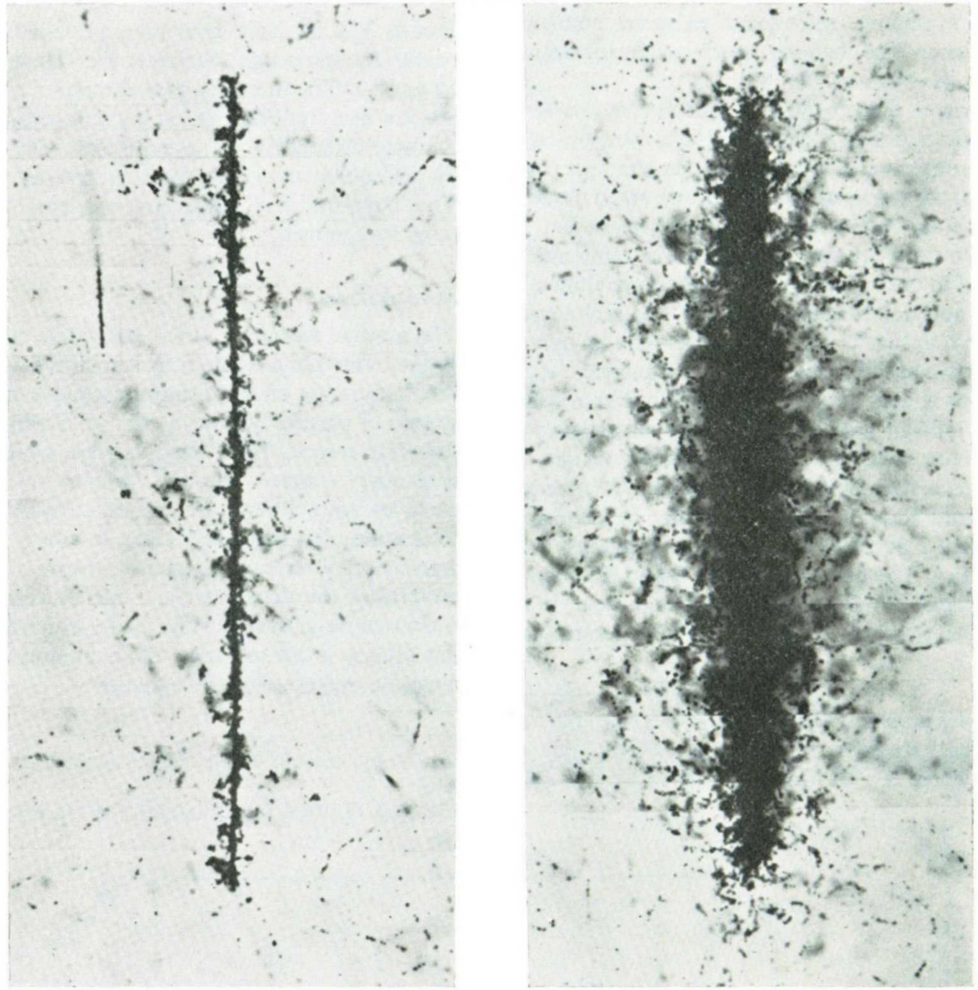
$E_\alpha = 19$ MeV with δ-ray

Heavy Nuclei in Cosmic Rays

Ionisation density of relativistic heavy ions from cosmic radiation in nuclear emulsions

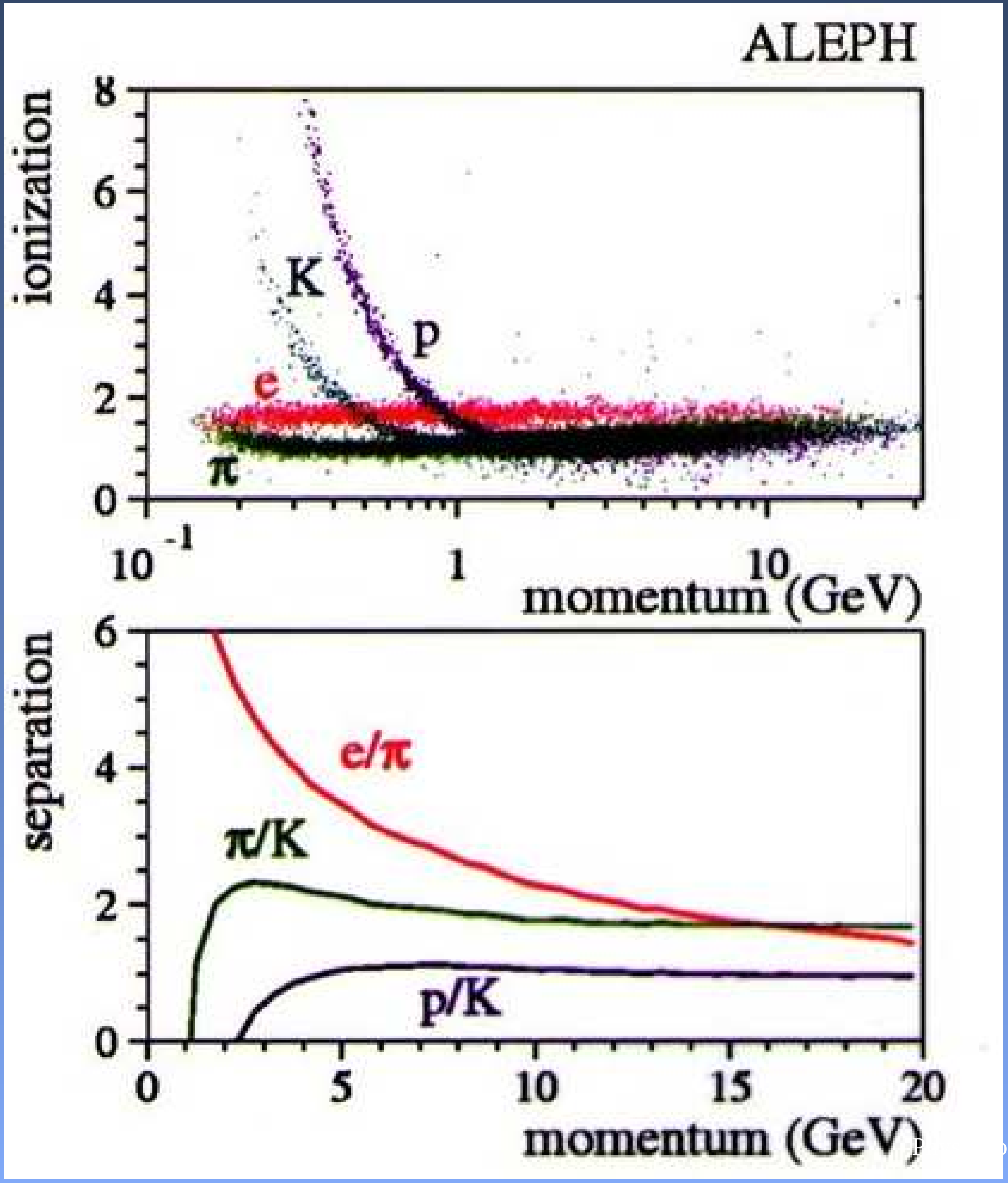
G. D. Rochester

Advancement of Science
Dec. 1970, p.183-194



Fe $Z = 26$ $Z = \sim 90$

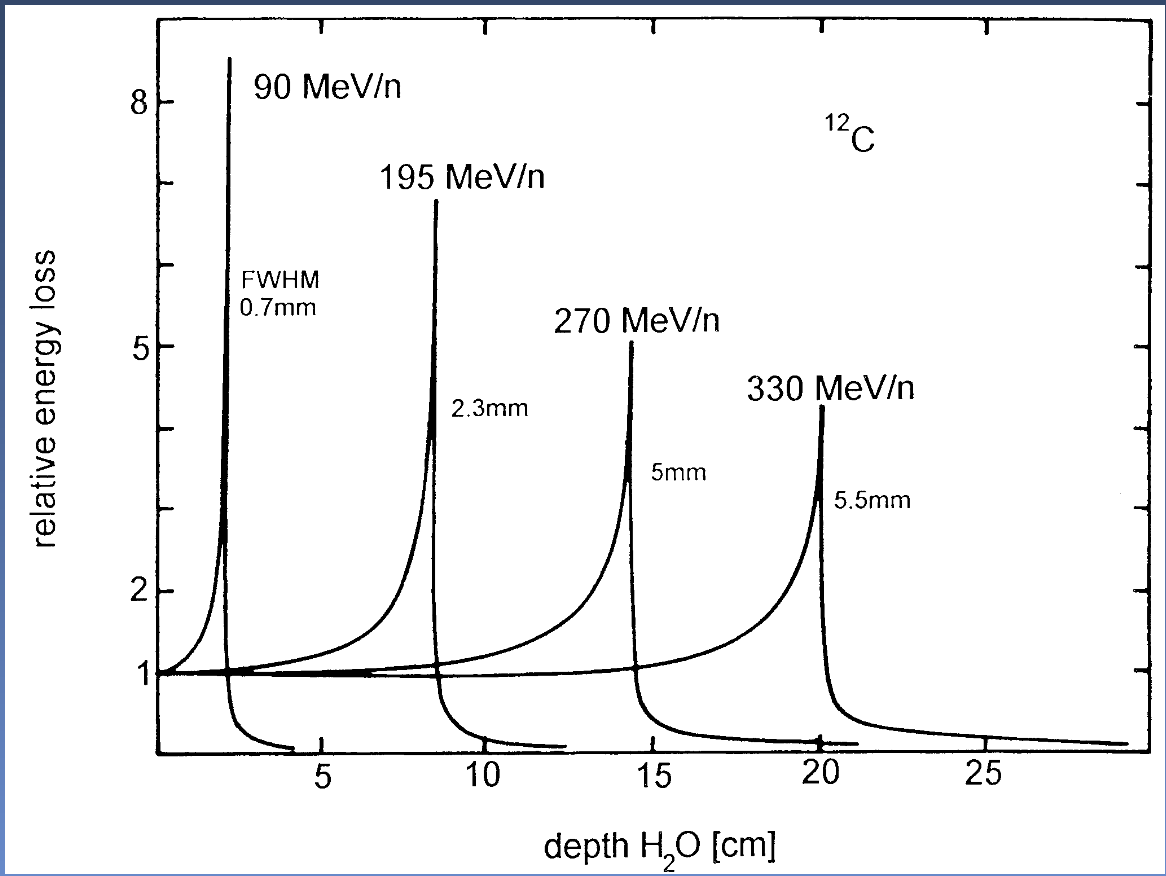
(ALEPH): Particle Identification with $\mathrm{d}E/\mathrm{d}x$



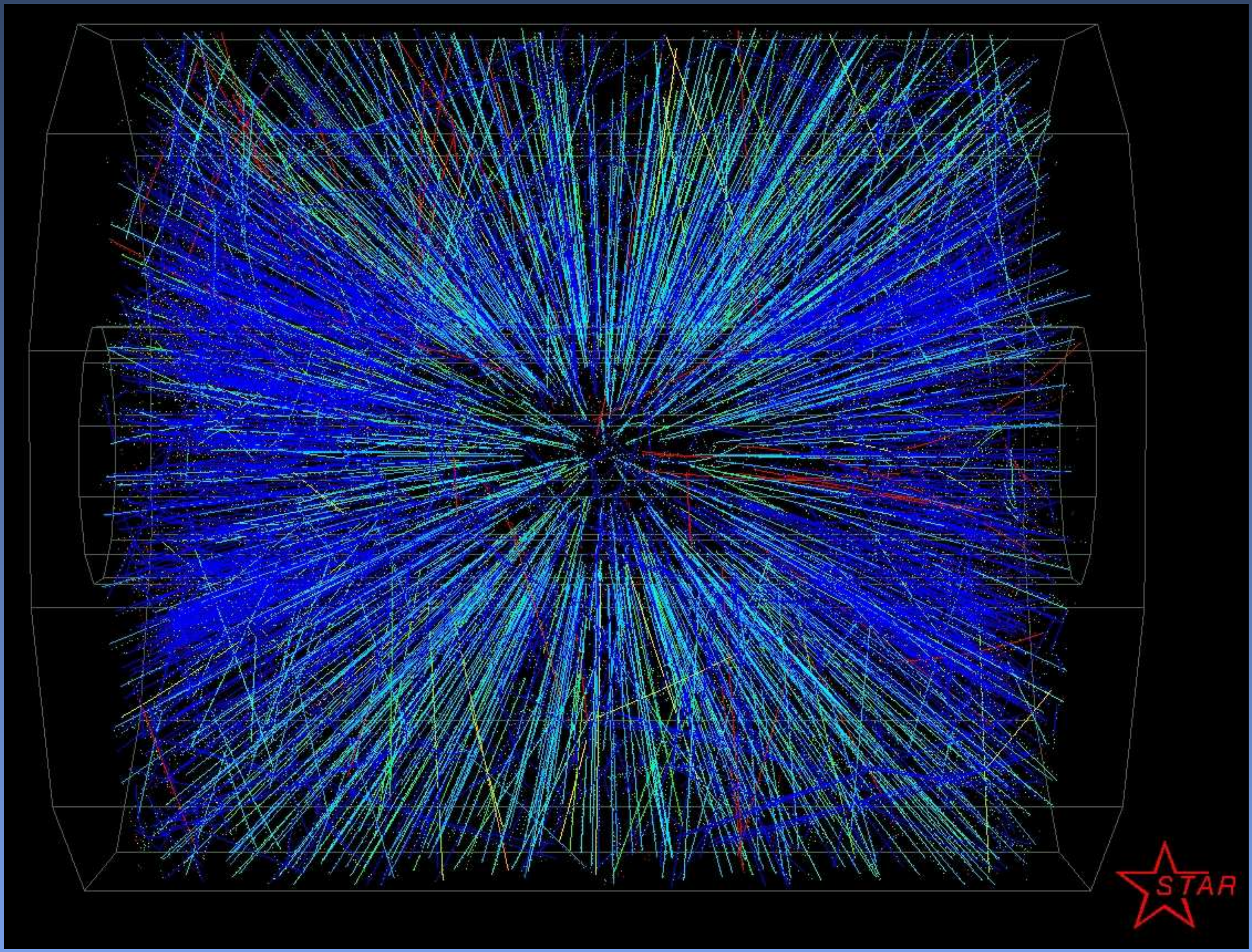
Bragg Curves

Bragg curves of heavy ions for medical applications

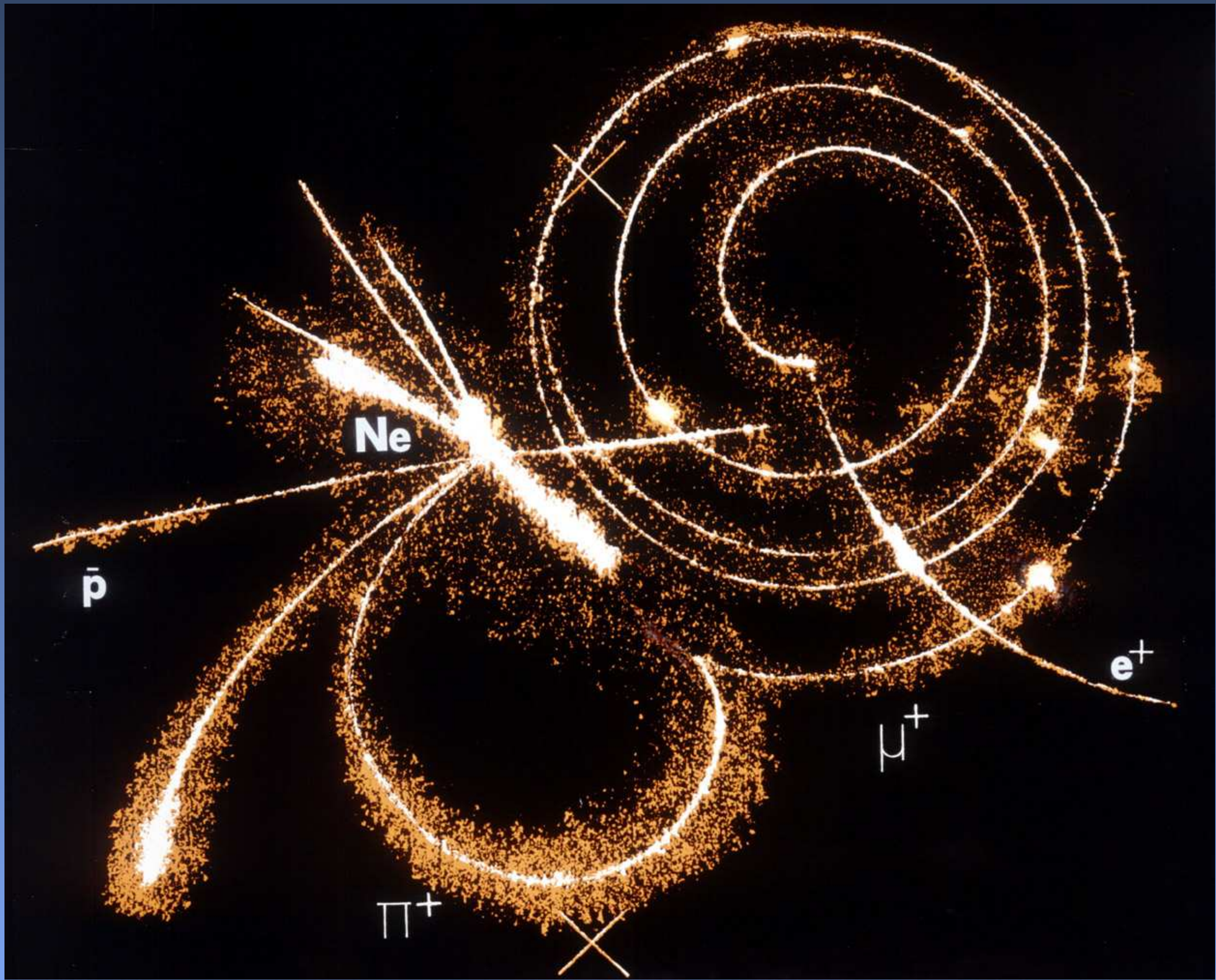
Kraft 1996
GSI Darmstadt



Heavy Ion Collision in STAR



$\pi \rightarrow \mu \rightarrow e$ decay chain



Landau Distributions (1)

Energy transfer probability: $\phi(\varepsilon) = \underbrace{\frac{2\pi N e^4}{m_e v^2} \cdot \frac{Z}{A}}_{\xi/x} \cdot \frac{1}{\varepsilon^2}$ for $z = 1$

with x: area density in $\mathrm{g/cm^2}$ ($x =$ density $\times$ length).
For $1\ \mathrm{cm\ Ar}$ and $\beta = 1 \Rightarrow \xi = 0.123\ \mathrm{keV}$.

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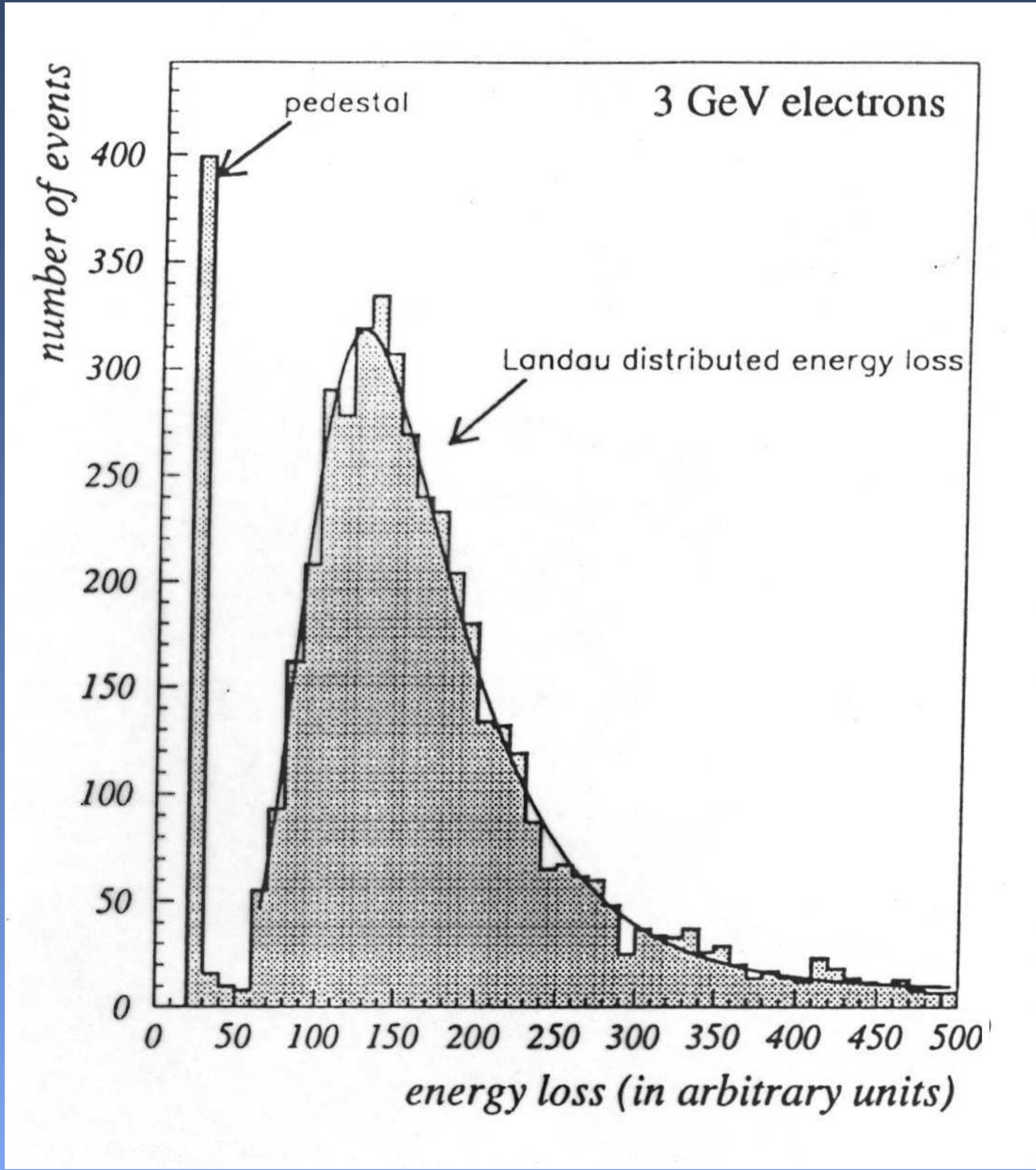
- This distribution is asymmetric due to close collisions with high energy transfers.
- Particularly important for gases and thin absorbers.
- In argon $(\beta\gamma = 4)$; $\Delta^{\mathrm{m.p.}} = 1.2\mathrm{keV/cm}$; $\langle\Delta\rangle = 2.69\ \mathrm{keV/cm}$.

Landau Distributions (2)

Electrons in Ar/CH_4 (80 : 20), gap: 0.5 cm

Affholderbach et al. 1996

NIM A 410 (1998) 166

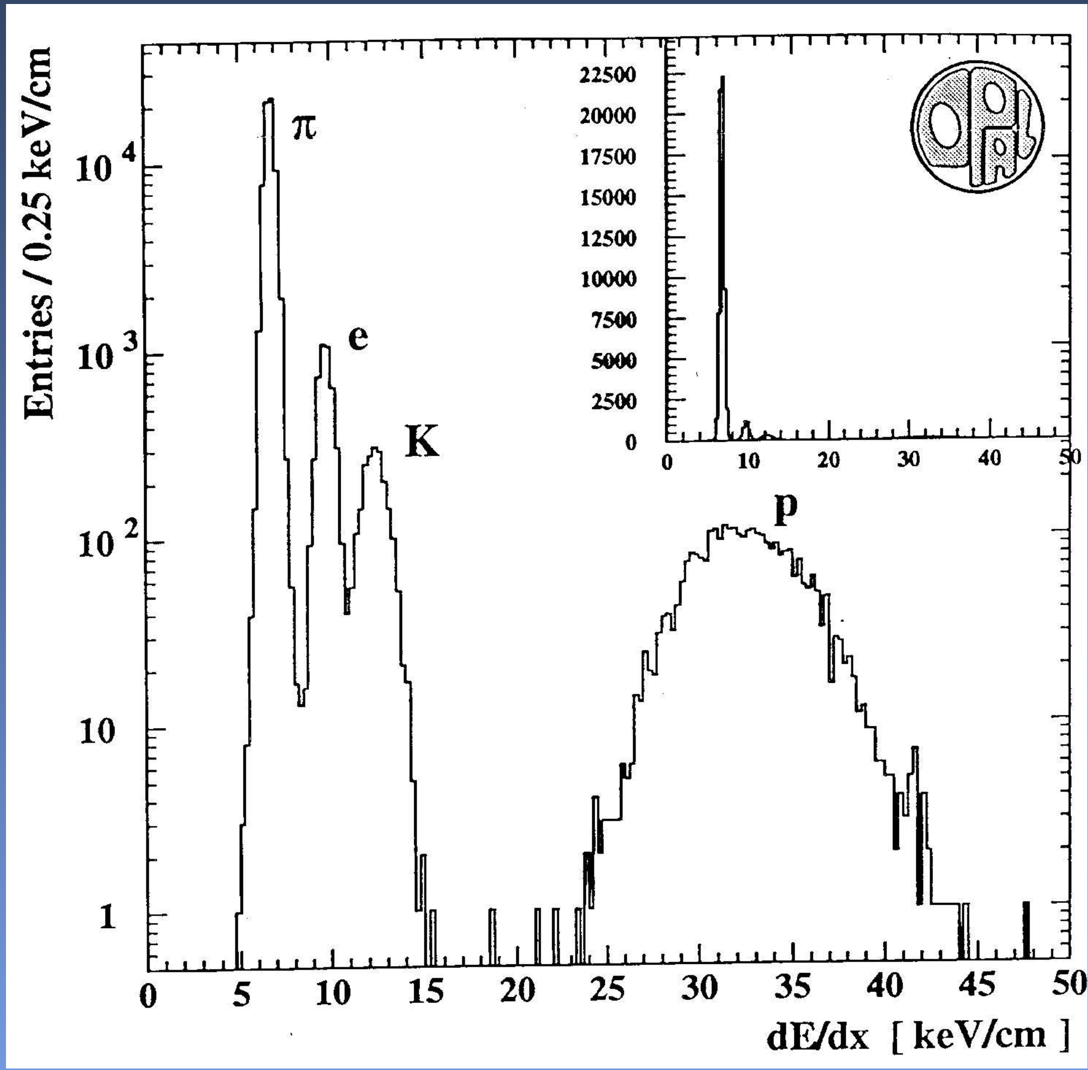


Landau Distributions (3)

OPAL detector at LEP/CERN

Momentum:
$\langle p \rangle = 0.465\ \mathrm{GeV/c}$

CERN-PPE 94-49



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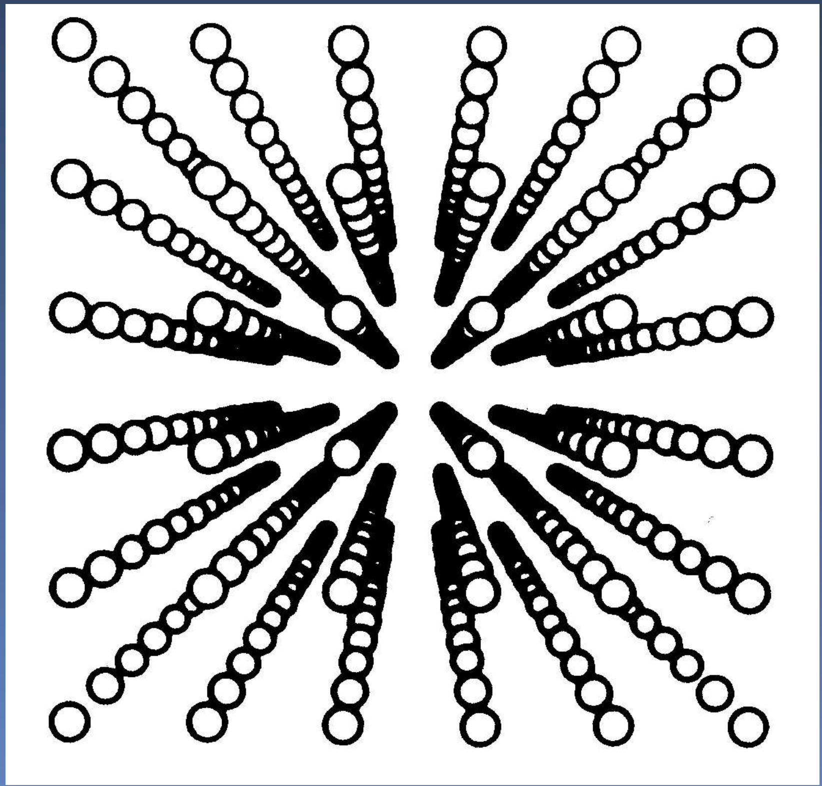
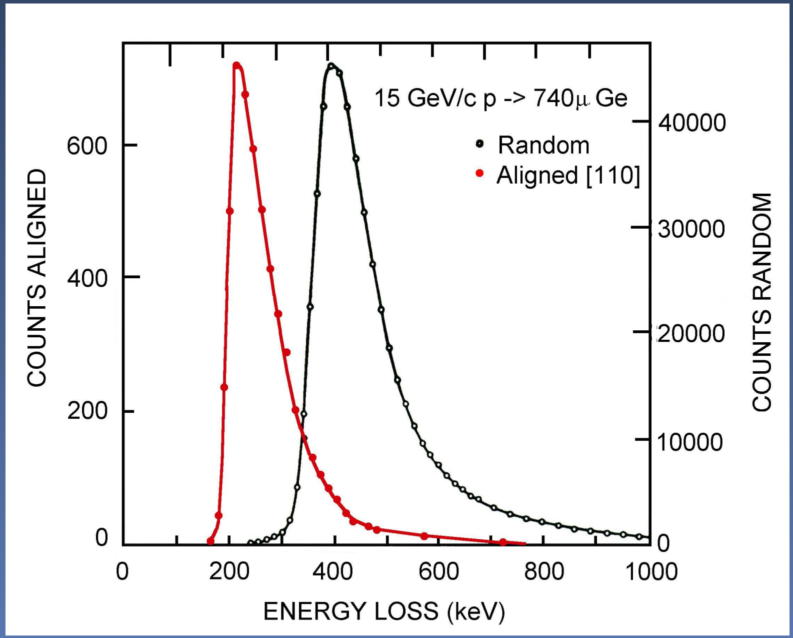
- Application: beam steering with bent crystals.

Channeling (2)





S. P. Möller CERN 94-05 (1994)

Scintillation

- **Inorganic crystals**:
  Effect of the lattice, electron-hole pair creation, excitation, de-excitation at activator centers.
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$*$: $C_{24}H_{16}N_2O_2$: 1.4-Bis-(2-[5-phenyloxa-zolile])-benzene

$\#$: $C_{27}H_{19}NO$: 2.5-di(4-biphenyl)-oxasole

$+$: $C_5H_8O_2$: PMMA-polymethylmethacralate

Birks formula

- Birk's formula for organic scintillators:
  light yield $N = N_0 \cdot \frac{\mathrm{d}E/\mathrm{d}x}{1+k_B \cdot \mathrm{d}E/\mathrm{d}x}$.

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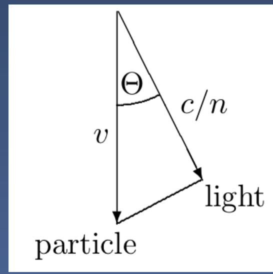
- Anti-correlation between ionisation and excitation (scintillation).

Adiabatic Light Guide

Cherenkov Radiation

Velocity of the particle: v.
Velocity of light in a medium of refractive index n : c/n.
threshold condition:



$$v_{\text{thresh}} \geqslant c/n \;\Rightarrow\; \beta_{\text{thresh}} = \frac{v_{\text{thresh}}}{c} \geqslant \frac{1}{n}.$$

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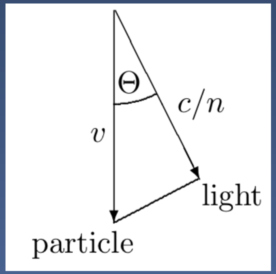

$$v_{\text{thresh}} \geqslant c/n \;\Rightarrow\; \beta_{\text{thresh}} = \frac{v_{\text{thresh}}}{c} \geqslant \frac{1}{n}.$$

- $\cos\Theta_C = \frac{1}{n\beta}$,
  $\beta = 1: \quad \Theta_C^{\max} = \arccos\frac{1}{n} = 42^\circ$ in water,
  $E_{\text{thresh}} = \gamma_{\text{thresh}} \cdot m_0c^2; \quad \gamma_{\text{thresh}} = \frac{1}{\sqrt{1-\beta^2_{\text{thresh}}}} = \frac{n}{\sqrt{n^2-1}}.$
- number of Cherenkov photons per unit path length:

$$\frac{\mathrm{d}N}{\mathrm{d}x} = 2\pi\alpha z^2 \cdot \int \left(1 - \frac{1}{n^2\beta^2}\right)\frac{\mathrm{d}\lambda}{\lambda^2} = 2\pi\alpha z^2 \frac{\lambda_2 - \lambda_1}{\lambda_1 \cdot \lambda_2} \sin^2\Theta_C$$

$$= 490 z^2 \sin^2\Theta_C\ [\mathrm{cm}^{-1}]$$

$$\approx 210\ \mathrm{cm}^{-1} \text{ in water for } z = 1,\ \beta = 1$$

Cherenkov Counters

- Threshold Cherenkov counter

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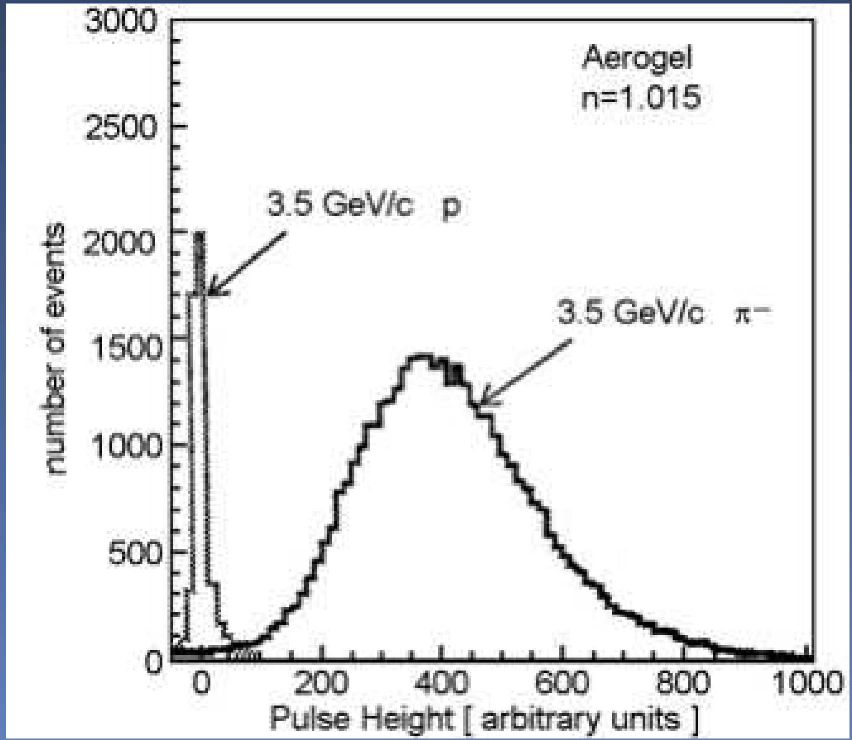
- RICH - Ring Imaging Cherenkov Counter

Threshold *Cherenkov* counter

Pulse height distribution for 3.5 GeV$/c$ pions and protons in an aerogel Cherenkov counter.

BELLE Collaboration
hep-ex/9903045 (1999)

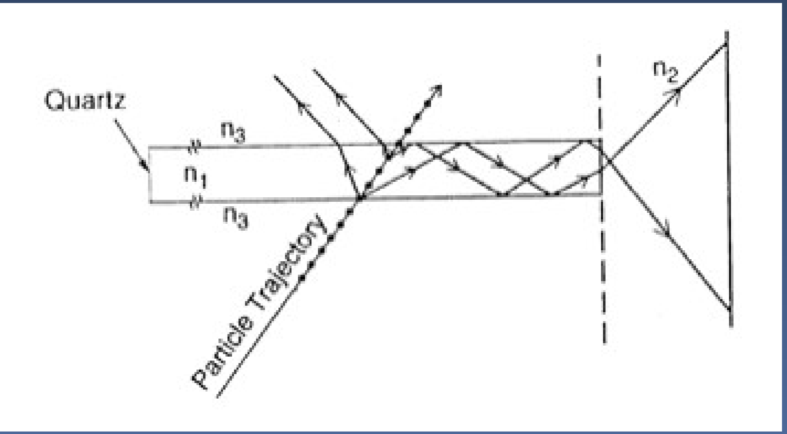


$\gamma_{\text{thresh}} = \frac{n}{\sqrt{n^2-1}} = 5.84$ for aerogel of $n = 1.015$

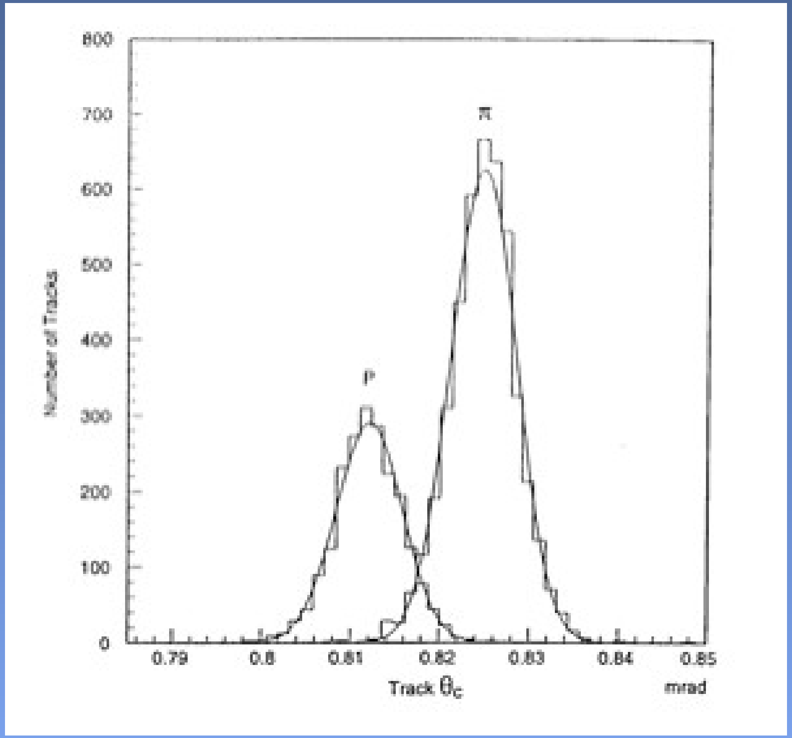
$$p = 3.5\ \text{GeV}/c = \begin{cases} E_\pi = 3.50\text{GeV}; & \gamma_\pi = 25.1 \\ E_p = 3.63\ \text{GeV}; & \gamma_p = 3.86 \end{cases}$$

$\gamma_\pi > \gamma_{\text{thres}};\quad \gamma_p < \gamma_{\text{thres}}.$

DIRC



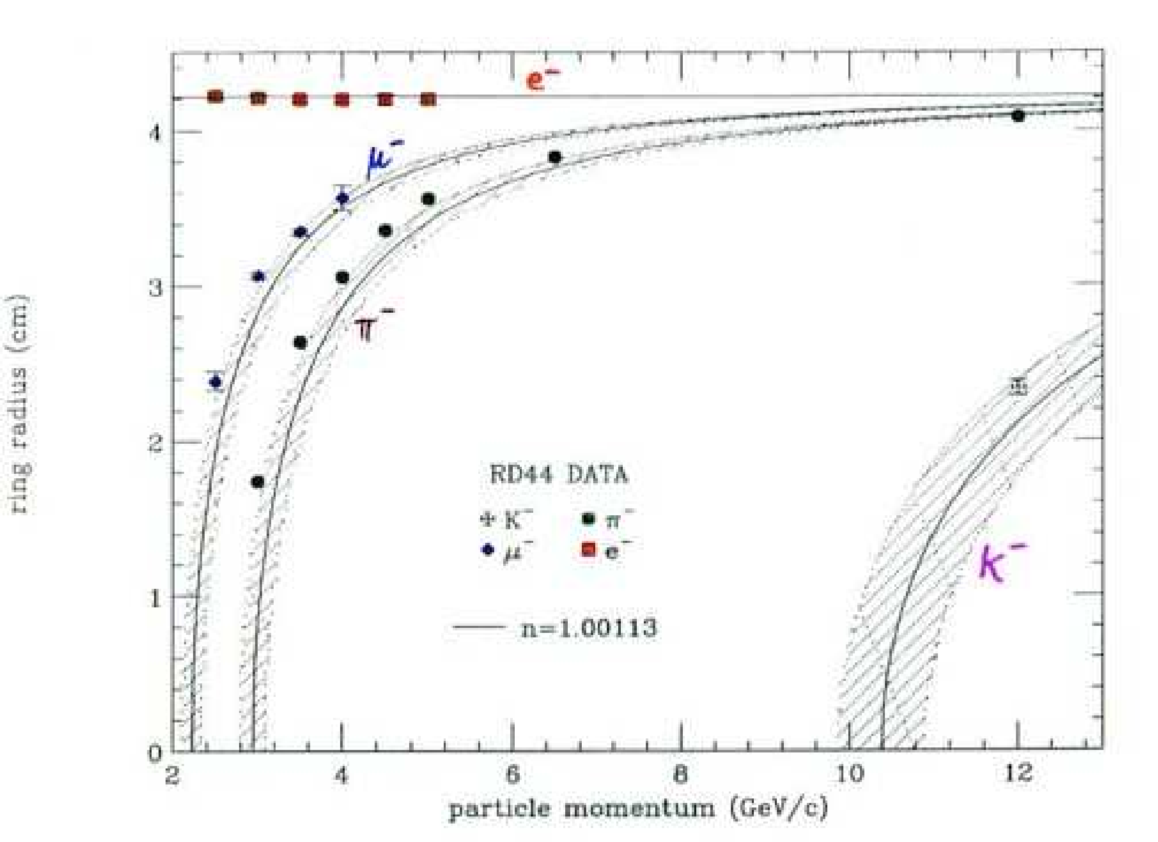
DIRC-counter
$5.4\ \mathrm{GeV}/c$
I. Adam et al. 1997



RICH (1)

RICH
$Ar + C_4F_{10} = 25/75$
100 channel PMT
$10 \times 10\ cm^2$

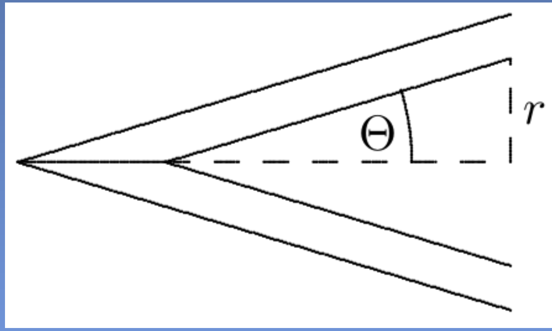
R. Debbe et al.
hep-ex/9503006



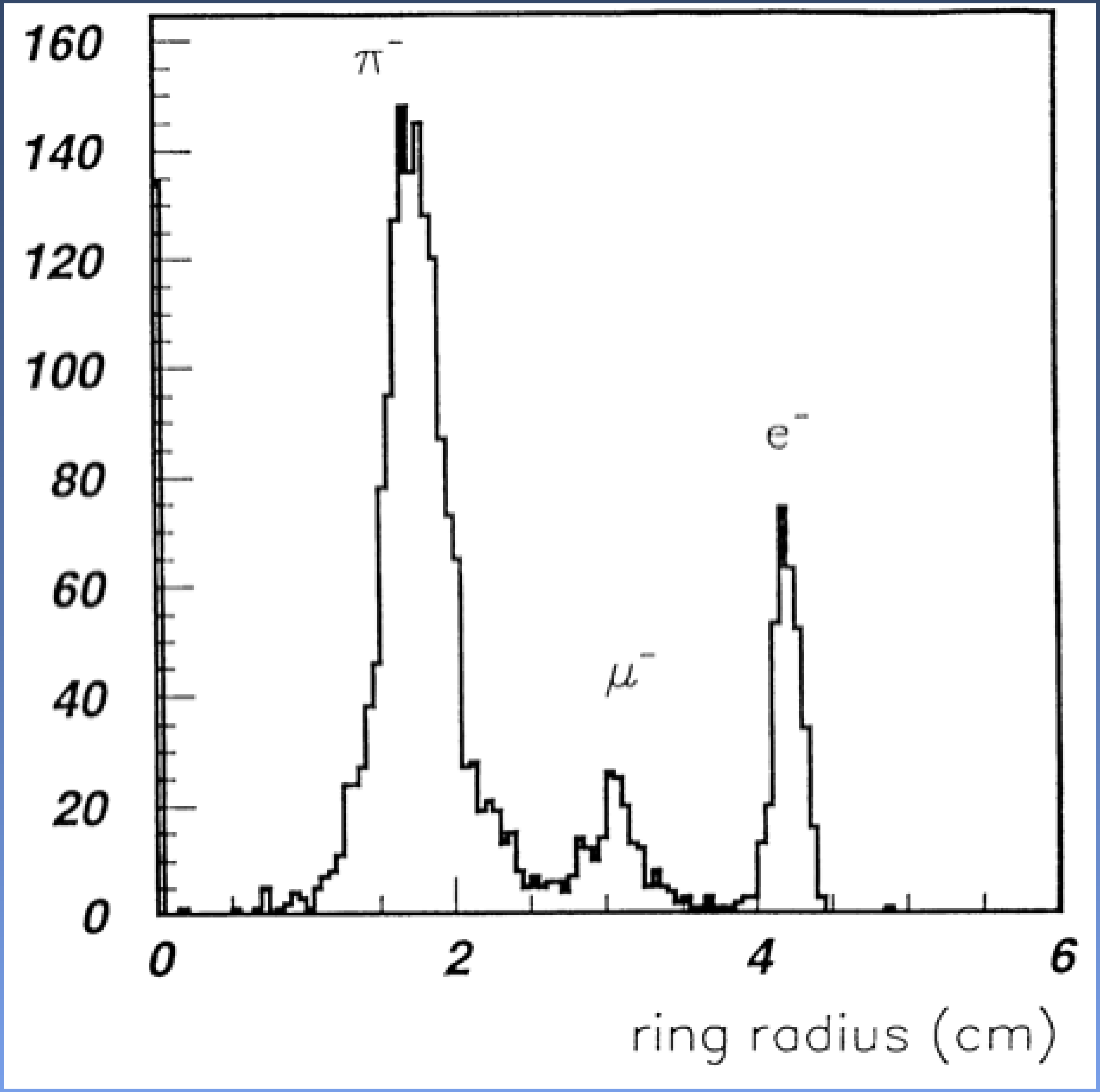
RICH (2)

RICH
$Ar + C_4F_{10} = 25/75$
100 channel PMT
$10 \times 10\ \mathrm{cm}^2$
$3\ \mathrm{GeV}/c$

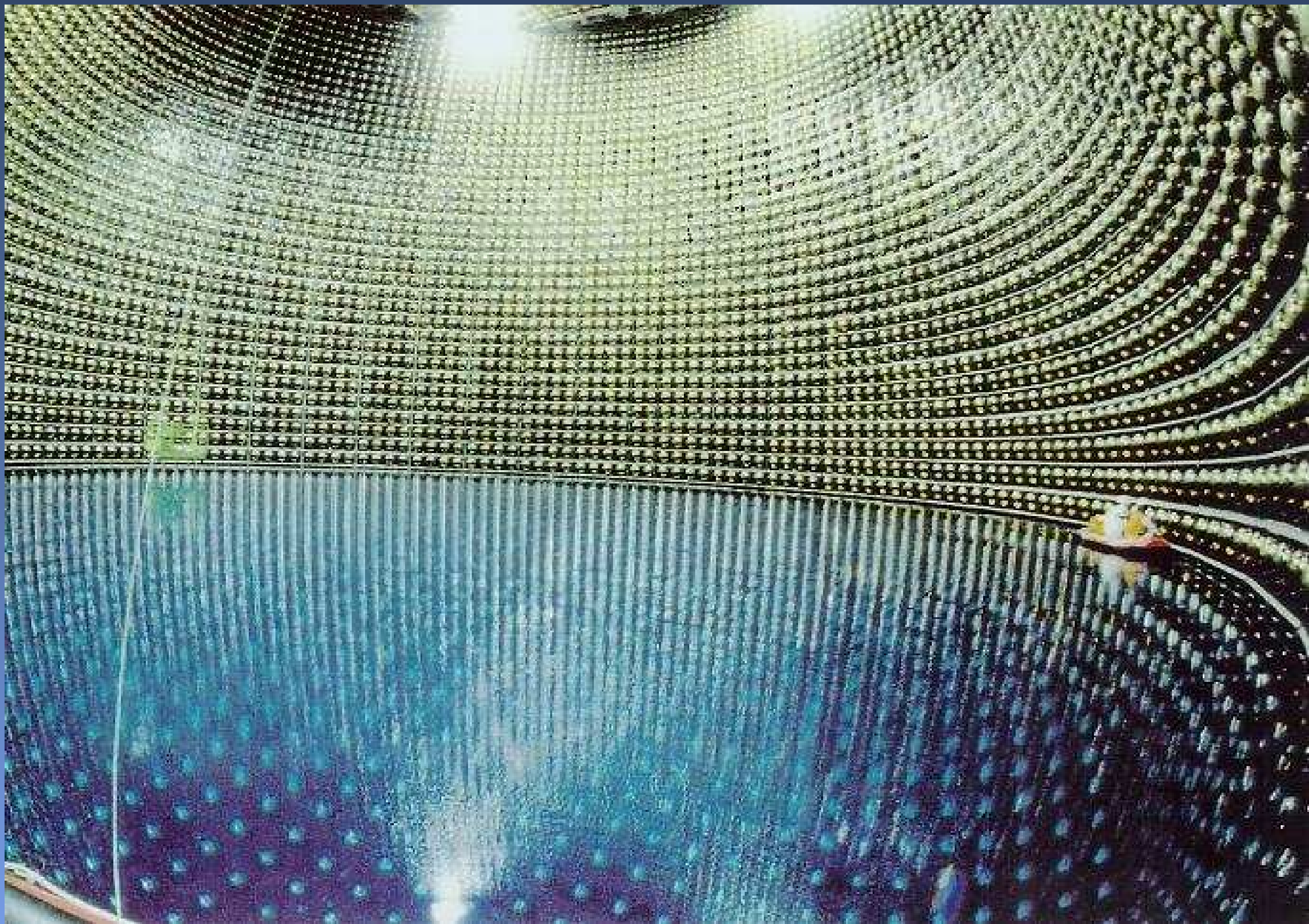
R. Debbe et al.
hep-ex/9503006



$r \sim \sin\Theta$



Super-Kamiokande



Filling the water Cherenkov counter.

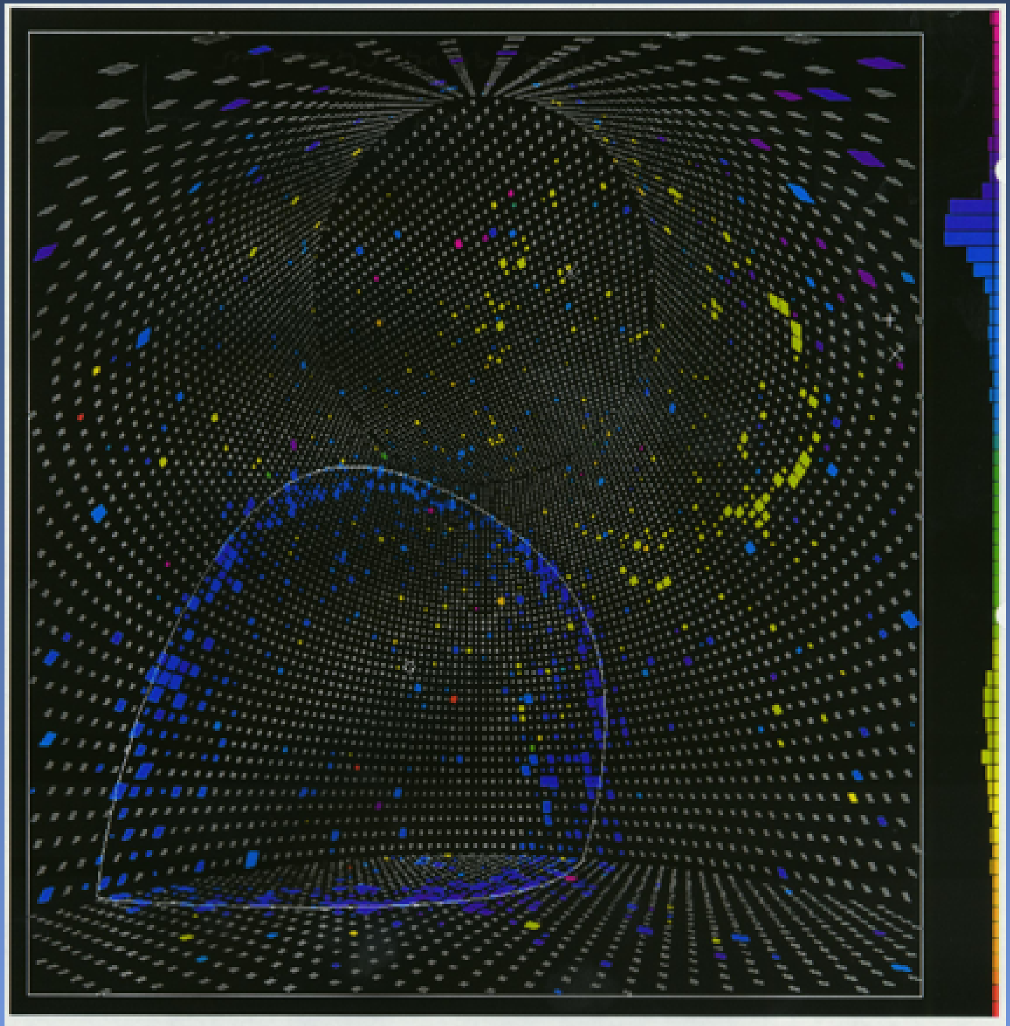
Super-Kamiokande

Event with a stopping muon.

$$\nu_\mu + N \rightarrow \mu^- + X$$

$$\hookrightarrow e^- + \overline{\nu}_e + \nu_\mu$$

$$E_{\nu_\mu} = 481 \text{ MeV}$$
$$E_\mu = 394 \text{ MeV}$$
$$E_e = 52 \text{ MeV}$$



Superkamiokande Photo Gallery

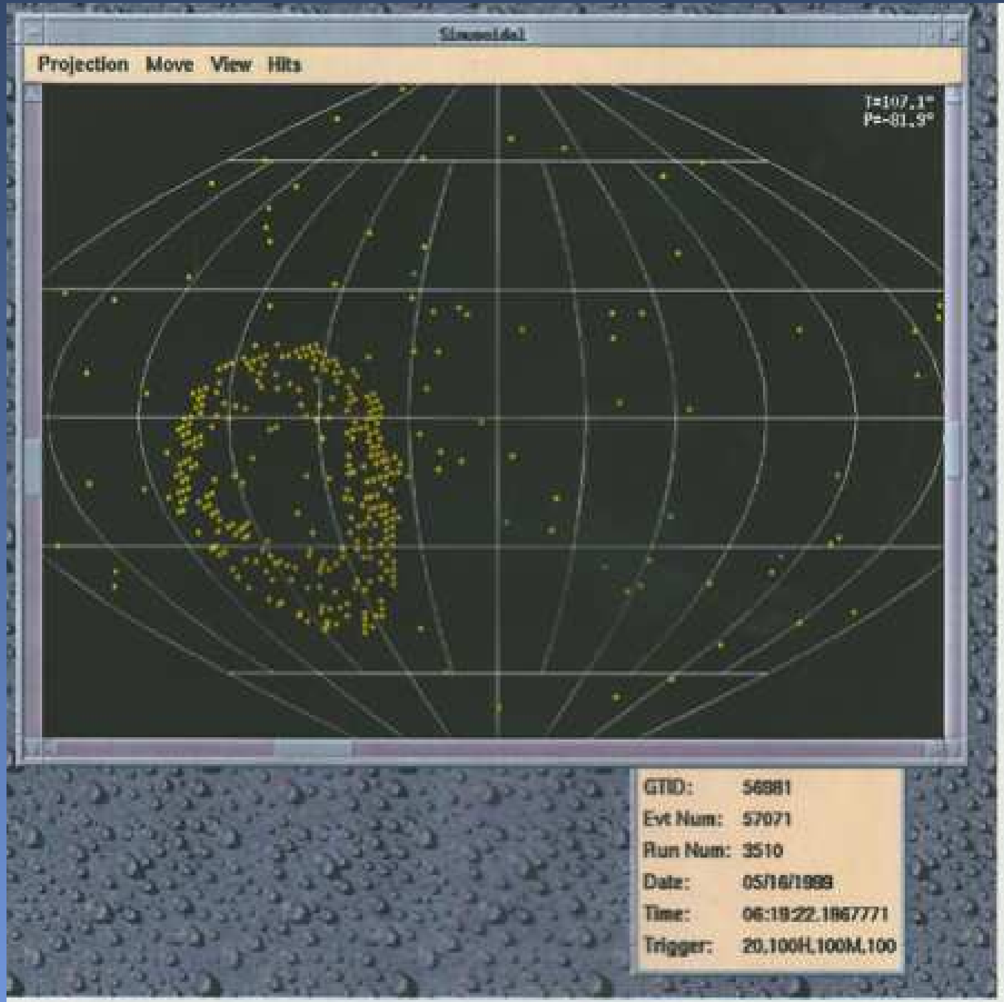
SNO -Sudbury Neutrino Observatory (1)

Event with a stopping muon.

$$\nu_\mu + N \rightarrow \mu^- + X$$

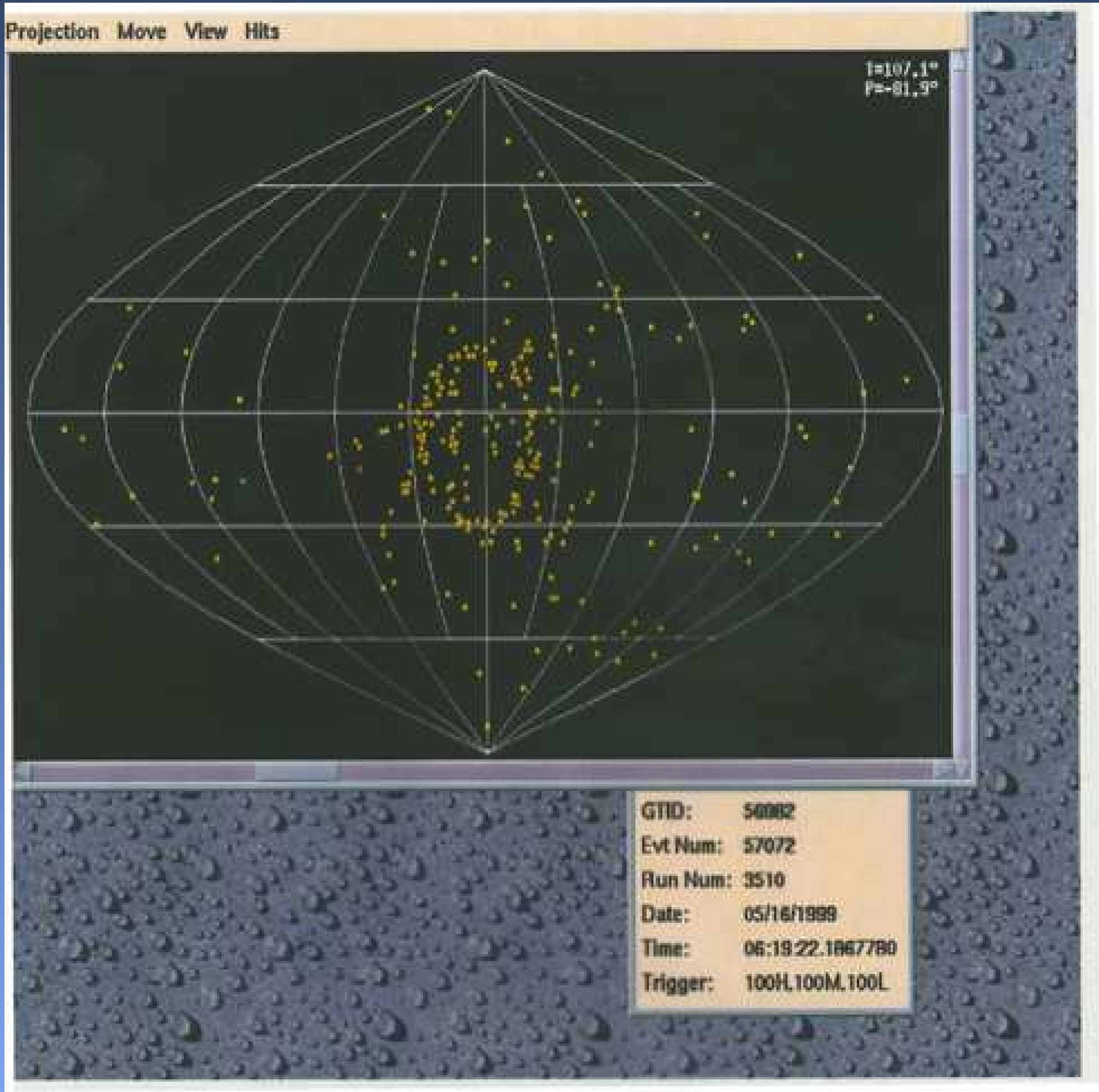
$$\hookrightarrow e^- + \overline{\nu}_e + \nu_\mu$$

two frames taken at
$\Delta t = 0.9\ \mu\text{s}$
time difference



SNO Photo Gallery

SNO -Sudbury Neutrino Observatory (2)



Transition Radiation

Energy radiated from a single boundary:

$$S = \frac{1}{3}\alpha z^2 \hbar\omega_p \gamma \propto \gamma$$

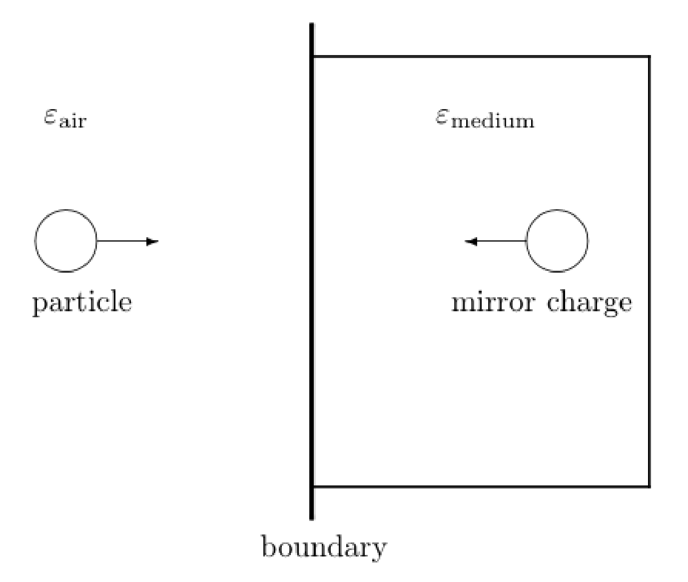
with $\hbar\omega_P$: plasma energy,
$\hbar\omega_P \approx 20\ eV$ for plastic radiators.

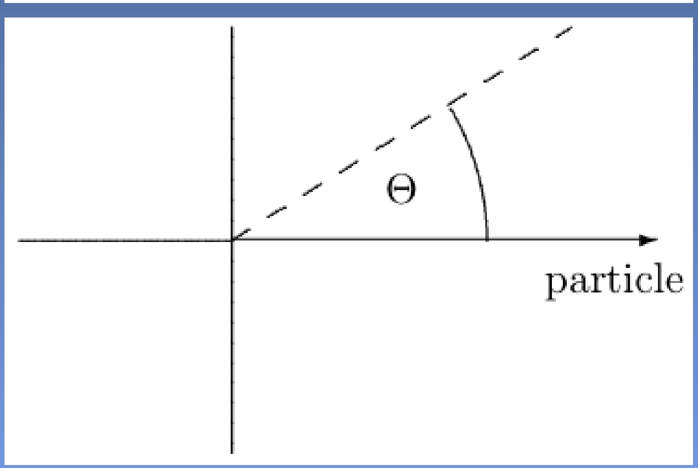
Typical emission angle: $\Theta = \frac{1}{\gamma}$,
energy of radiated photons $\sim \gamma$,
$\rightsquigarrow$ number of radiated photons: αz^2 .

Effective threshold: $\gamma \approx 1000$.

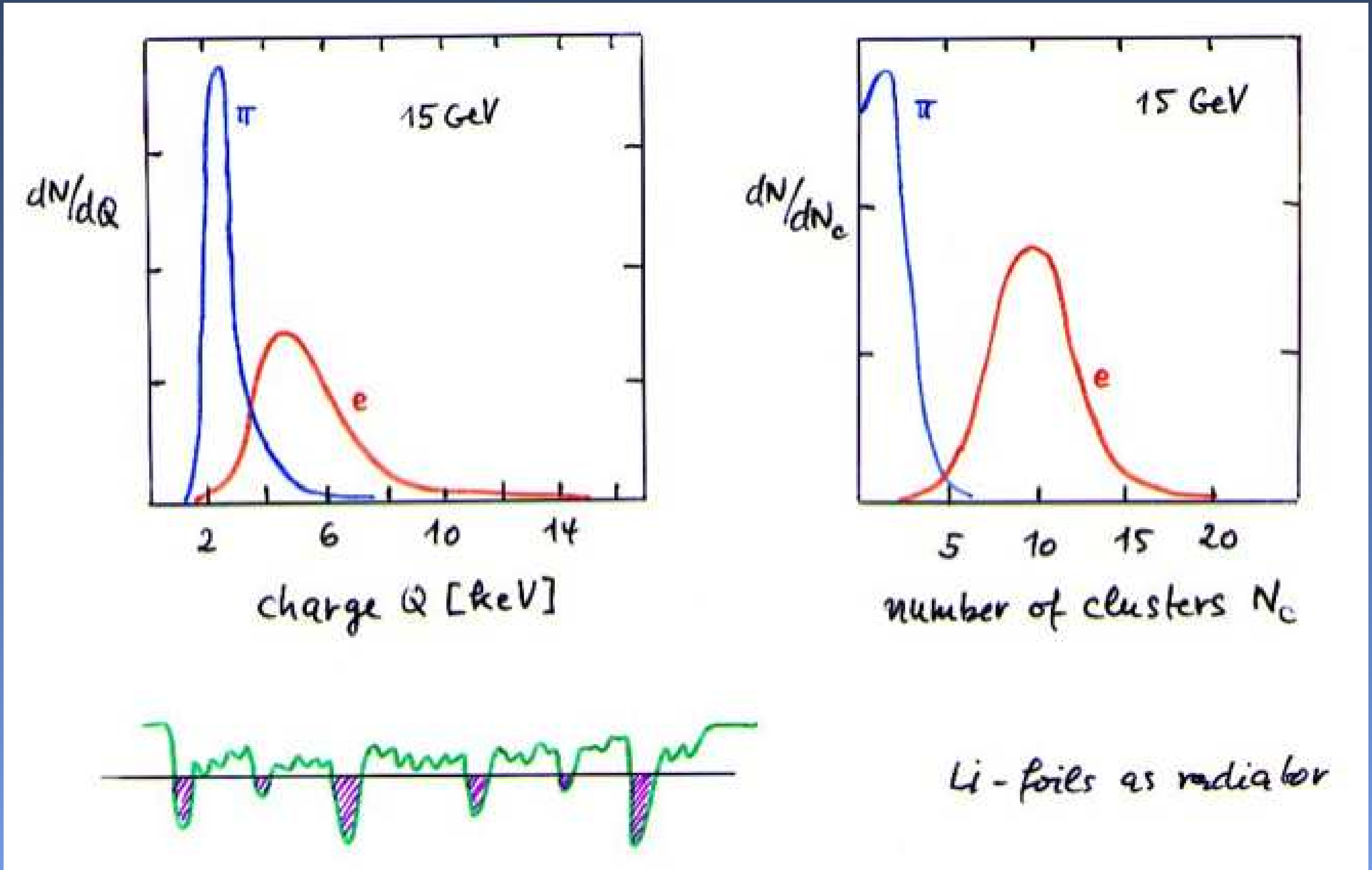
Use stacked assemblies of low Z - material with many transitions.

Detector with high Z gas.



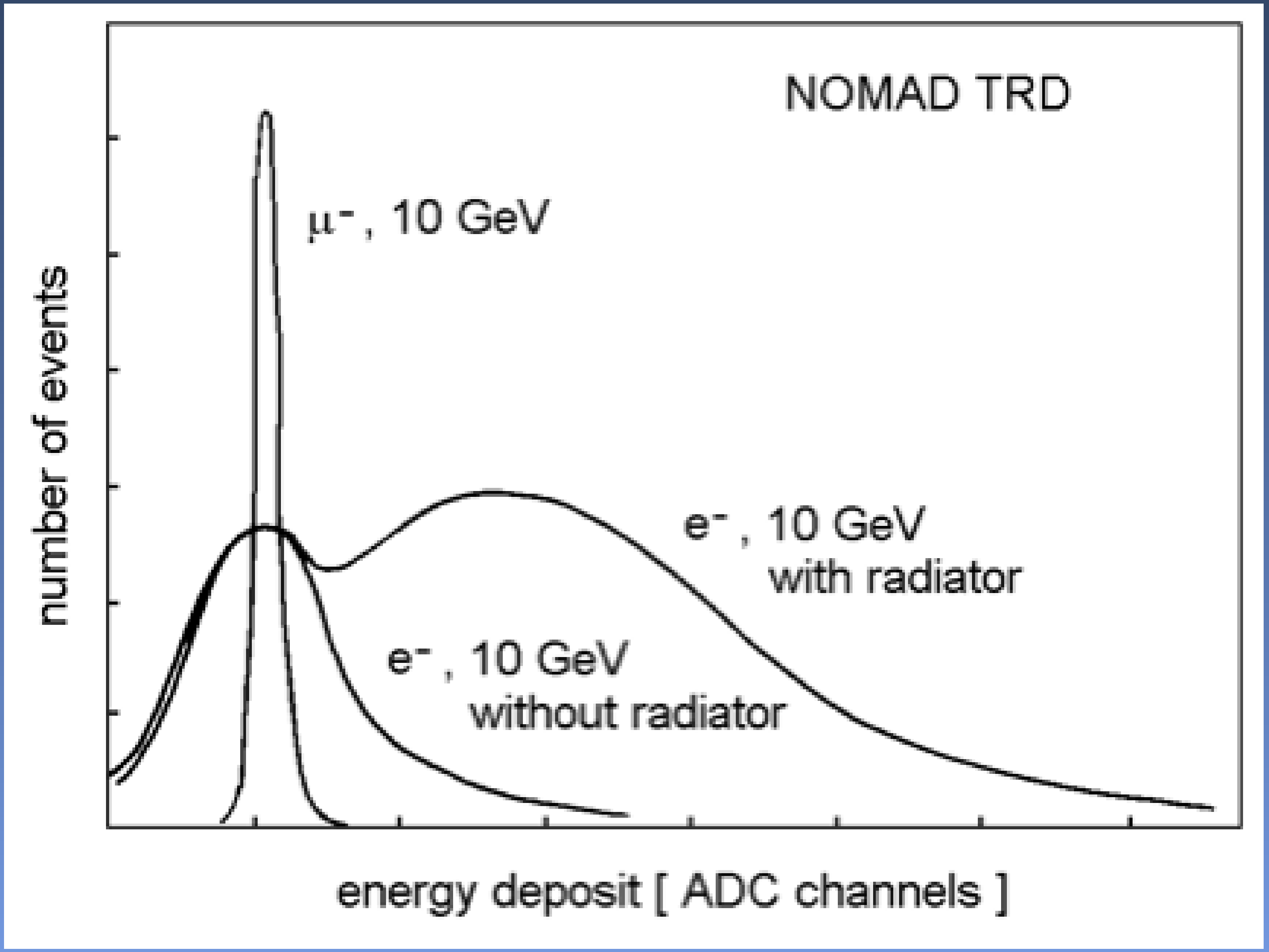


Li-foils as radiator



Fabjan et al. 1980

NOMAD TRD



NOMAD TRD, G. Bassompierre et al., NIM A 403 (1998) 363

test beam performance: e/μ-separation at 10 GeV

Bremsstrahlung (1)

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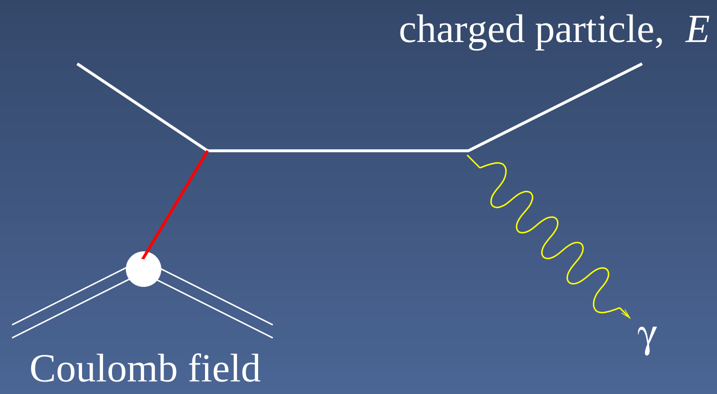

- N: Avogadro number,
- A, Z: target, z: particle,
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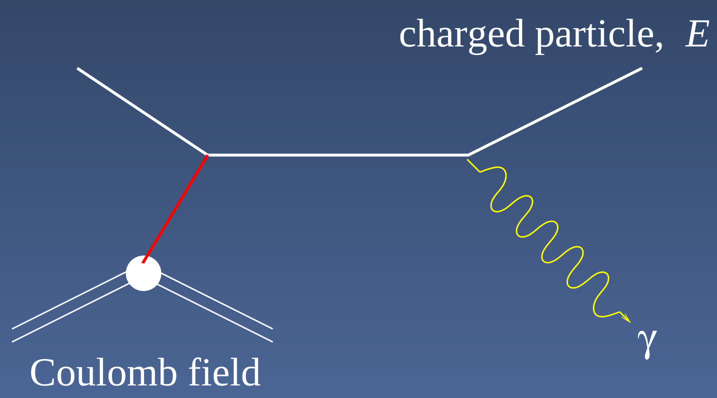

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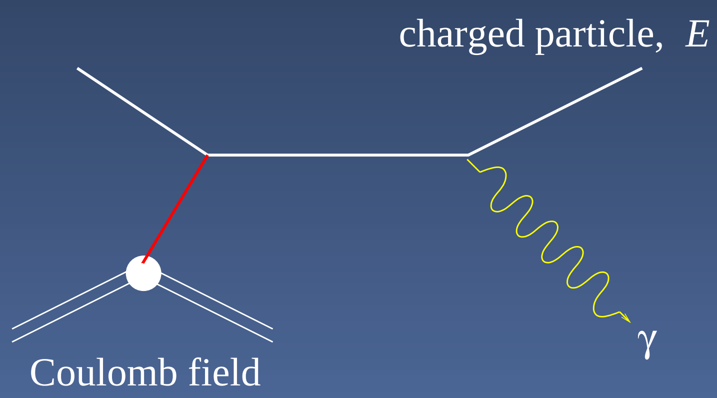

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Usual definition for the critical energy E_c^e:

$$\left(\frac{\mathrm{d}E}{\mathrm{d}x}\right)_\mathrm{ionisation} = \left(\frac{\mathrm{d}E}{\mathrm{d}x}\right)_\mathrm{bremsstrahlung}$$

$$E_\mathrm{c}^e = \begin{cases} \frac{610\ \mathrm{MeV}}{Z+1.24} & \text{for solids and liquids} \\ \frac{710\ \mathrm{MeV}}{Z+0.92} & \text{for gases} \end{cases}$$

Bremsstrahlung (2)

material	$X_0[\mathrm{g/cm^2}]$	$X_0[\mathrm{cm}]$	$E_c[\mathrm{MeV}]$
air	37	30000	84
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$$E_{\mathrm{c}}^{\mu} = E_{\mathrm{c}}^{e} \cdot \left(\frac{m_\mu}{m_e}\right)^2 = 960\ \mathrm{GeV}.$$

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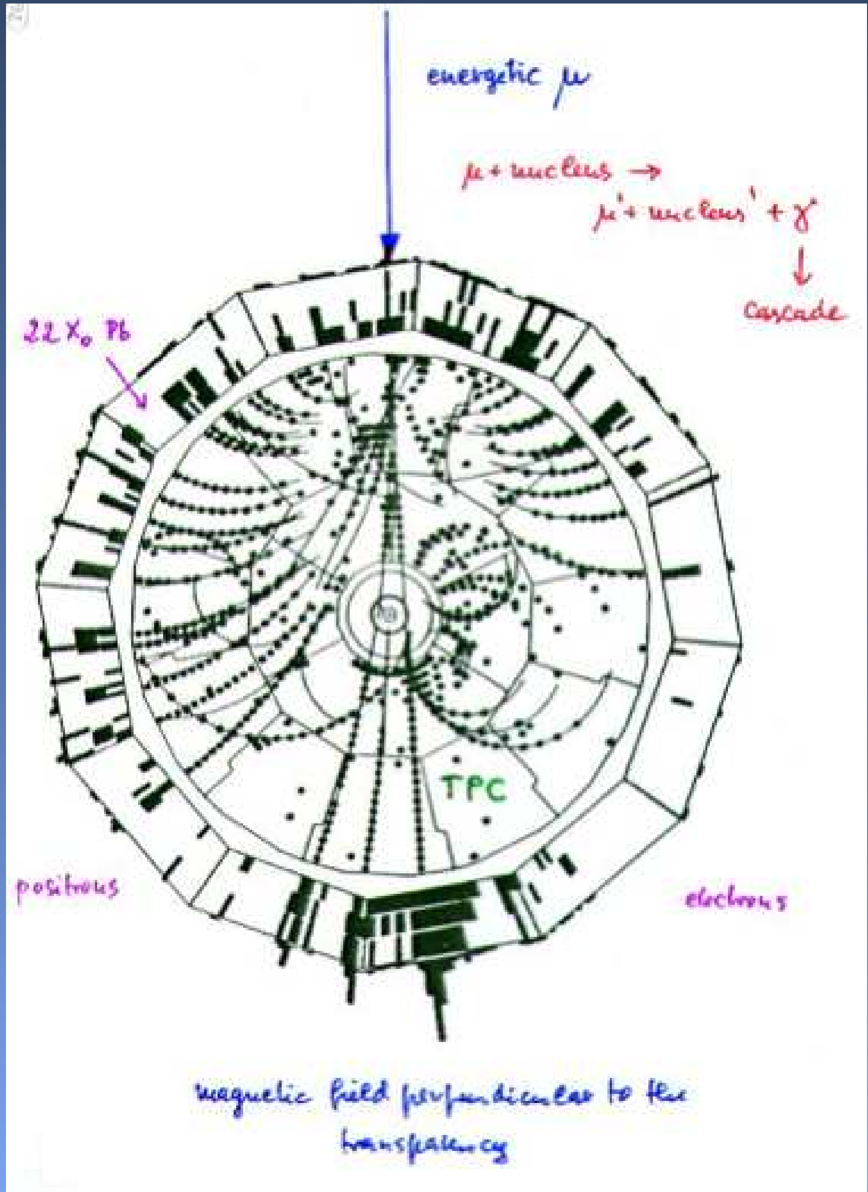
Bremsstrahlung is important for electromagnetic cascades.

Bremsstrahlung (3)

C. Grupen
ALEPH

Magnetic field perpendicular to the transparency.



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Range of muons:

$$R = \int_E^0 \frac{\mathrm{d}E}{-\mathrm{d}E/\mathrm{d}x} = \frac{1}{b} \ln\left(1 + \frac{b}{a}E\right) \begin{cases} 140\ \text{m} & \text{rock for } E = 100\ \text{GeV} \\ 800\ \text{m} & \text{rock for } E = 1\ \text{TeV} \\ 2300\ \text{m} & \text{rock for } E = 10\ \text{TeV} \end{cases}$$

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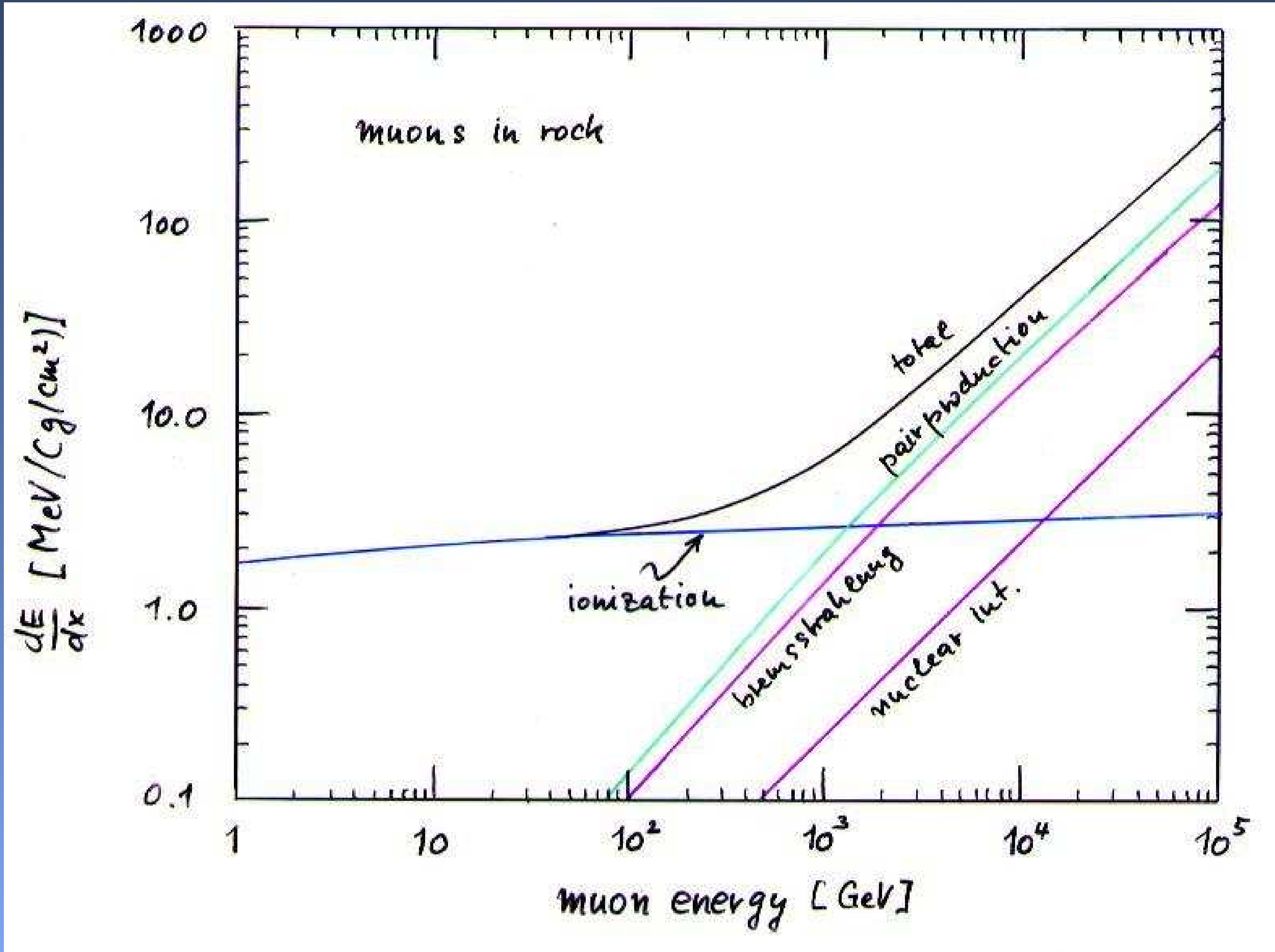

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material	Al	Fe	Pb	air
λ_i/cm	26.2	10.6	10.4	48000
$\lambda_i/\ (\mathrm{g/cm^2})$	70.6	82.8	116.2	62.0

for most materials $\lambda_i, \lambda_a > X_0$.

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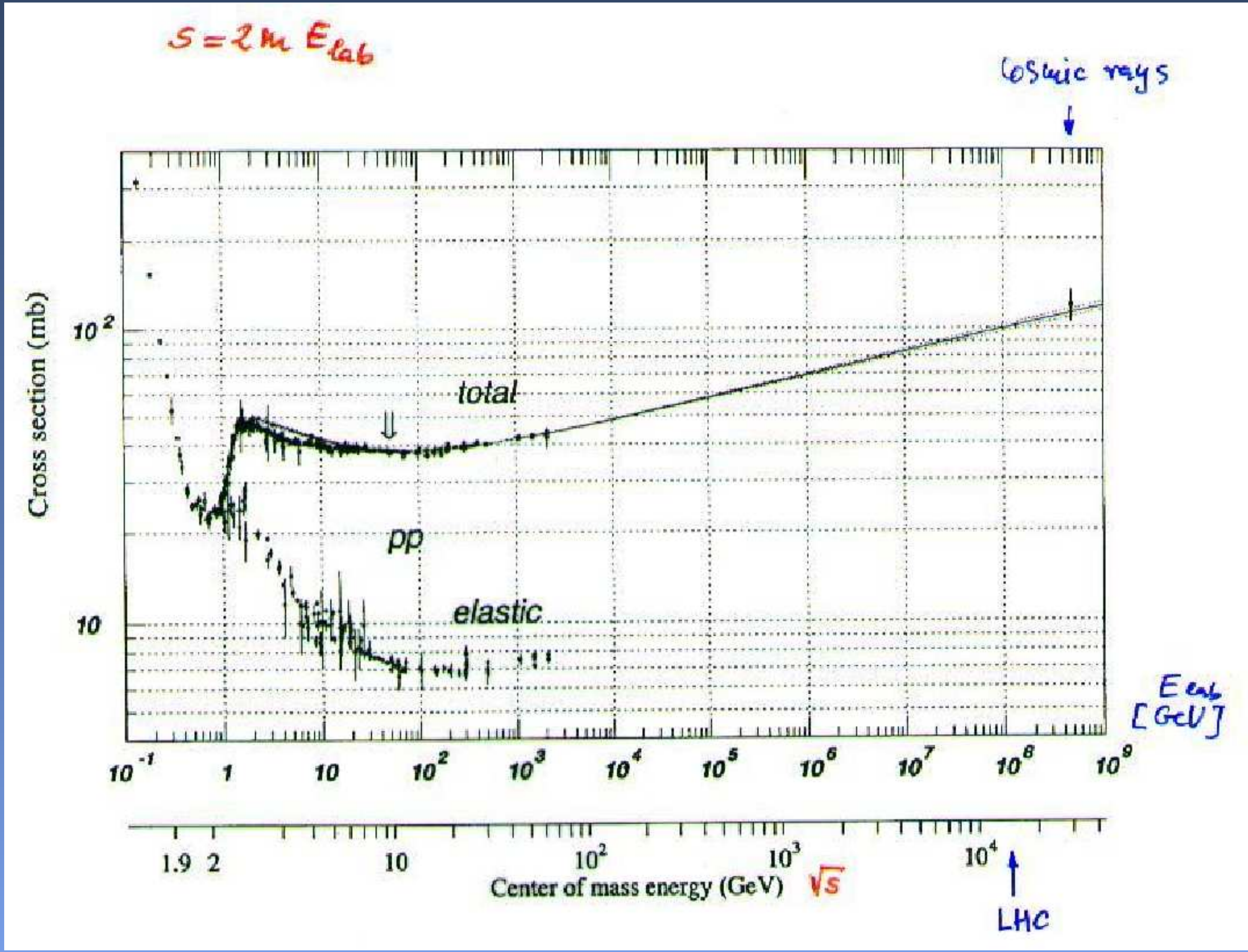

Particle Data Group, Eur. Phys. J. C 15 (2000) 1

Interactions of Photons (1)

$I = I_0 e^{-\mu x}$ with

$\mu = \frac{N}{A} \sum_{i=1}^{3} \sigma_i$

(mass attenuation coefficient).

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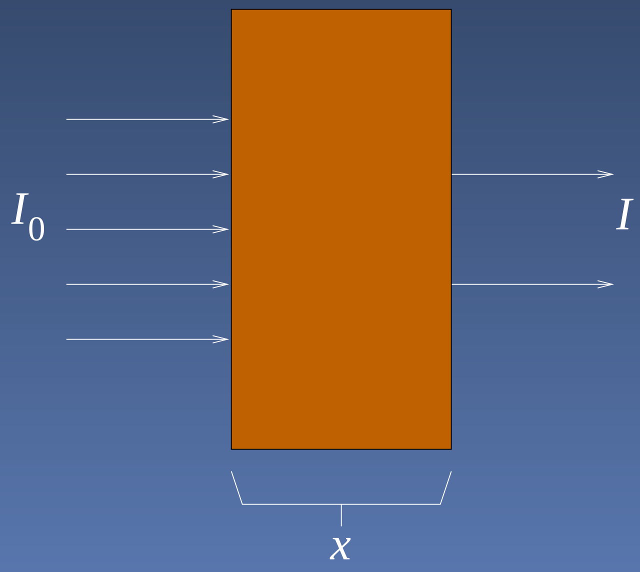

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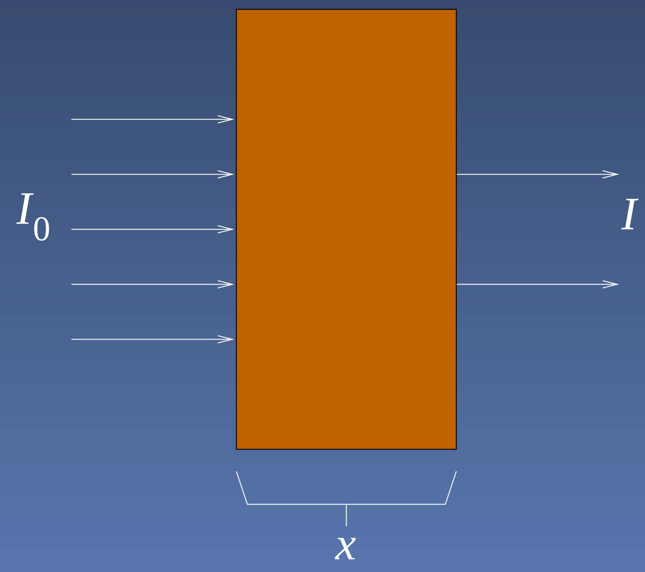

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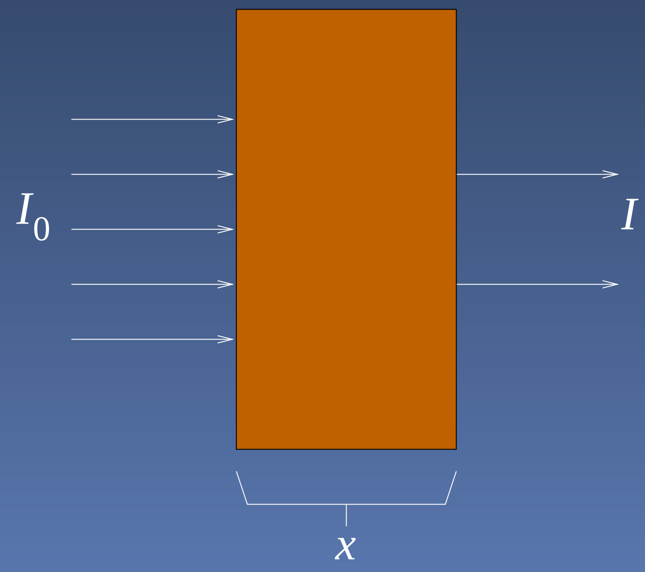

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$$\sigma^{\mathrm{K}}_{\mathrm{Photo}} = \left(\frac{32}{\varepsilon^7}\right)^{1/2} \alpha^4 Z^5 \sigma_{\mathrm{Thomson}} [\mathrm{cm}^2/\mathrm{atom}]; \quad \varepsilon = \frac{E_\gamma}{m_e c^2},$$

$$\sigma_{\mathrm{Thomson}} = \frac{8}{3}\pi r_e^2 = 665 \text{ mb}.$$

For high energies: $\sigma^{\mathrm{K}}_{\mathrm{Photo}} = 4\pi r_e^2 Z^5 \alpha^4 \cdot \frac{1}{\varepsilon}$.

Interactions of Photons (2)

Compton Scattering:

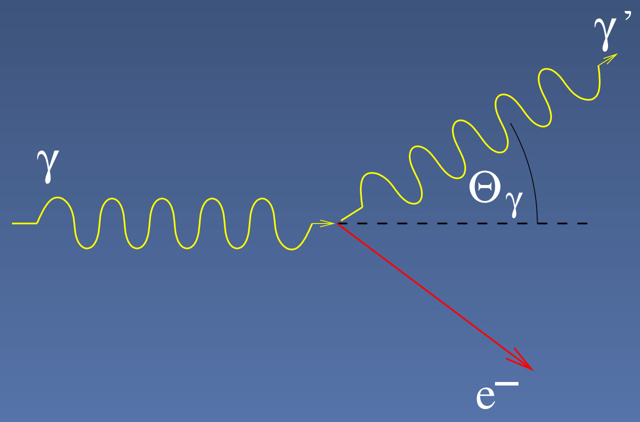
$\sigma_{\mathrm{C}} \propto \frac{\ln \varepsilon}{\varepsilon} \cdot Z$

The photon counts the number of electrons in the atom:

$$\frac{E'_\gamma}{E_\gamma} = \frac{1}{1+\varepsilon(1-\cos\Theta_\gamma)}.$$

Maximum energy transfer for backscattering ($\Theta_\gamma = \pi$):

$$E^{\mathrm{kin}}_{\mathrm{max}} = \frac{2\varepsilon^2}{1+2\varepsilon} m_e c^2 \xrightarrow[\varepsilon>>1]{} E_\gamma.$$



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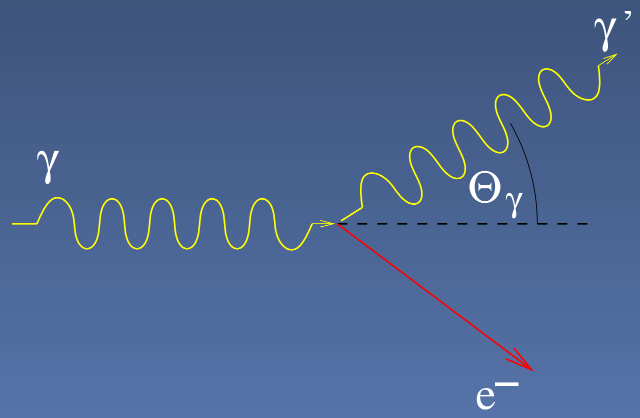

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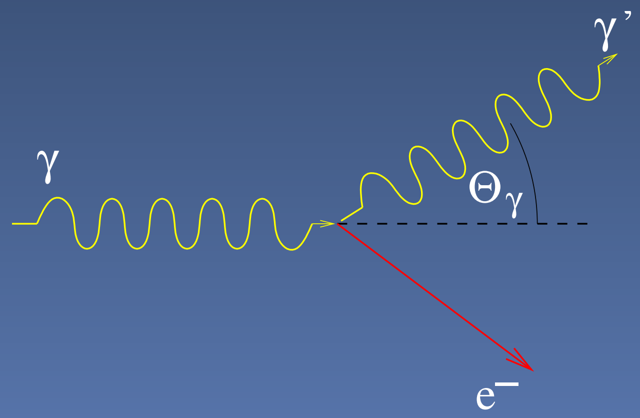

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$\Rightarrow$ Distinction between mass attenuation coefficient (relates to σ_{C})

and mass absorption coefficient (relates to σ_{CA})

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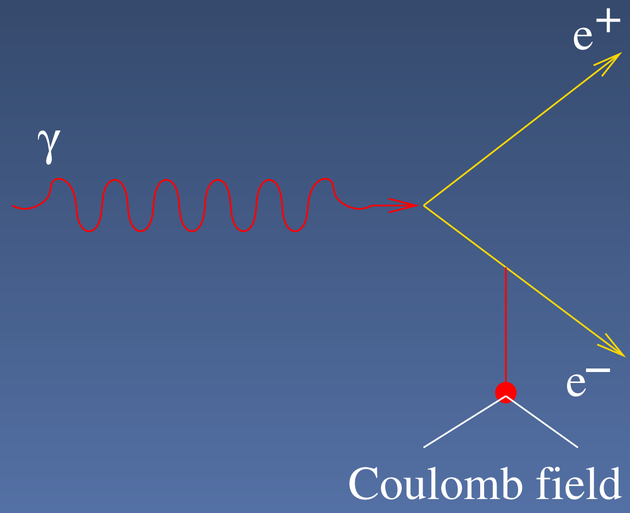
Pair Production:

$$\gamma + \text{nucleus} \rightarrow \text{nucleus}' + e^+ + e^-$$



Threshold energy:

$$E_\gamma = 2m_ec^2 + \frac{2m_e^2c^2}{m_{\text{target}}}$$

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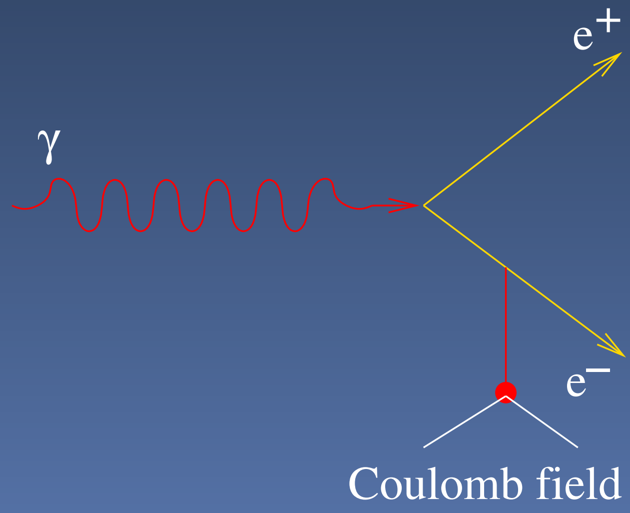

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For $\varepsilon >> \frac{1}{\alpha Z^{1/3}}$ i.e. $E_\gamma >> 20$ MeV (complete screening):

$$\sigma_{\text{pair}} = 4\alpha r_e^2 Z^2 \left(\frac{7}{9} \ln \frac{183}{Z^{1/3}} - \frac{1}{54}\right) \; [\text{cm}^2/\text{atom}] \approx \frac{7}{9}\frac{A}{N} \cdot \frac{1}{X_0}.$$

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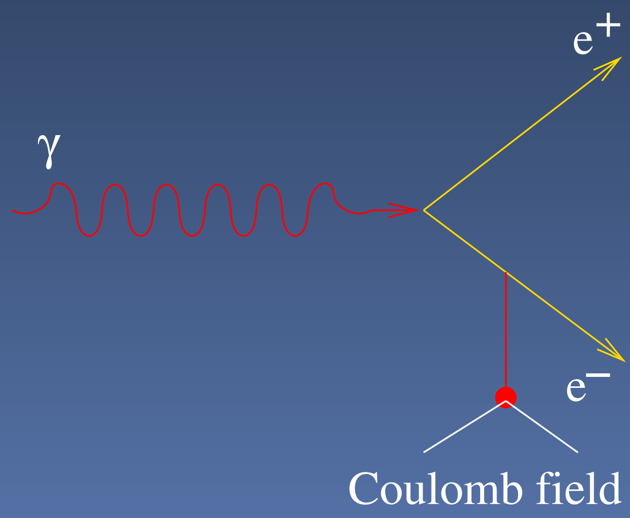

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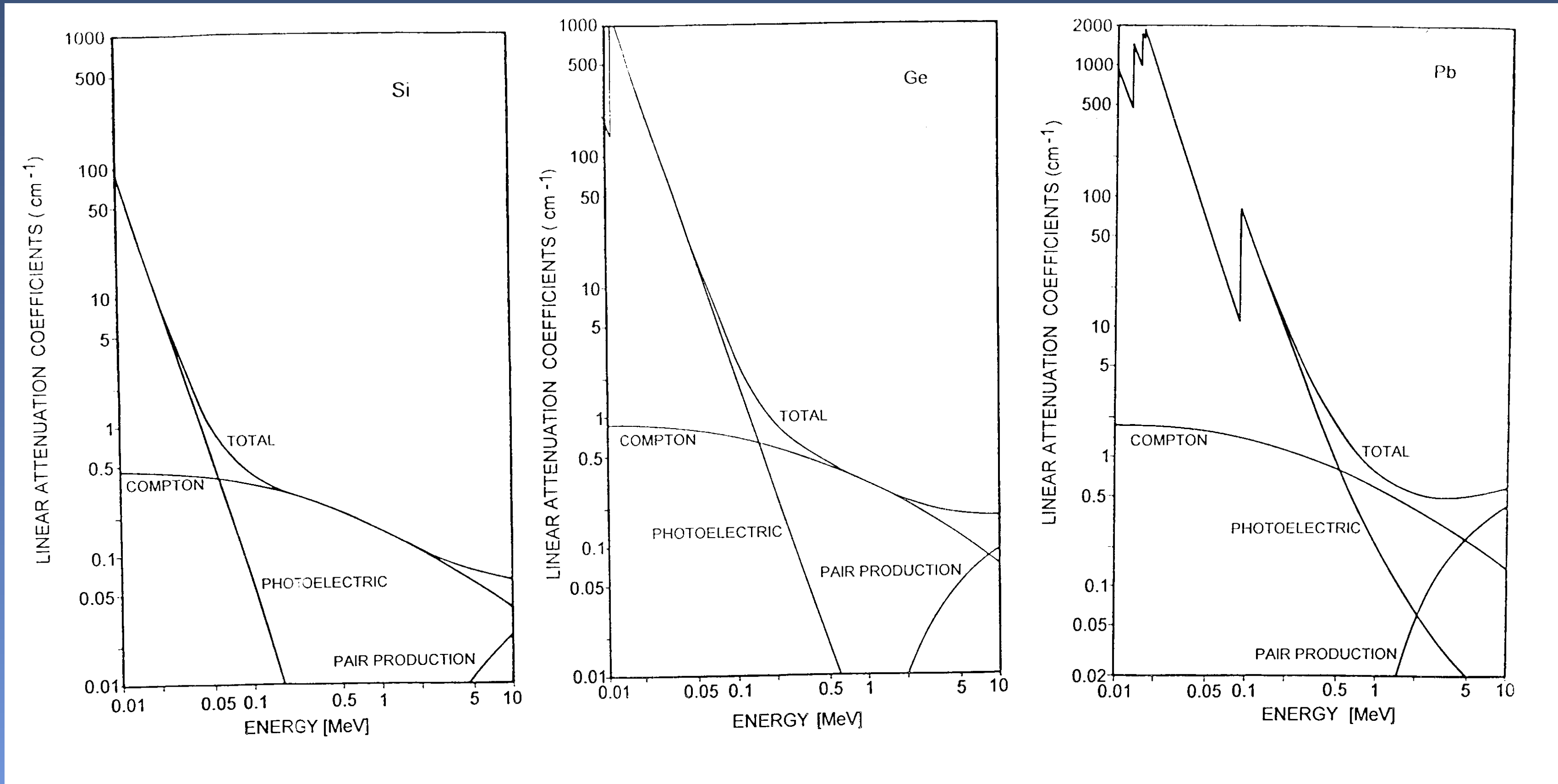
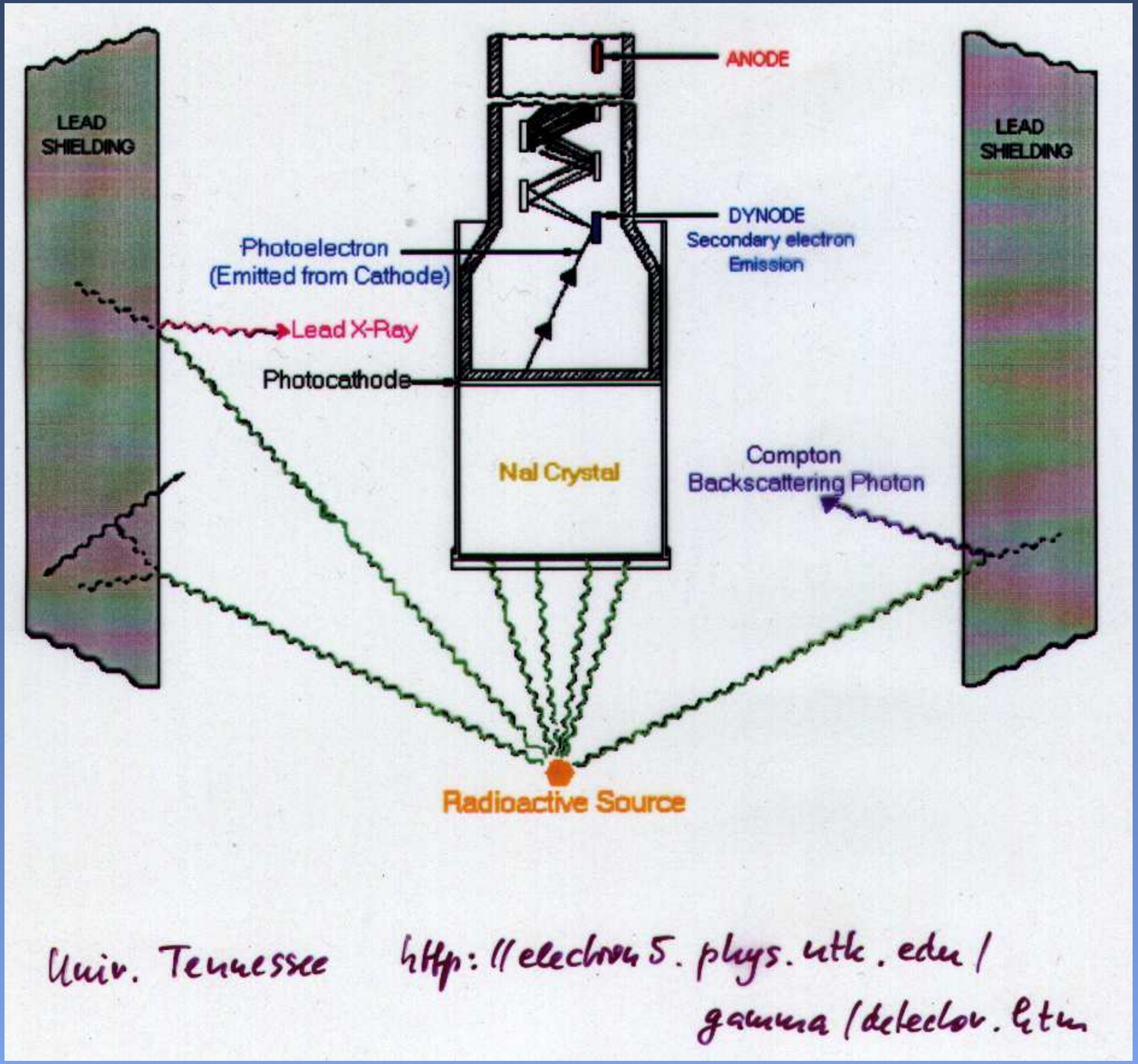
At high energies ($E_\gamma > 1$ GeV) asymmetric energy sharing between e^+ and e^-, important for electromagnetic cascades.

Interactions of Photons (4)

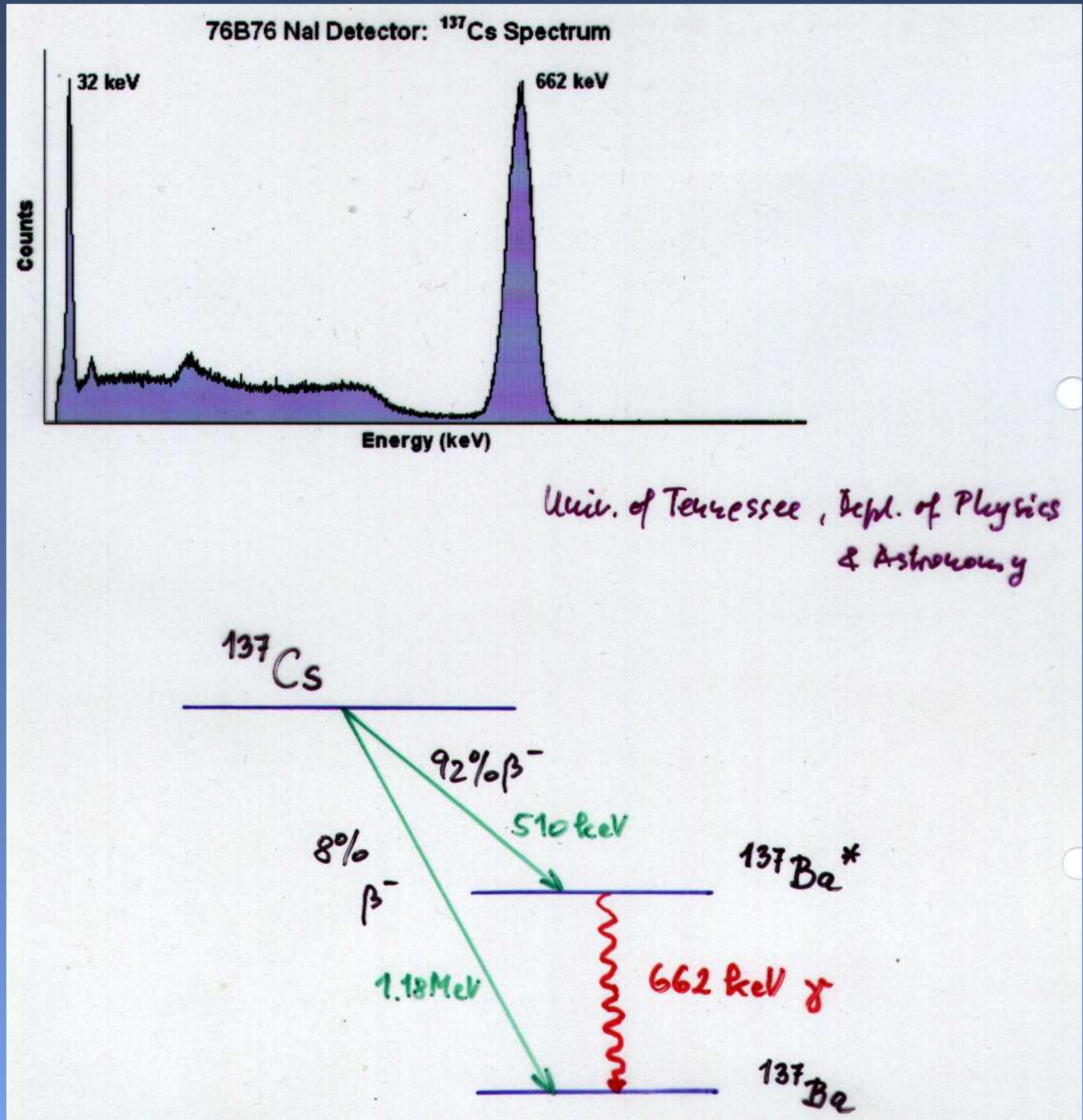


Harshaw 1969

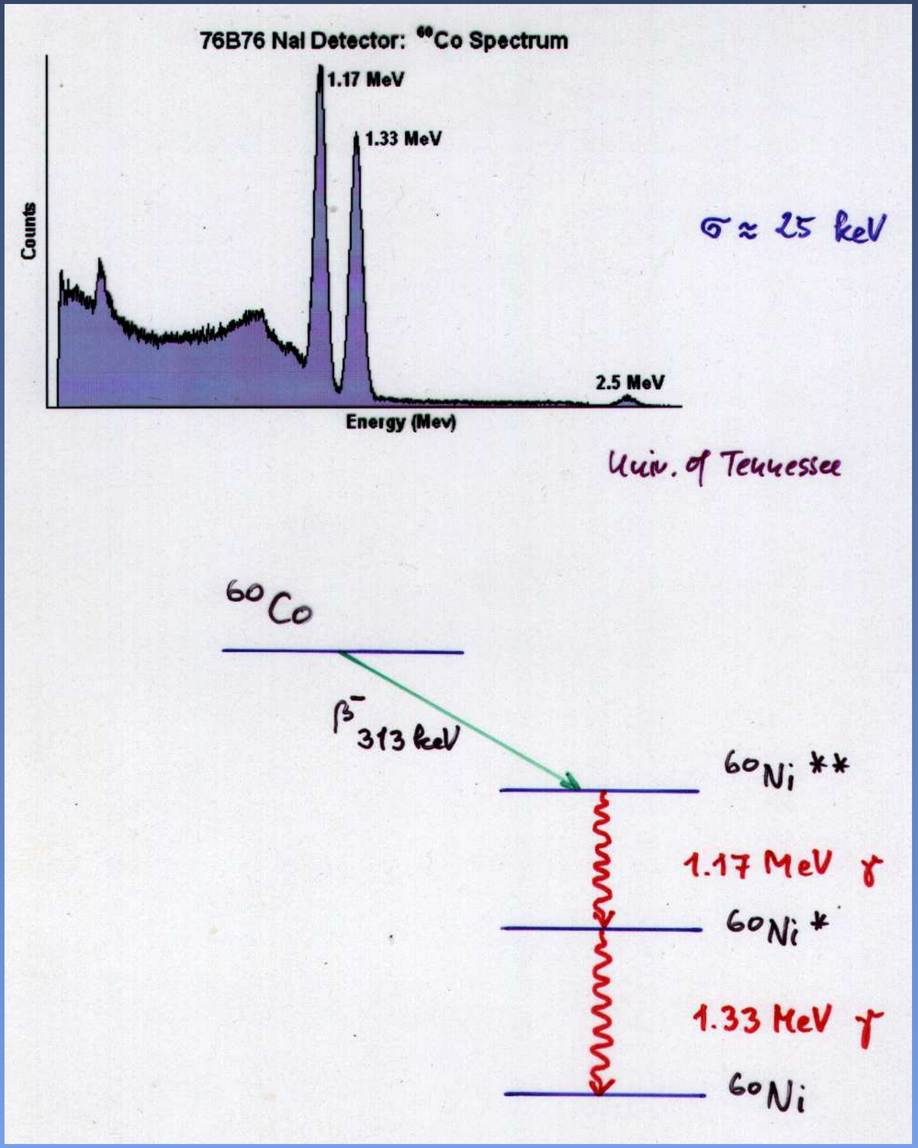
Setup for γ Ray Spectroscopy



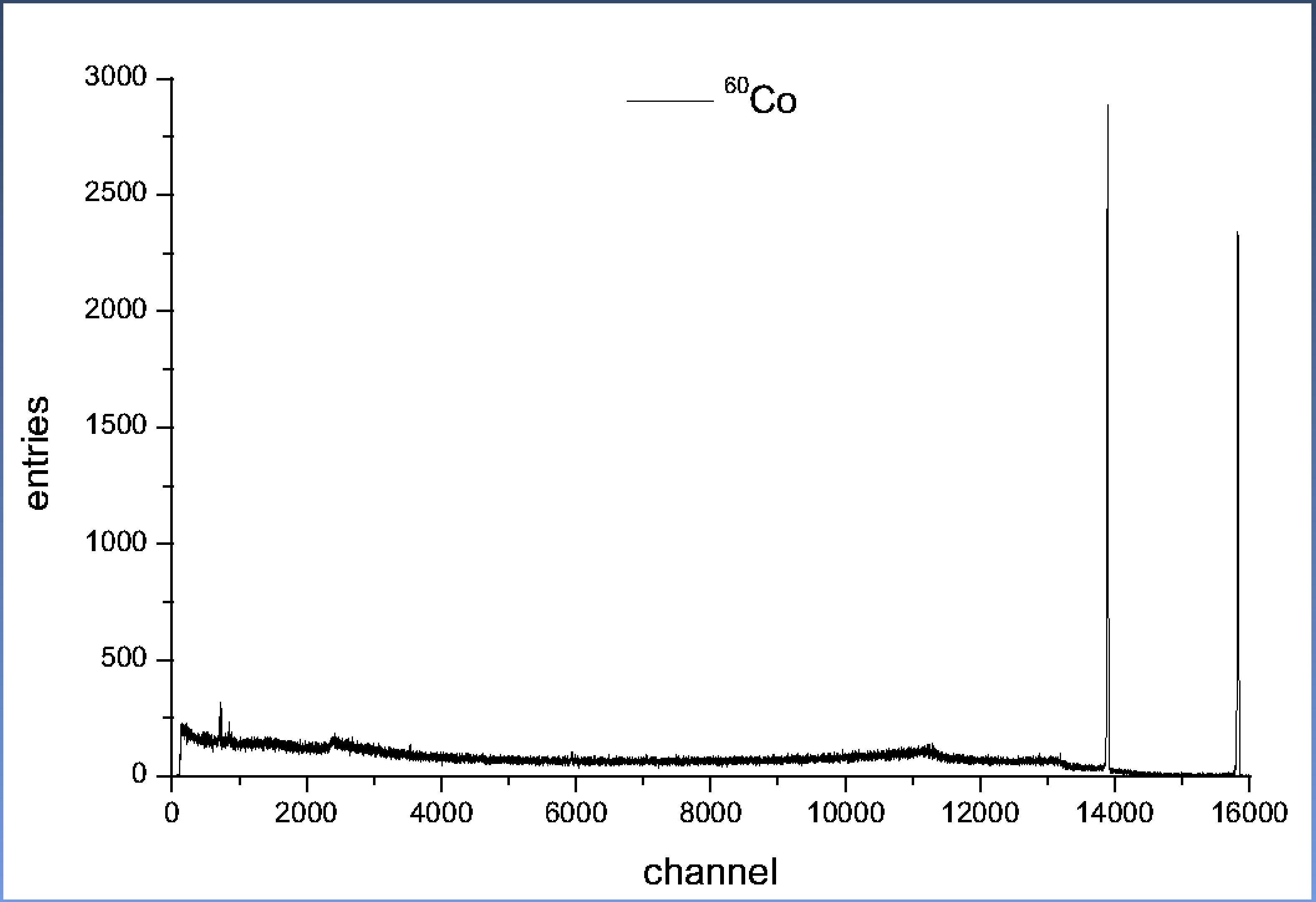
γ Spectrum of ^{137}Cs



γ Spectrum of ^{60}Co with NaI(Tl)



γ **Spectrum of** ^{60}Co **with** HPGe

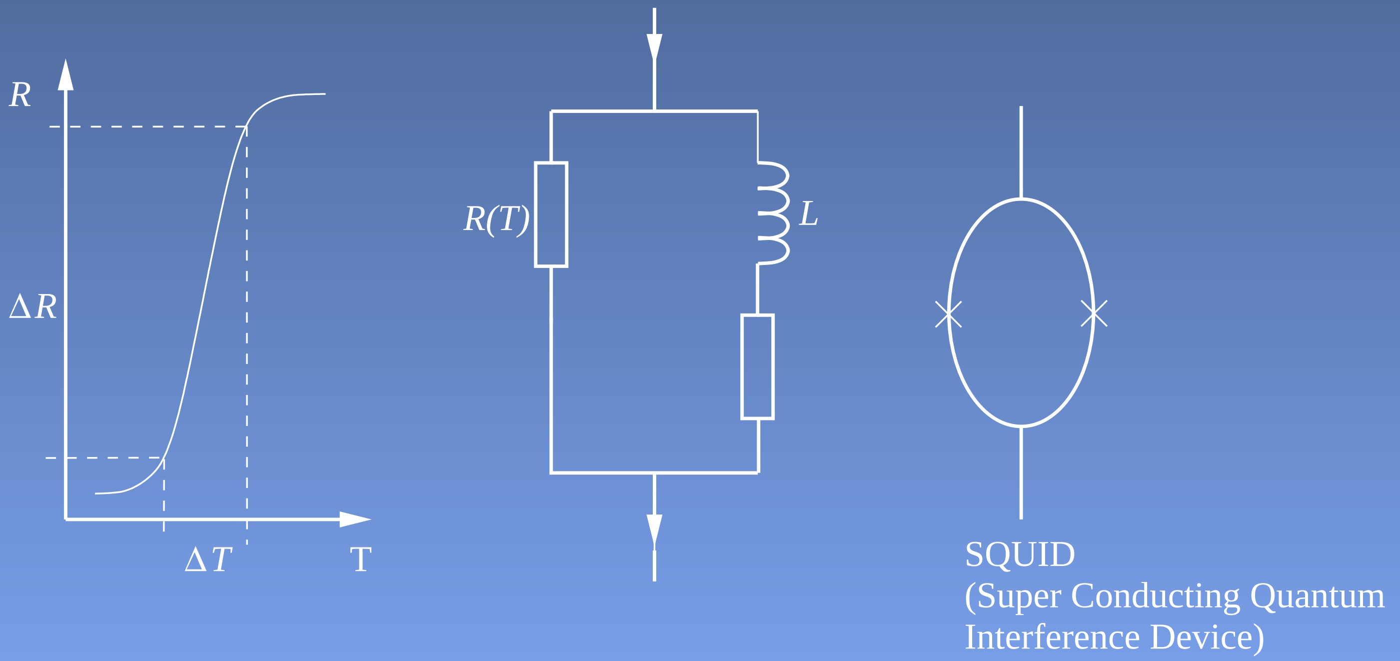


High resolution photon detector

CRESST - Cryogenic Rare Event Search with Superconducting Thermometers

NIM A 354 (1995) 408
avmp01.mppmu.mpg.de/cresst/

Superconducting phase transition thermometer
Principle:



$$\frac{\Delta R}{\Delta T} \Rightarrow \frac{\mathrm{d}R}{\mathrm{d}t} \rightarrow U_{\mathrm{ind}} \Rightarrow \frac{\mathrm{d}H}{\mathrm{d}t}$$

Trident Production / Pair Production

Trident production:

$\gamma + e^- \rightarrow e^- + e^+ + e^-$

Pair production:

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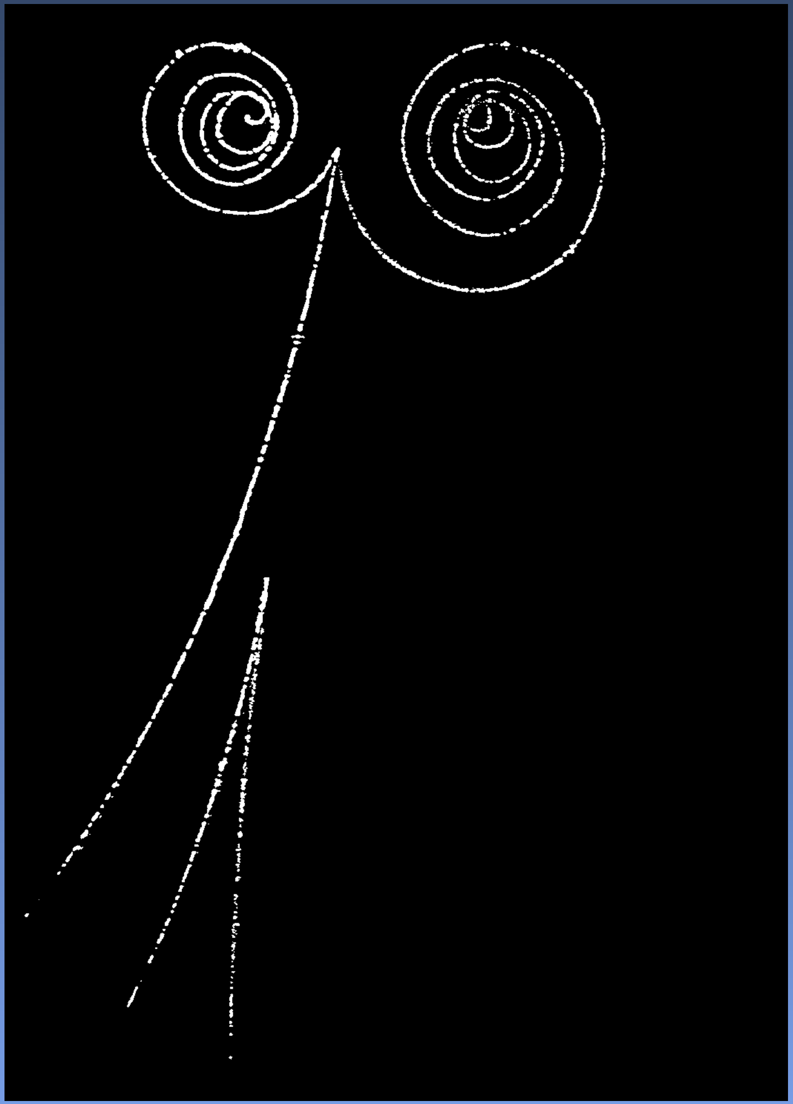
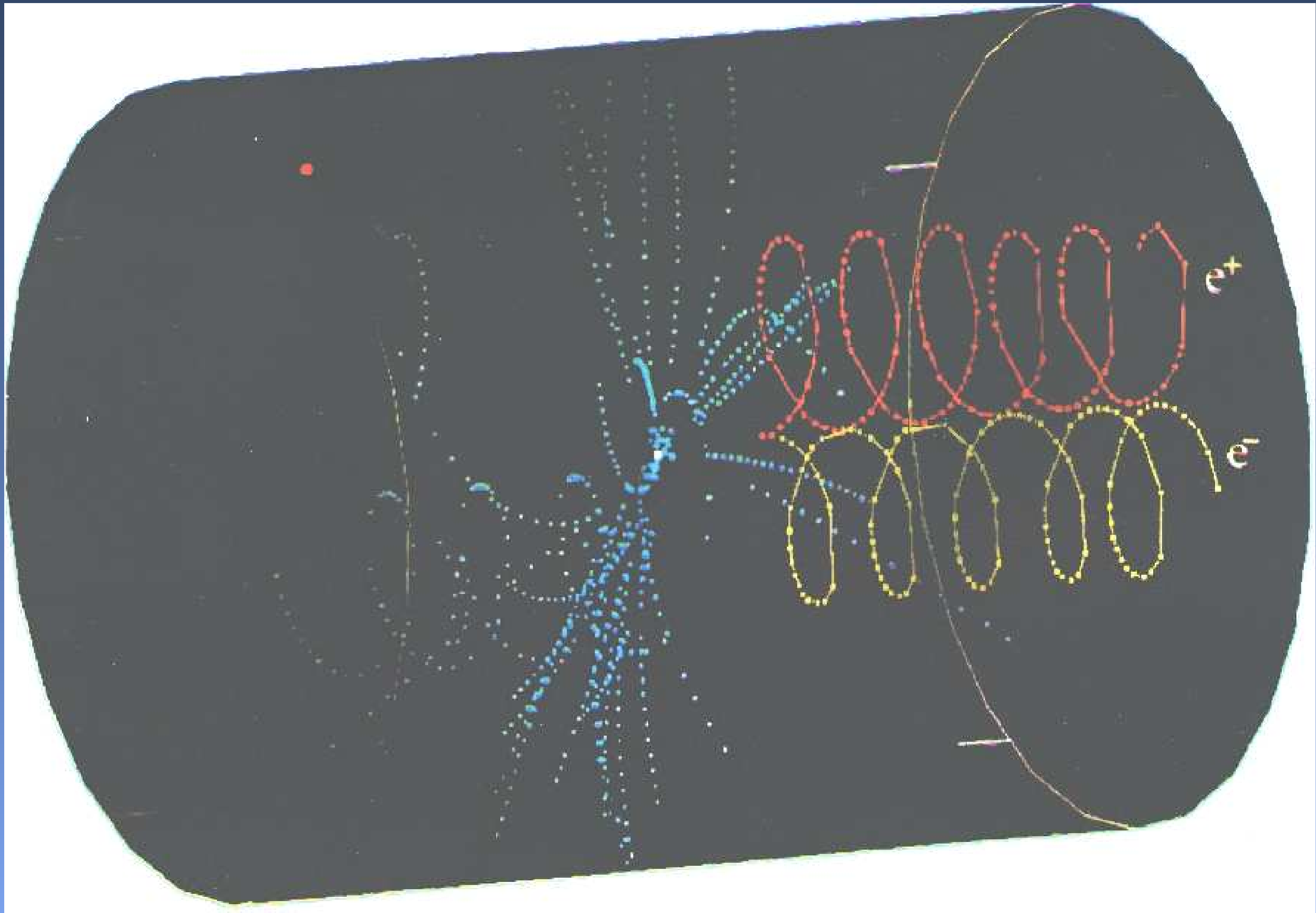
F. Close et al. 1987



ALEPH



Interaction of Neutrons

Indirect detection technique: induce neutrons to interact and produce charged particles:

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Neutron detection and identification is important in the field of radiation protection because the relative biological effectiveness (quality factor) is high and depends on the neutron energy.

$$H\;[\mathrm{Sievert}] = q \cdot D[\mathrm{Gray}]$$

Interaction of Neutrinos

$\nu_e + n \rightarrow p + e^-$

$\overline{\nu}_e + p \rightarrow n + e^+$ (discovery of the neutrino)

$\nu_\mu + n \rightarrow p + \mu^-; \quad \nu_\tau + n \rightarrow p + \tau^-$

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for MeV neutrinos:

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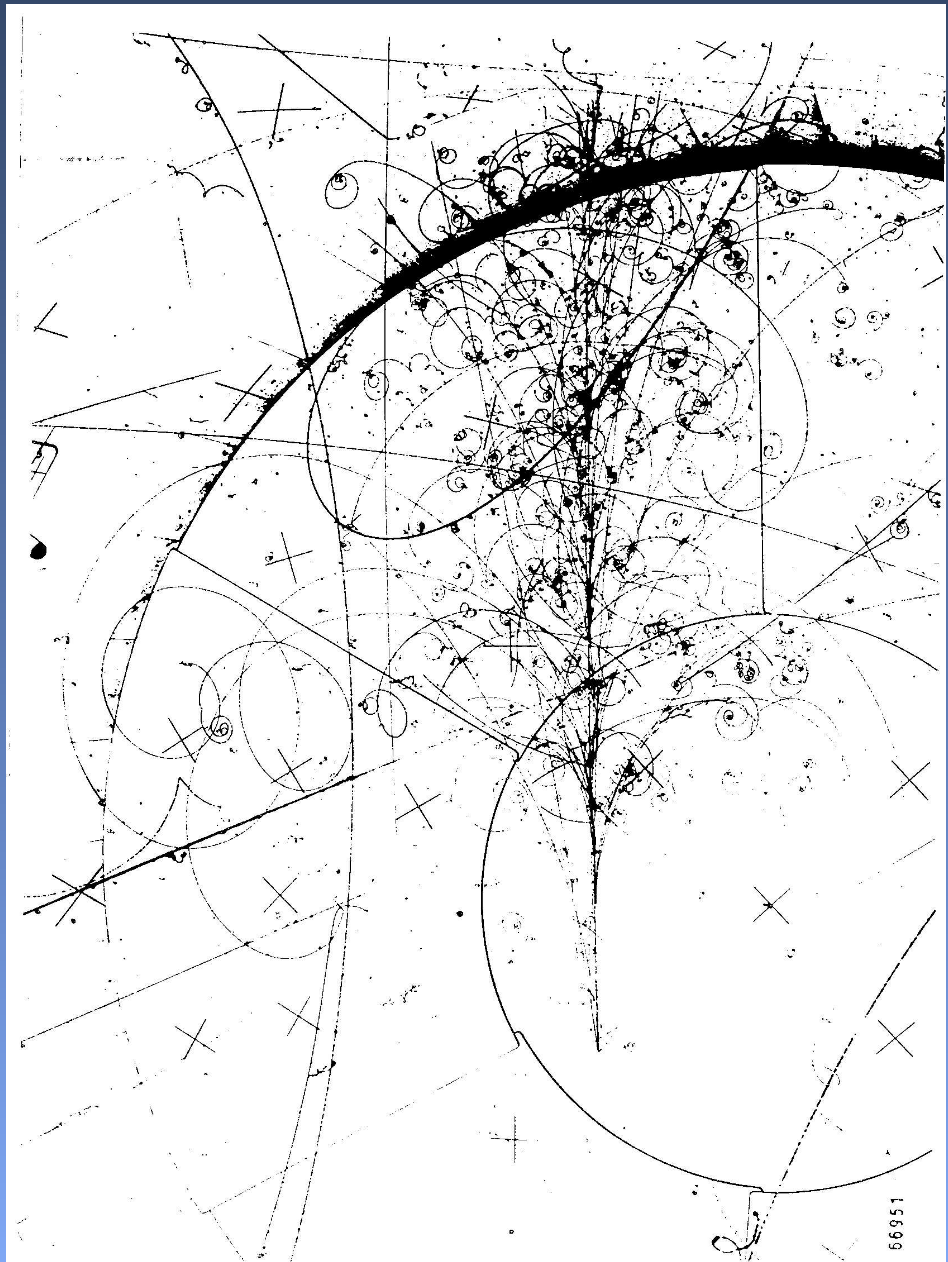
Measurement by missing momentum and missing energy technique.

Electromagnetic Cascade (1)

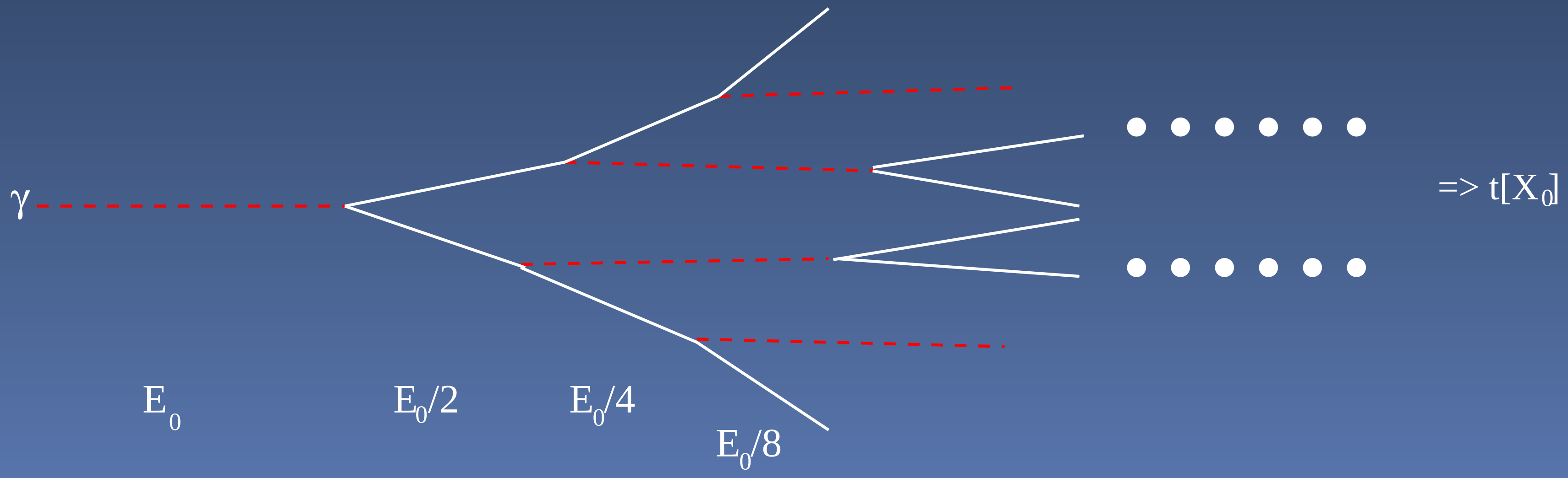


$\nu_e +$ nucleon $\rightarrow e^- +$ hadrons
electromagnetic cascade

H. Wachsmuth, CERN 1998

Electromagnetic Cascade (2)

The basic features can already be learned from a very simple model:



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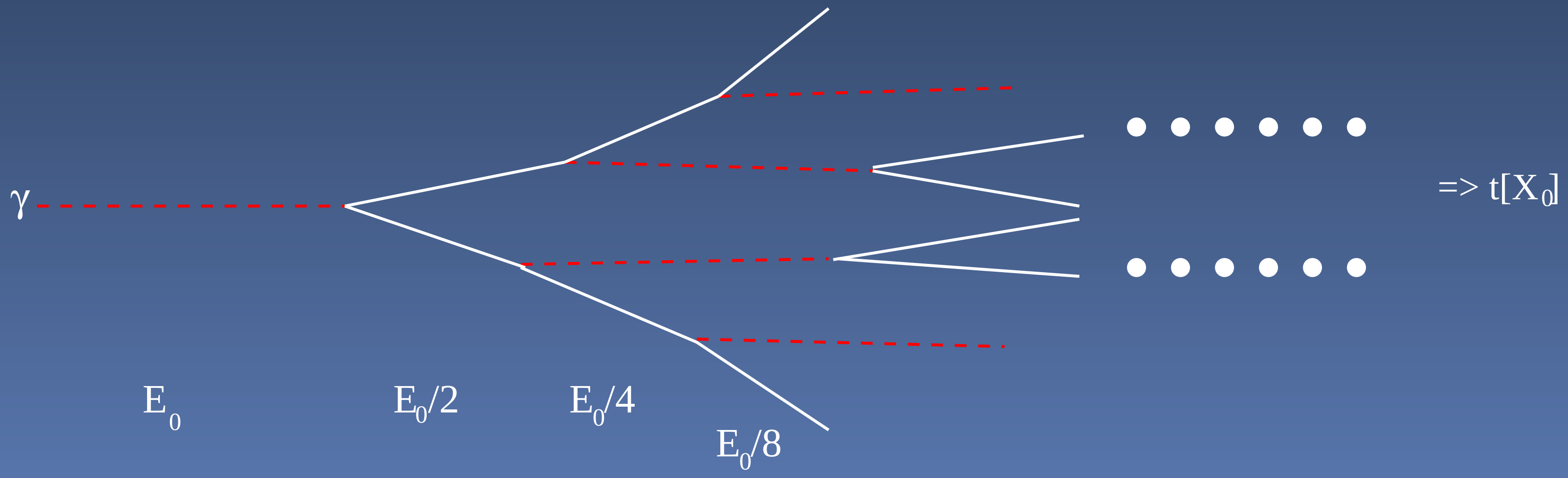

Number of particles (e^+, e^-, γ): $N(t) = 2^t$.

Energy of particles: $E(t) = E_0 \cdot 2^{-t}$.

Particle multiplication stops if: $E(t) < E_c$: $E_c = E_0 \cdot 2^{-t_{\max}}$.

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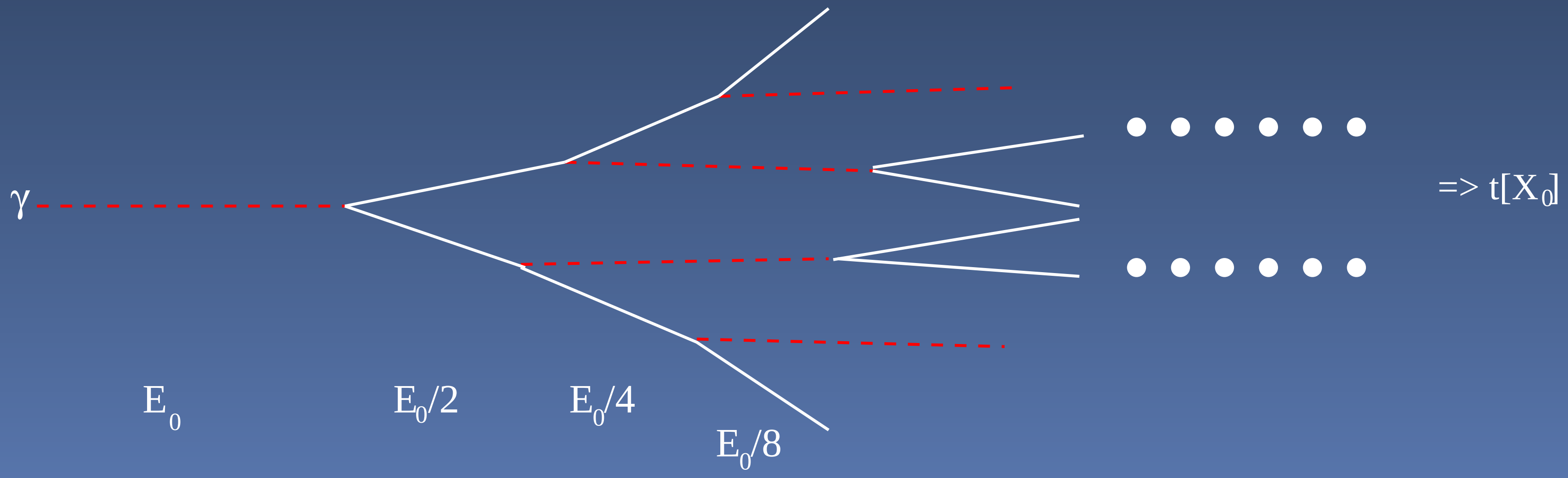

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Total number of shower particles:

$$S = \sum N(t) = \sum 2^t = 2^{t_{\max}+1} - 1 \approx 2 \cdot 2^{t_{\max}} = 2 \cdot \frac{E_0}{E_c} \propto E_0.$$

Energy Resolution of Electromagnetic Calorimeters

Total track length (sampling step t):

$$S^* = \frac{S}{t} = 2 \cdot \frac{E_0}{E_c} \cdot \frac{1}{t},$$

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$$\frac{\mathrm{d}E}{\mathrm{d}t} = \mathrm{const.} \cdot t^{\alpha} \mathrm{e}^{-bt} \quad (a,\ b\text{: fit parameters}).$$

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For homogenous calorimeters: $R_m = \begin{cases} 14\ \text{g/cm}^2 \hat{=} 1.8\ \text{cm} & \text{Fe} \\ 18\ \text{g/cm}^2 \hat{=} 1.6\ \text{cm} & \text{Pb} \end{cases}$

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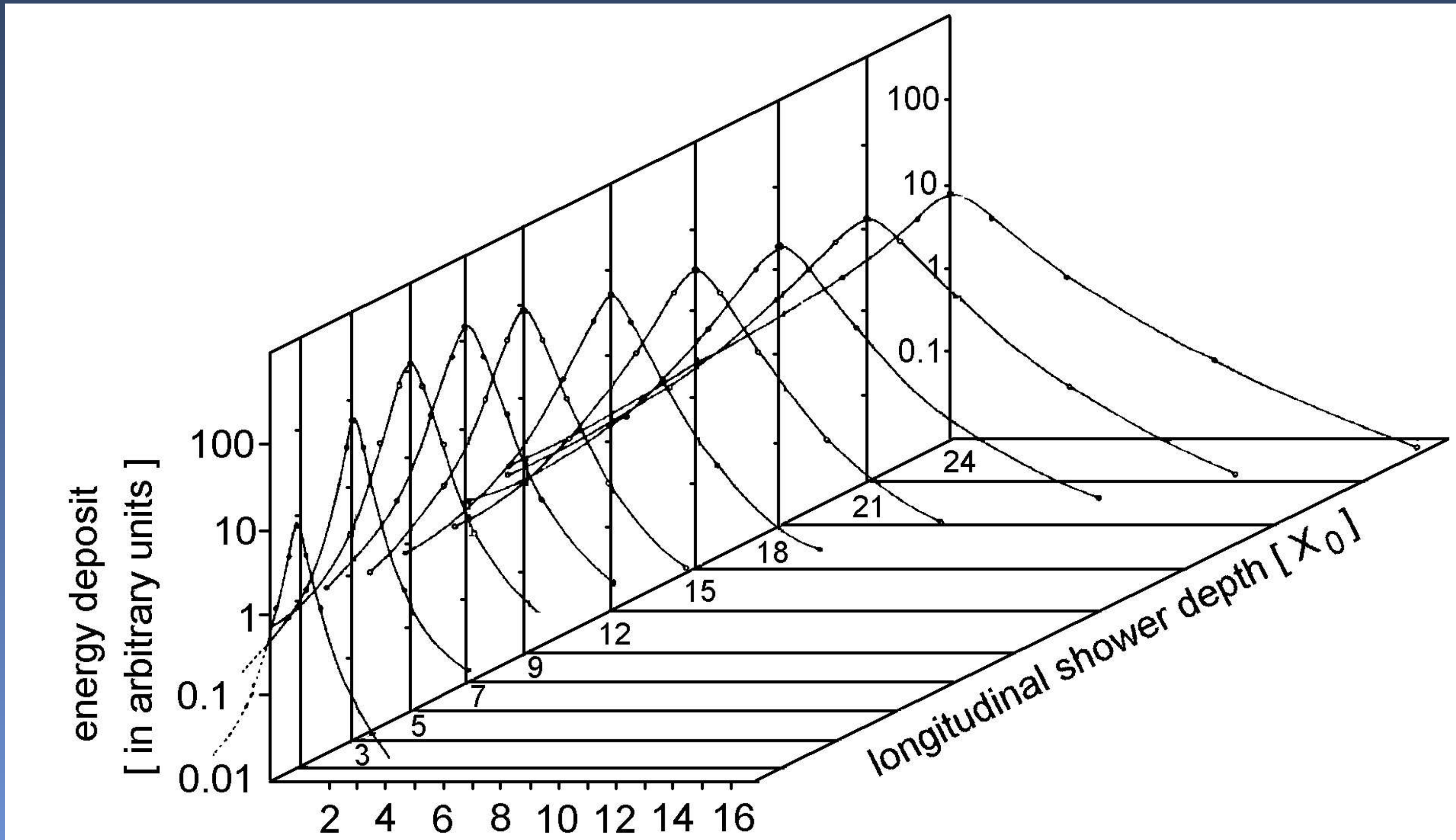
Attractive alternative: *sampling calorimeters*.

Logitudinal and Lateral Profile of an Electron Shower



6 GeV electrons, Grupen 1996

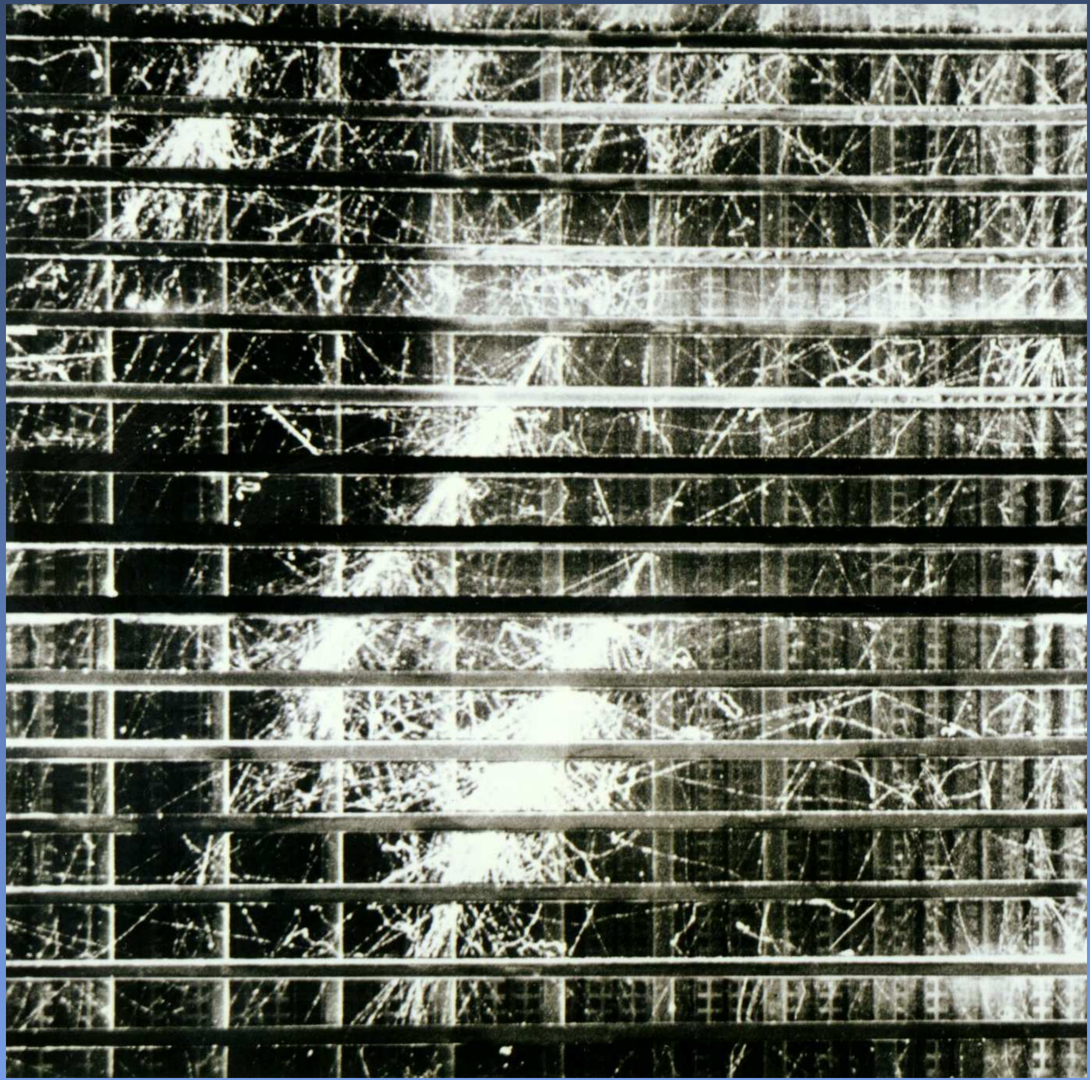
Multi-Plate Cloud Chamber (1)

$\mu^- + \text{nucleus} \rightarrow \mu^- + \text{nucleus}' + \gamma$
$\gamma \rightarrow$ electromagnetic cascade

Rochester 1981

Multi-Plate Cloud Chamber (2)

Multi-plate cloud chamber in an air shower experiment below $3\ \mathrm{m}$ of concrete



electromagnetic showers initiated by muon *bremsstrahlung*.

Wolter 1970

Hadron Cascades

Longitudinal development: interaction length.
Lateral spread: transverse momentum p_t
since $\lambda > X_0$ and $\langle p_t \rangle >> \langle p_t \rangle_{\text{multiple scattering}}$

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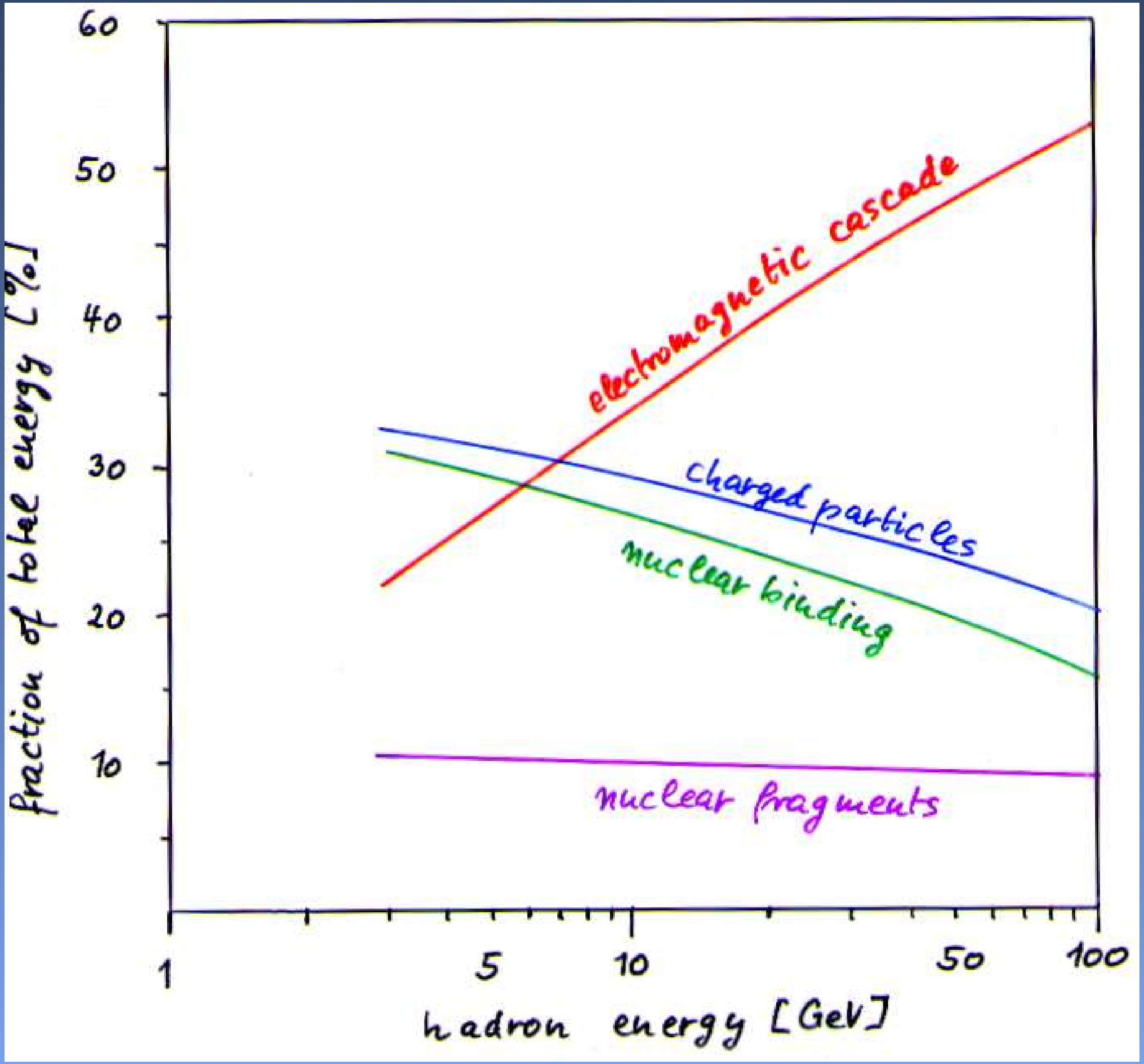
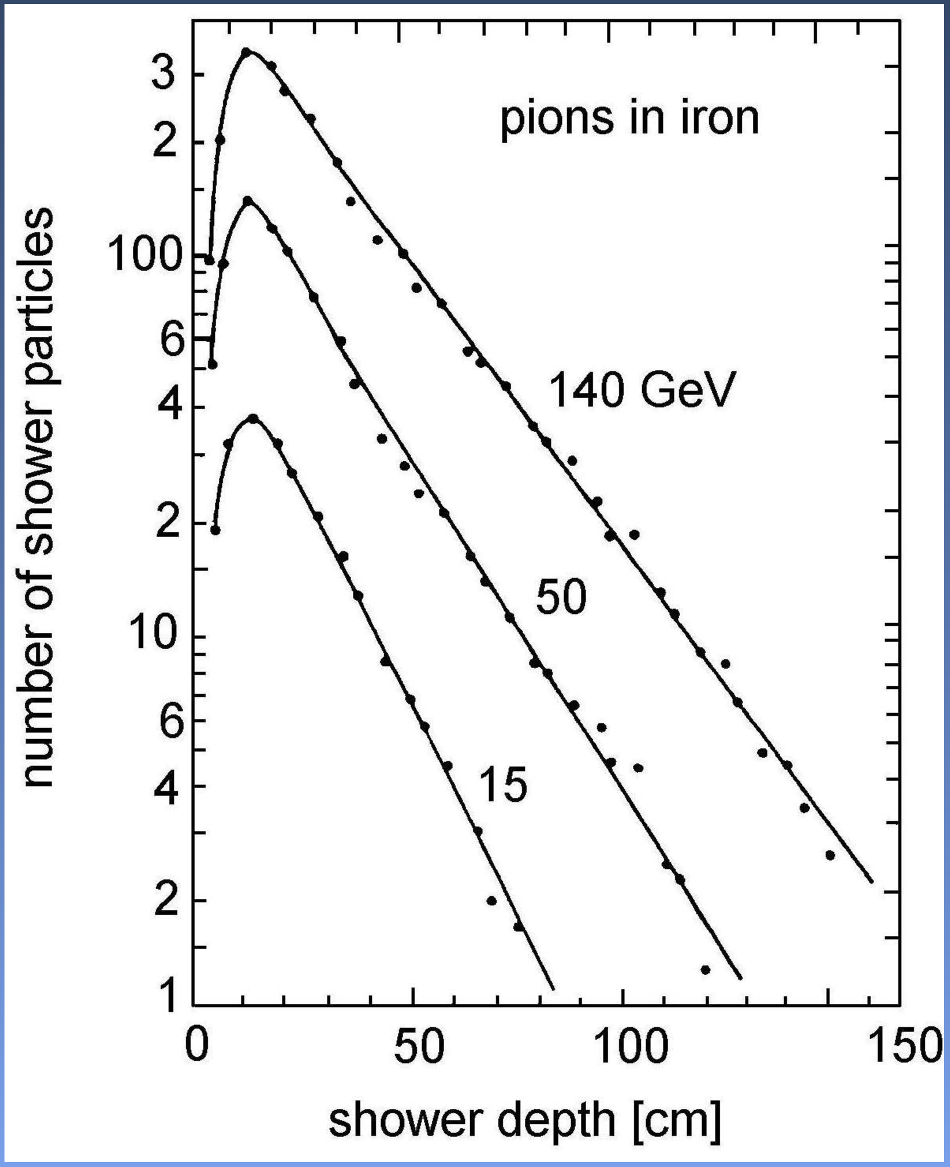
Problem of compensation: different response to electrons and hadrons, aim at balanced response $e/\pi = 1$.

Energy Sharing in a Hadron Cascade

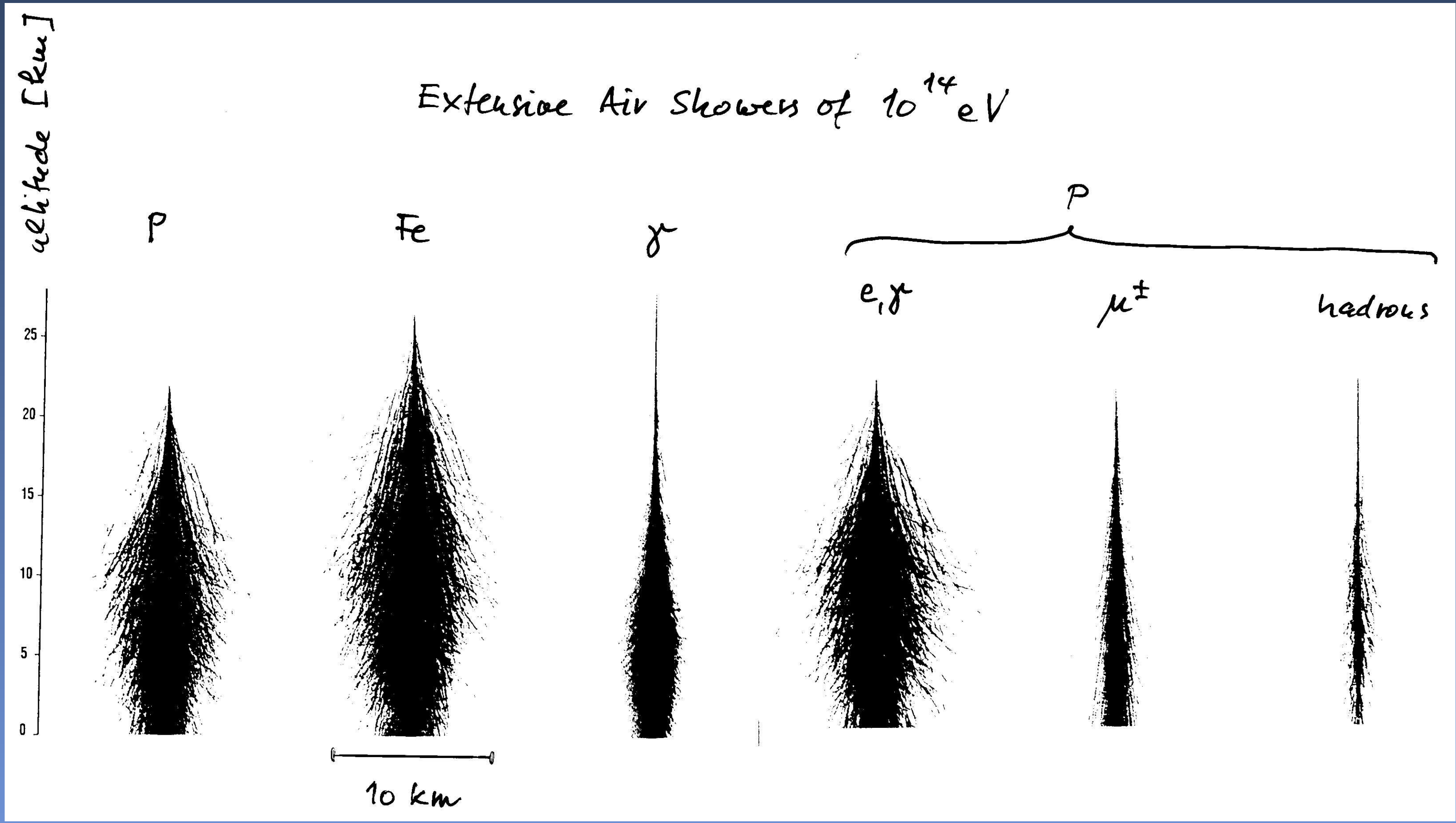


Longidudinal Development of a Hadron Cascade

Holder 1978
NIM 151 (1978) 69

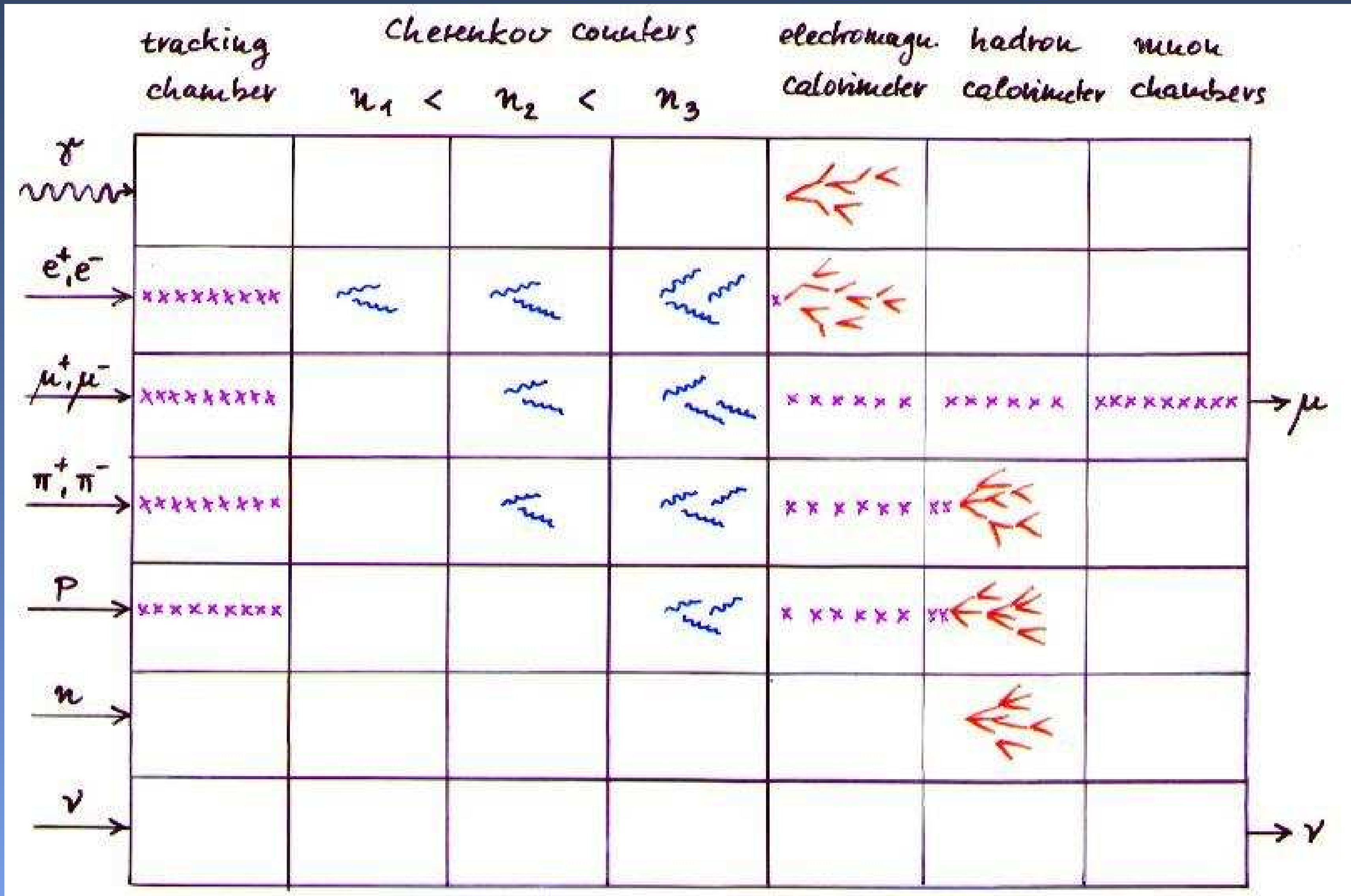


Extensive Air Showers of 10^{14} eV



J. Knapp, D. Heck, Karlsruhe 1998

Methods of Particle Identification



Particle Identification with Time of Flight (TOF)

$\Delta t = L\left(\frac{1}{v_1} - \frac{1}{v_2}\right) = \frac{L}{c}\left(\frac{1}{\beta_1} - \frac{1}{\beta_2}\right)$

using $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ this gives:

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Since in this case $\approx pc$ one gets for a momentum defined beam:

$$\Delta t = \frac{Lc}{2p^2}(m_1^2 - m_2^2).$$

Example: $e/\mu/\pi$-separation

Example 1:

$e/\mu/\pi$-separation for $L = 149.5\text{ cm}$
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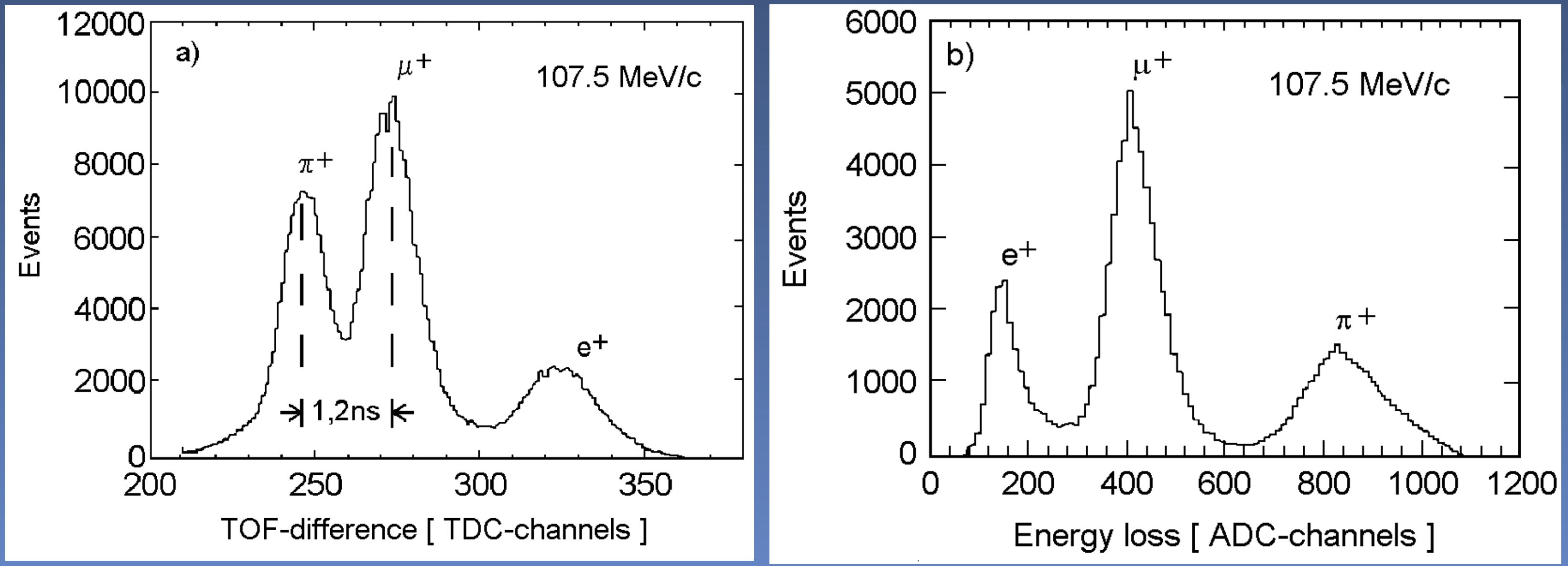

E. Fragiacomo et al. NIM A 439 (2000) 45

Examples: TOF-resolution π/p-separation

Example 1:
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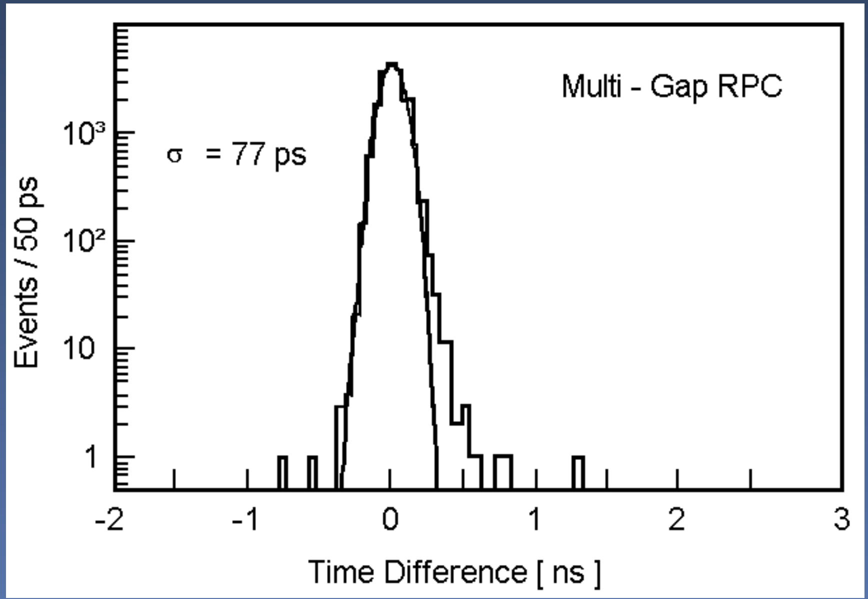

F. Sauli CERN-EP 2000/080

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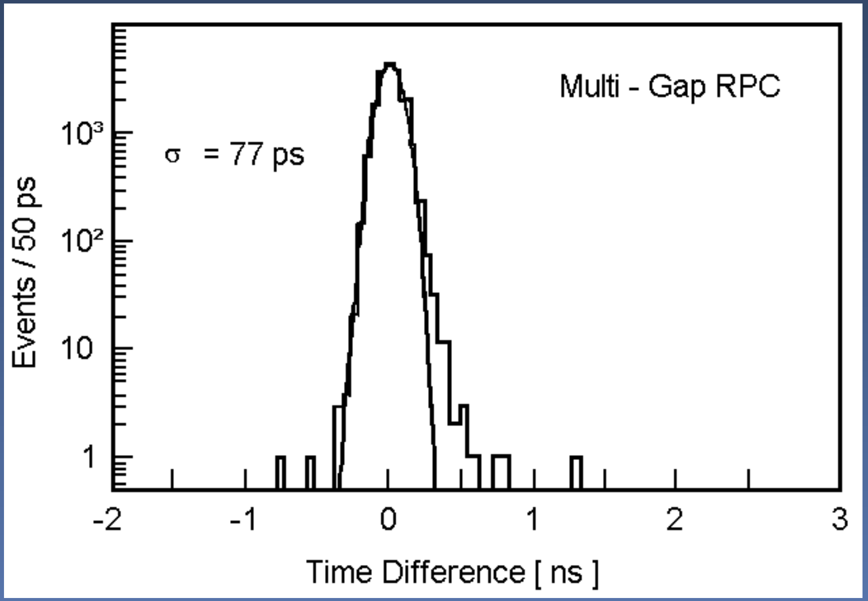
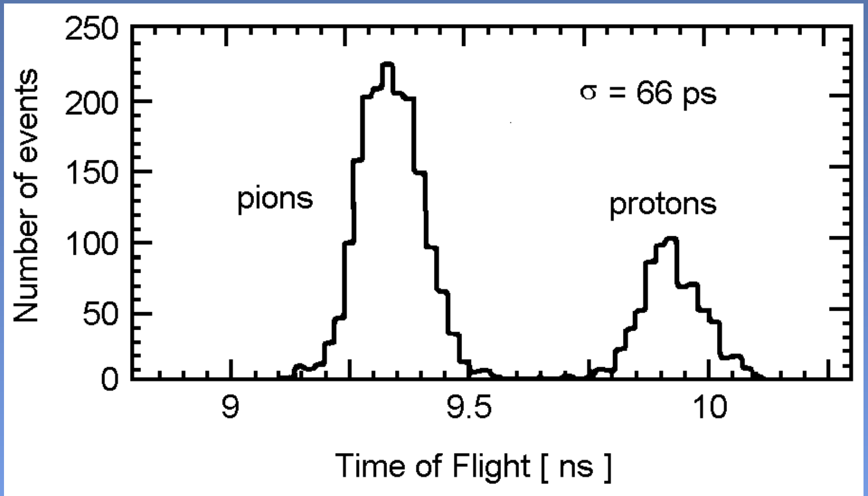


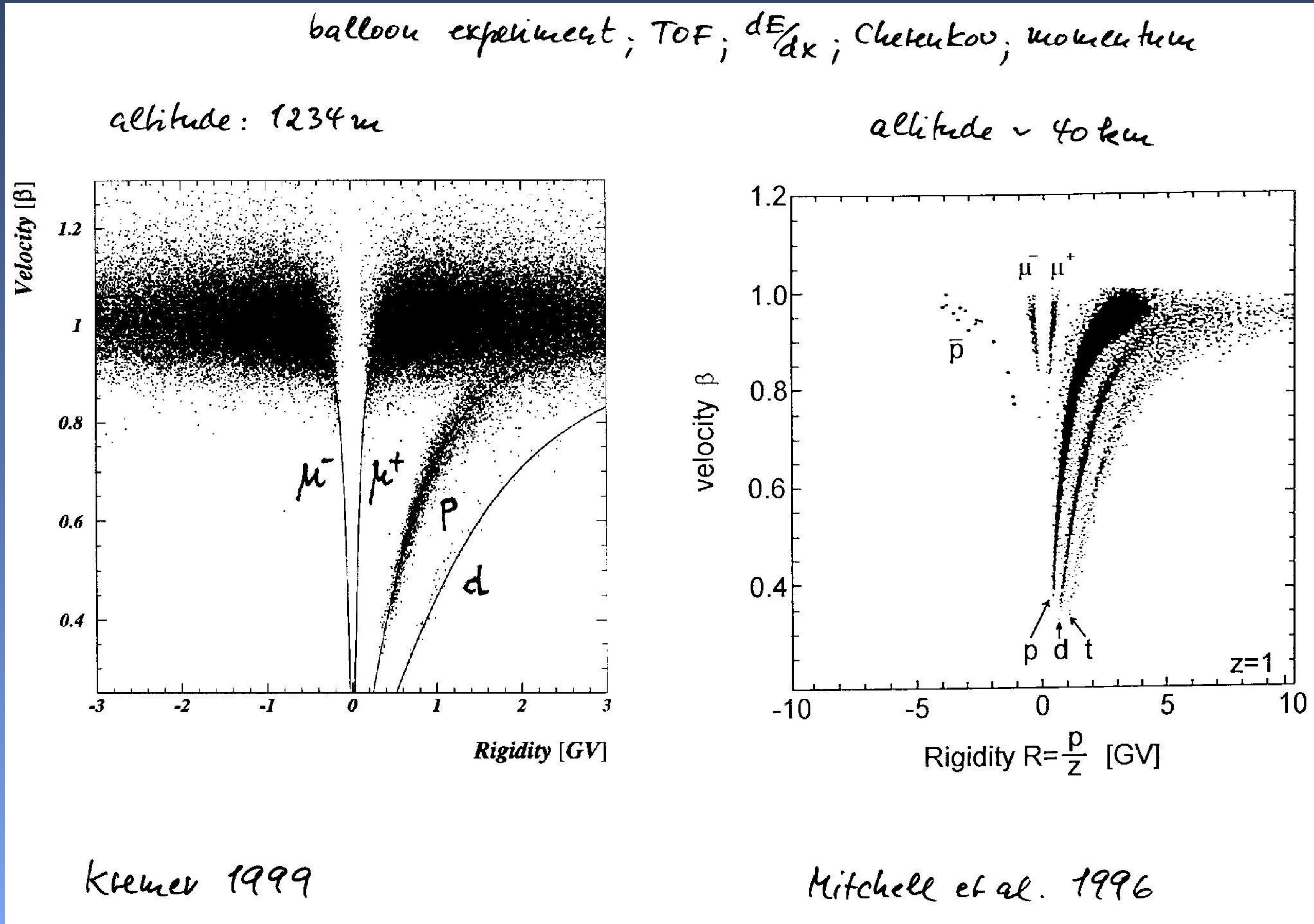
F. Sauli CERN-EP 2000/080

π/p-separation in a $p = 2\ \mathrm{GeV}/c$ scintillator system.

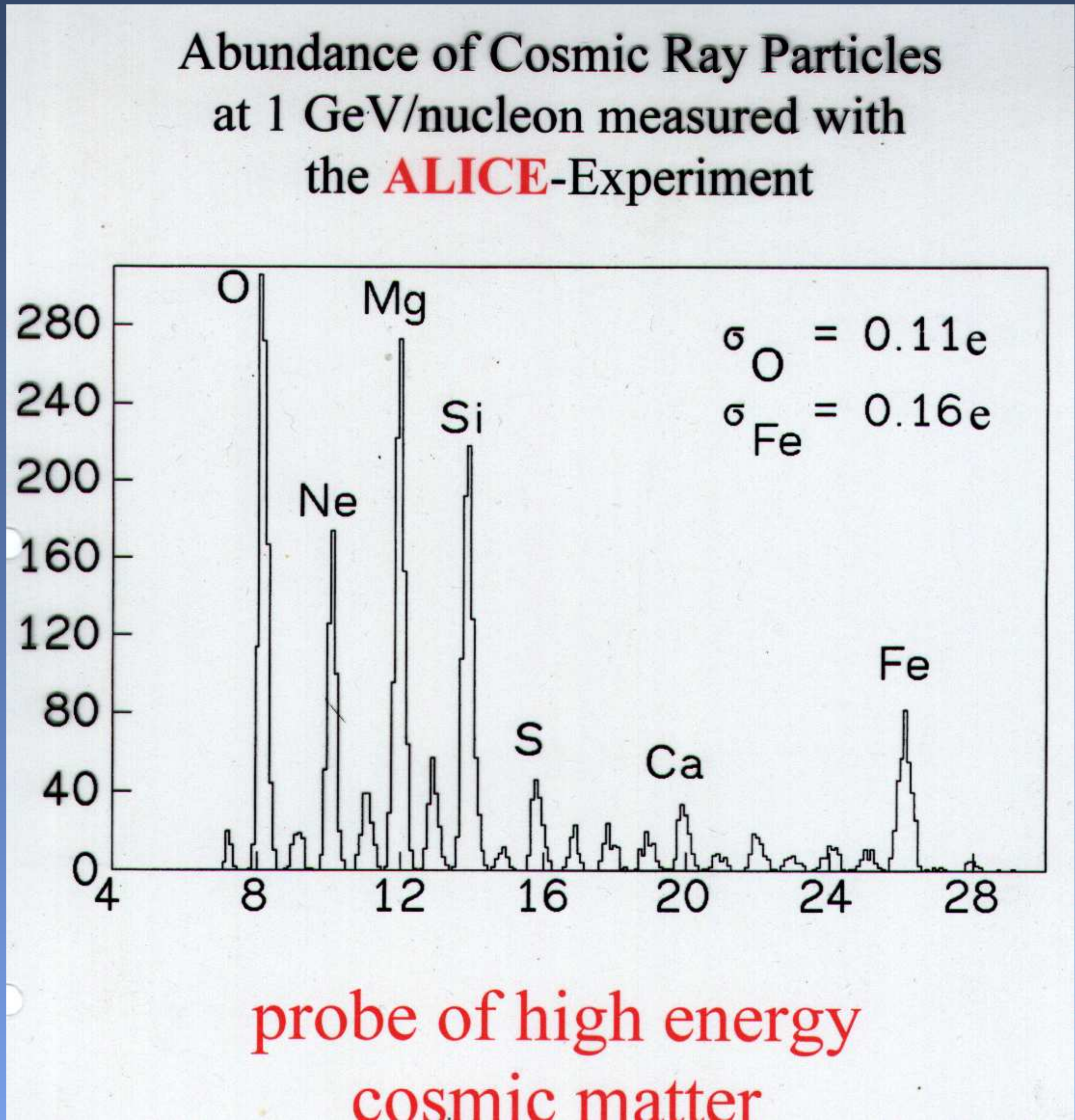


A. Sapathy et al., BELLE 1999

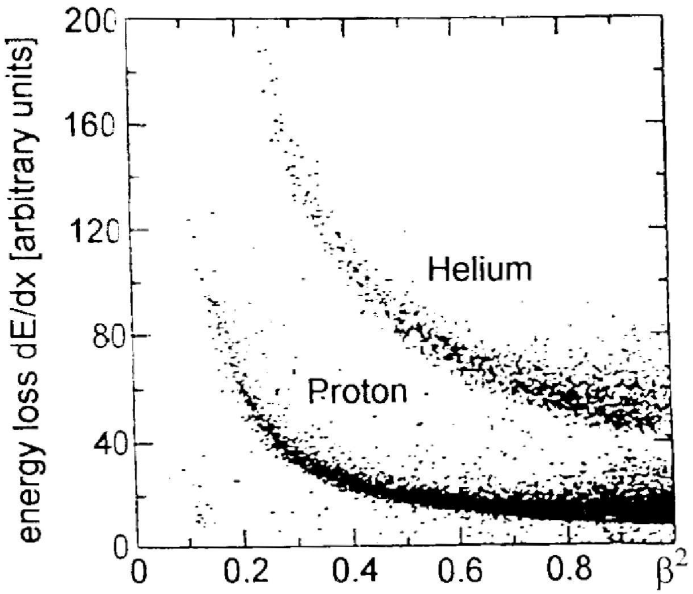
Balloon Experiment; $\mathrm{d}E/\mathrm{d}x$; Cherenkov; momentum

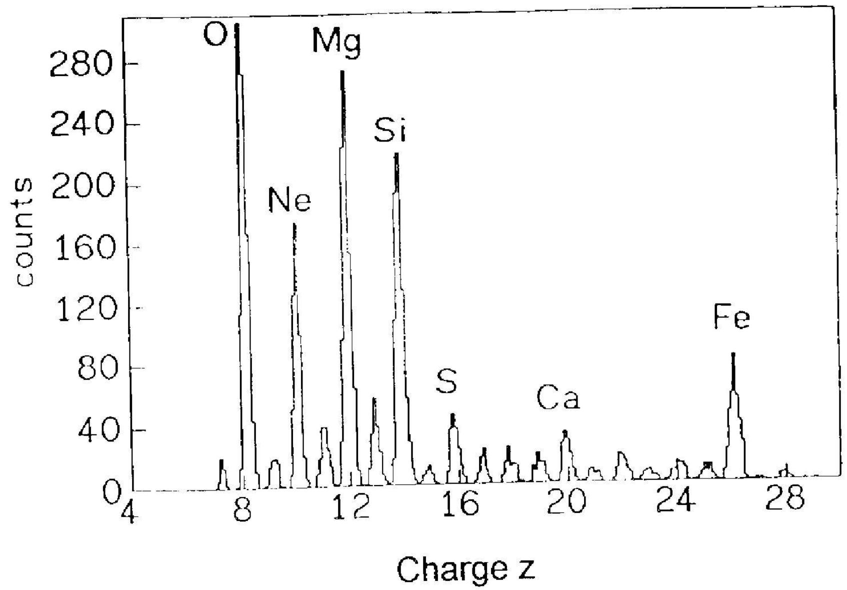


ALICE


Abundance of Cosmic Ray Particles
at 1 GeV/nucleon measured with
the ALICE-Experiment
O
Mg
Si
Ne
S
Ca
Fe
σ O = 0.11e
σ Fe = 0.16e
280
240
200
160
120
80
40
0
4
8
12
16
20
24
28
probe of high energy
cosmic matter

Balloon Experiment $\sim$ 40 km



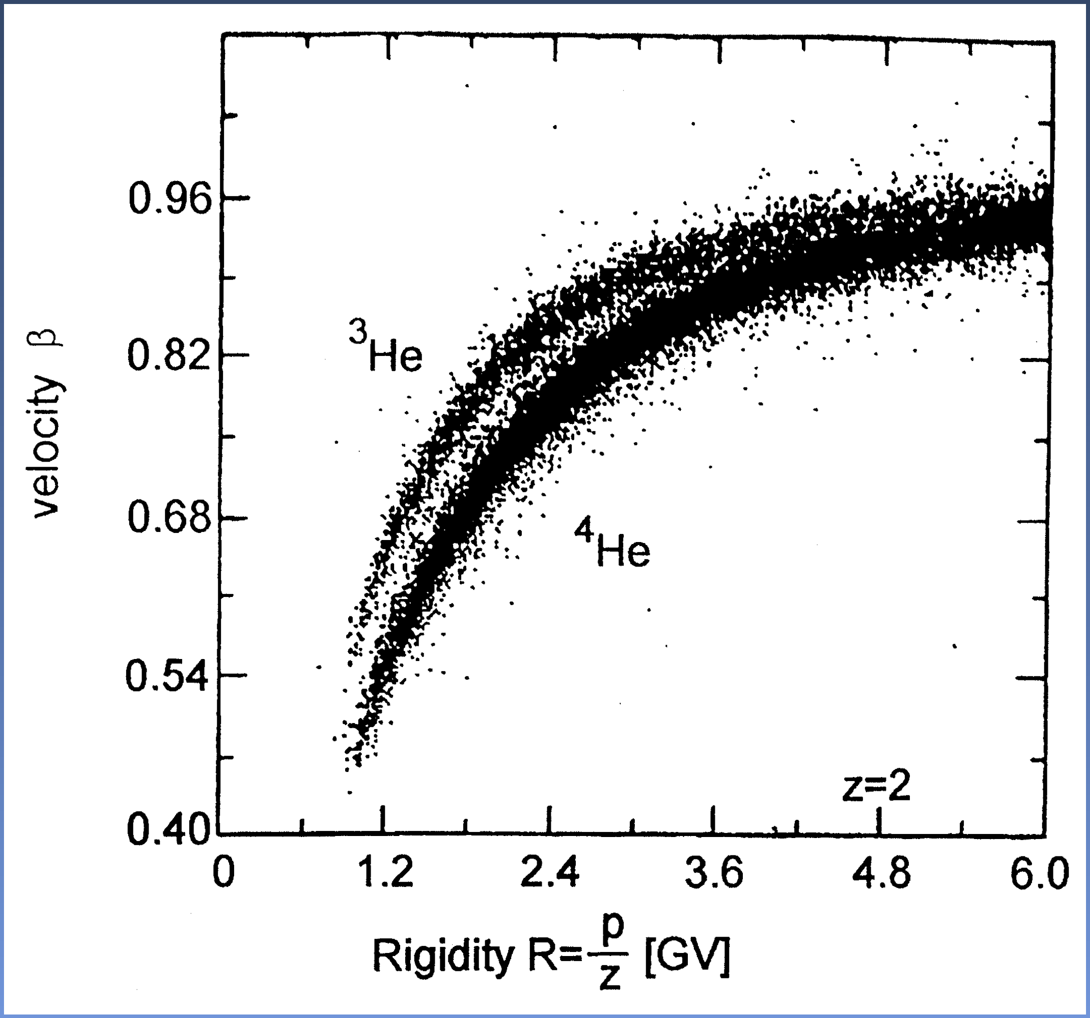


Reimer 1995
Ph. D. Thesis Siegen

Hesse 1991
Proc. ICRC Dublin, Vol. 1, p. 596

Balloon Experiment

Balloon flight 40 km,
TOF, $\mathrm{d}E/\mathrm{d}x$,
momentum,
Cherenkov.



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Ph. D. Thesis Siegen

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- Particle identification:
  - direct identification: $(\pi, K, p, \mu, e, \gamma, \ldots)$,
  - indirect identification using invariant mass technique $(K_s^0 \to \pi^+ + \pi^-, \Lambda \to p + \pi^-)$,
  - very indirect identification using missing energy and momentum (neutrinos and other “invisible” particles (γ, LSP)).
- Essential for:
  - elementary particles,
  - astroparticle physics,
  - cosmic rays,
  - medicine,
  - radiation protection,
  - $\ldots$.