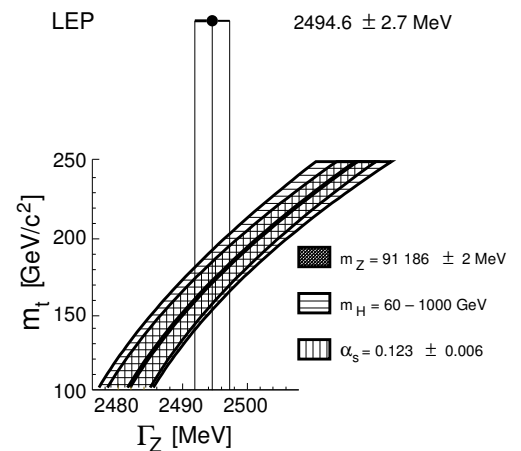
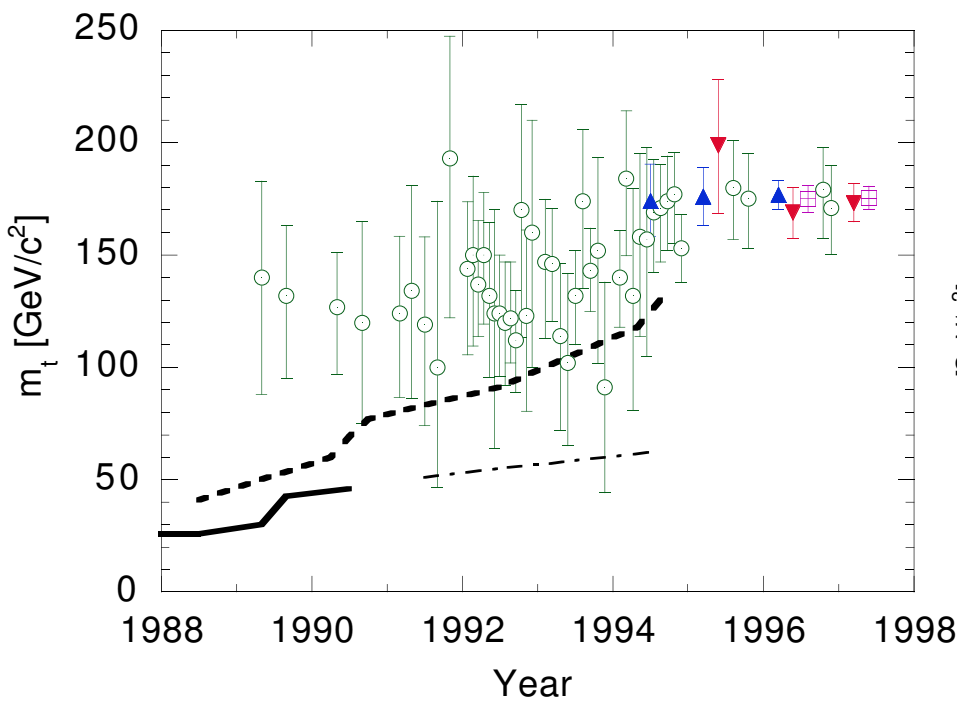


Global fits ...

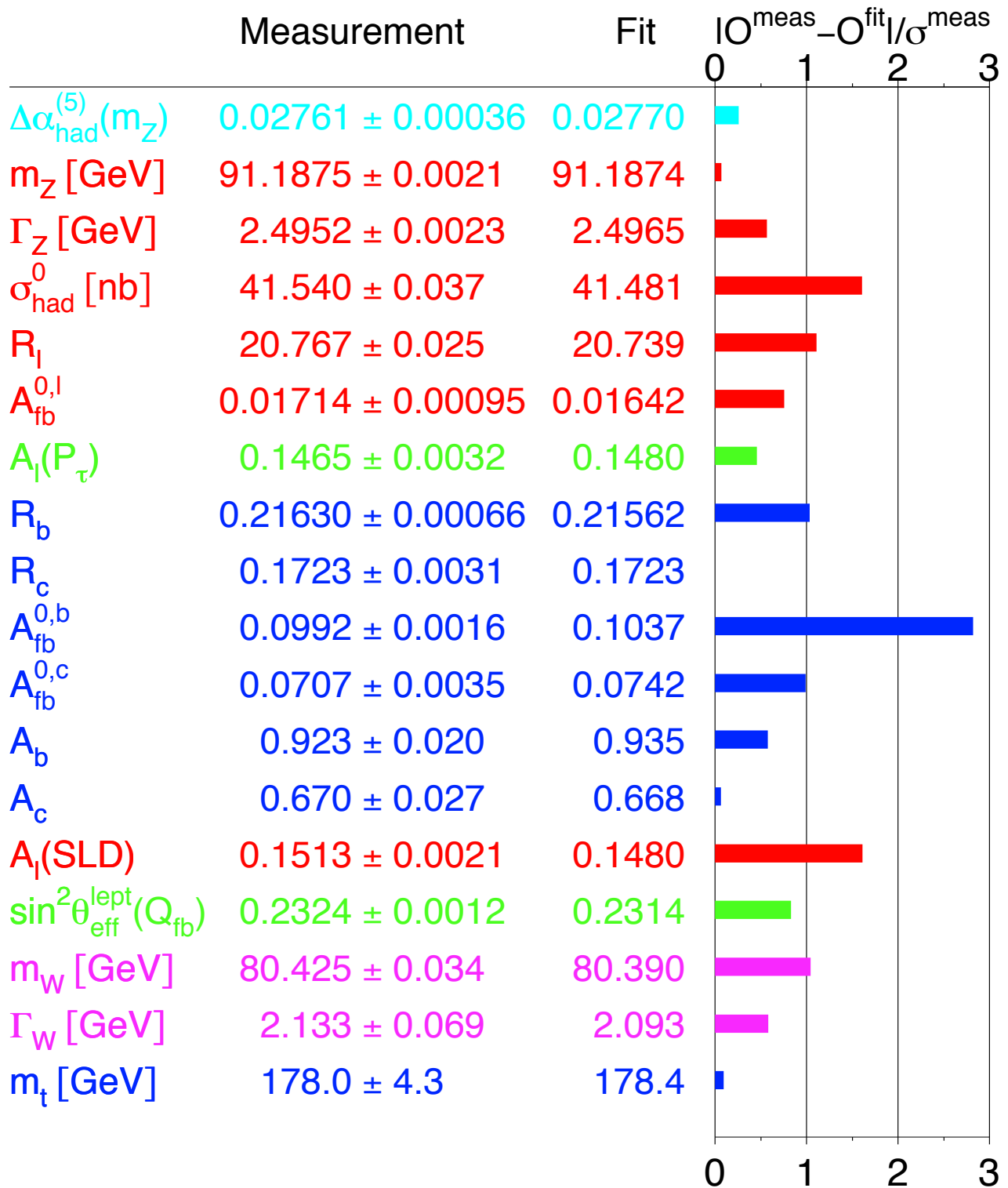
to precision EW measurements:

- ▷ precision improves with time
- ▷ calculations improve with time



11.94, LEPEWWG: $m_t = 178 \pm 11^{+18}_{-19} \text{ GeV}/c^2$

Direct measurements: $m_t = 174.3 \pm 5.1 \text{ GeV}/c^2$



LEP Electroweak Working Group, Winter 2005

Parity violation in atoms

Nucleon appears elementary at very low Q^2 ; effective Lagrangian for nucleon β -decay

$$\mathcal{L}_\beta = - \frac{G_F}{\sqrt{2}} \bar{e} \gamma_\lambda (1 - \gamma_5) \nu \bar{p} \gamma^\lambda (1 - g_A \gamma_5) n$$

$g_A \approx 1.26$: axial charge

NC interactions ($x_W \equiv \sin^2 \theta_W$):

$$\mathcal{L}_{ep} = \frac{G_F}{2\sqrt{2}} \bar{e} \gamma_\lambda (1 - 4x_W - \gamma_5) e \bar{p} \gamma^\lambda (1 - 4x_W - \gamma_5) p ,$$

$$\mathcal{L}_{en} = \frac{G_F}{2\sqrt{2}} \bar{e} \gamma_\lambda (1 - 4x_W - \gamma_5) e \bar{n} \gamma^\lambda (1 - \gamma_5) n$$

▷ Regard nucleus as a **noninteracting collection** of Z protons and N neutrons ▷ Perform NR reduction; nucleons contribute coherently to $A_e V_N$ coupling, so dominant **P-violating** contribution to eN amplitude is

$$\mathcal{M}_{pv} = \frac{-iG_F}{2\sqrt{2}} Q^W \bar{e} \rho_N(\mathbf{r}) \gamma_5 e$$

$\rho_N(\mathbf{r})$: nucleon density at e^- coordinate \mathbf{r}

$Q^W \equiv Z(1 - 4x_W) - N$: weak charge

Bennett & Wieman (Boulder) determined weak charge of Cesium by measuring 6S-7S transition polarizability

$$Q_W(\text{Cs}) = -72.06 \pm 0.28 \text{ (expt)} \pm 0.34 \text{ (theory)}$$

about 2.5σ above SM prediction

The vacuum energy problem

$$\text{Higgs potential } V(\varphi^\dagger \varphi) = \mu^2(\varphi^\dagger \varphi) + |\lambda|(\varphi^\dagger \varphi)^2$$

At the minimum,

$$V(\langle \varphi^\dagger \varphi \rangle_0) = \frac{\mu^2 v^2}{4} = -\frac{|\lambda| v^4}{4} < 0.$$

$$\text{Identify } M_H^2 = -2\mu^2$$

contributes field-independent vacuum energy density

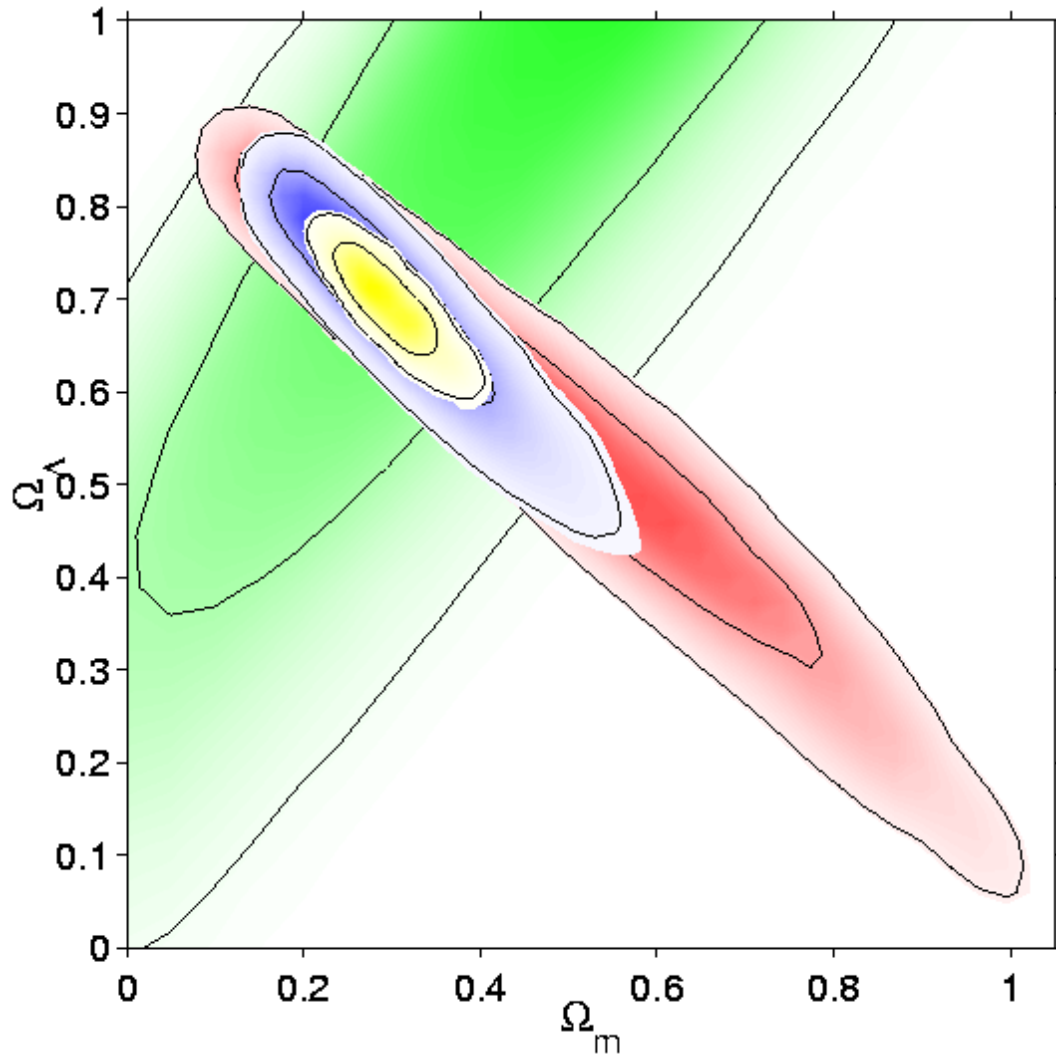
$$\rho_H \equiv \frac{M_H^2 v^2}{8}$$

Adding vacuum energy density ρ_{vac} \Leftrightarrow adding cosmological constant Λ to Einstein's equation

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G_N}{c^4}T_{\mu\nu} + \Lambda g_{\mu\nu}$$

$$\Lambda = \frac{8\pi G_N}{c^4} \rho_{\text{vac}}$$

observed vacuum energy density $\rho_{\text{vac}} \lesssim 10^{-46} \text{ GeV}^4$



Lewis & Bridle, astro-ph/0205436

But $M_H \gtrsim 114 \text{ GeV}/c^2 \Rightarrow$

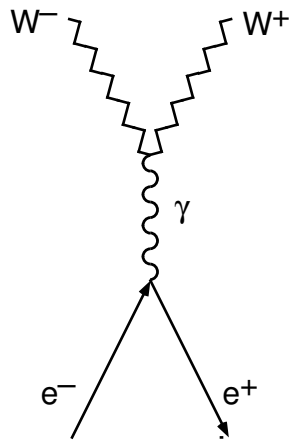
$$\rho_H \gtrsim 10^8 \text{ GeV}^4$$

MISMATCH BY 54 ORDERS OR MAGNITUDE

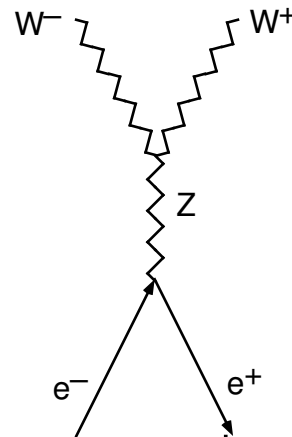
Why a Higgs Boson Must Exist

▷ Role in canceling high-energy divergences

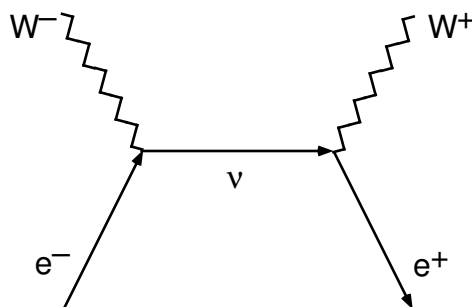
S -matrix analysis of $e^+e^- \rightarrow W^+W^-$



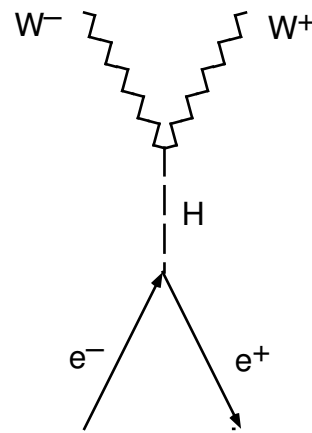
(a)



(b)



(c)

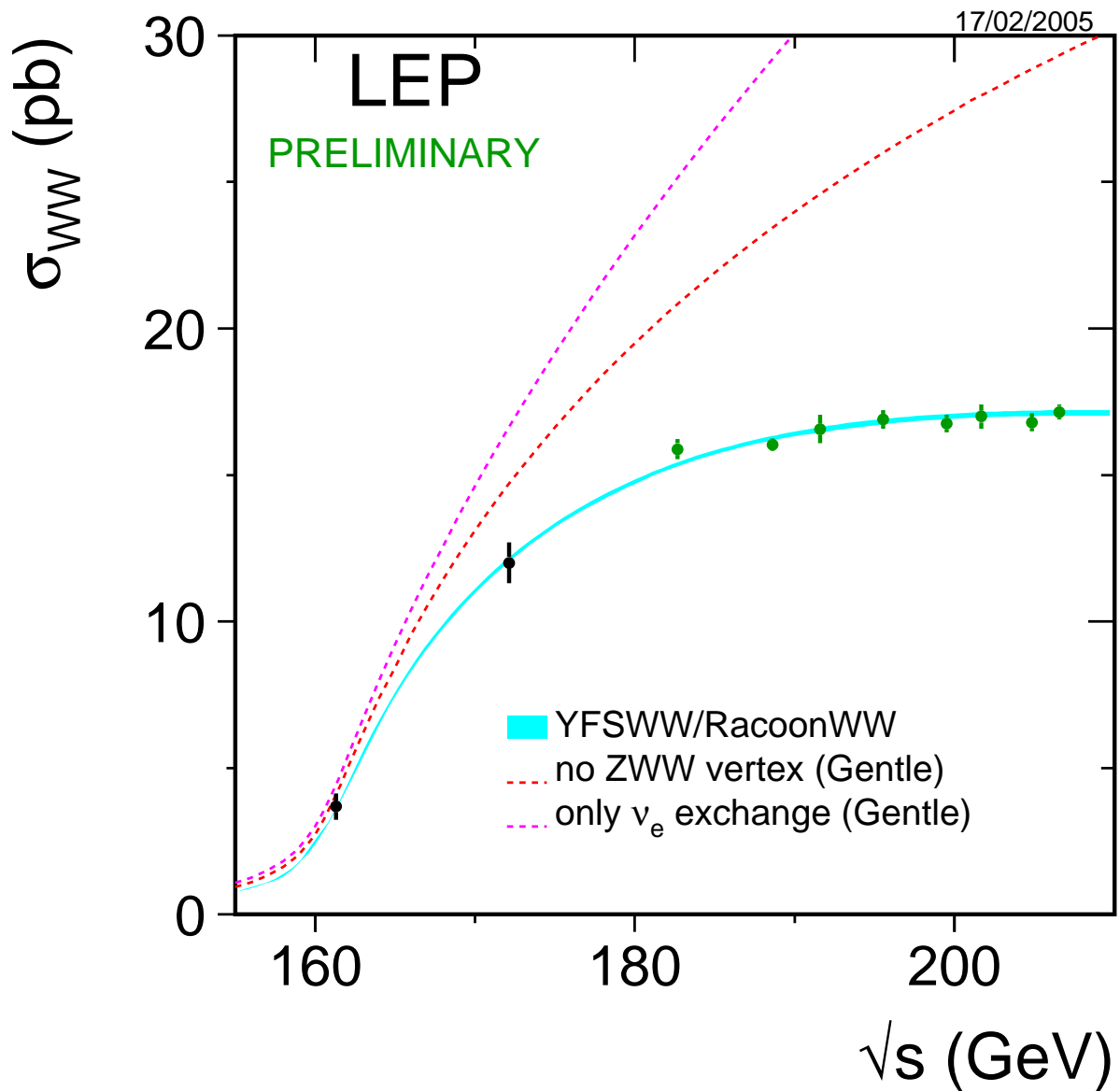


(d)

$J = 1$ partial-wave amplitudes $\mathcal{M}_\gamma^{(1)}$, $\mathcal{M}_Z^{(1)}$, $\mathcal{M}_\nu^{(1)}$ have—individually—unacceptable high-energy behavior ($\propto s$)

... But sum is well-behaved

“Gauge cancellation” observed at LEP2, Tevatron



$J = 0$ amplitude exists because electrons have mass, and can be found in “wrong” helicity state

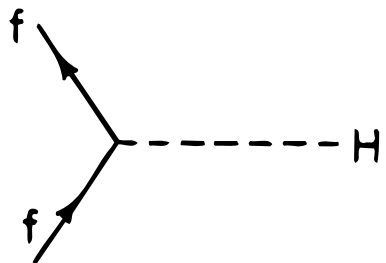
$$\mathcal{M}_\nu^{(0)} \propto s^{\frac{1}{2}} : \text{unacceptable HE behavior}$$

(no contributions from γ and Z)

This divergence is canceled by the Higgs-boson contribution

$$\Rightarrow He\bar{e} \text{ coupling must be } \propto m_e,$$

because “wrong-helicity” amplitudes $\propto m_e$



A Feynman diagram showing two fermion lines (labeled 'f') entering from the left and meeting at a vertex. From this vertex, a dashed line representing a Higgs boson (labeled 'H') extends to the right.

$$\frac{-im_f}{v} = -im_f(G_F \sqrt{2})^{1/2}$$

If the Higgs boson did not exist, *something else* would have to cure divergent behavior

IF gauge symmetry were unbroken . . .

- ▷ no Higgs boson
- ▷ no longitudinal gauge bosons
- ▷ no extreme divergences
- ▷ no wrong-helicity amplitudes

. . . and no viable low-energy phenomenology

In spontaneously broken theory . . .

- ▷ gauge structure of couplings eliminates the most severe divergences
- ▷ lesser—but potentially fatal—divergence arises because the electron has mass
 - . . . due to the Higgs mechanism
- ▷ SSB provides its own cure—the Higgs boson

A similar interplay and compensation *must exist* in any acceptable theory

Bounds on M_H

EW theory does not predict Higgs-boson mass

Self-consistency \Rightarrow plausible lower and upper bounds

▷ Conditional *upper bound* from Unitarity

Compute amplitudes \mathcal{M} for gauge boson scattering at high energies, make a partial-wave decomposition

$$\mathcal{M}(s, t) = 16\pi \sum_J (2J + 1) a_J(s) P_J(\cos \theta)$$

Most channels decouple—pw amplitudes are small at all energies (except very near the particle poles, or at exponentially large energies)—for any M_H .

Four interesting channels:

$$W_L^+ W_L^- \quad Z_L^0 Z_L^0 / \sqrt{2} \quad HH / \sqrt{2} \quad H Z_L^0$$

L : longitudinal, $1/\sqrt{2}$ for identical particles

In HE limit,^a s -wave amplitudes $\propto G_F M_H^2$

$$\lim_{s \gg M_H^2} (a_0) \rightarrow \frac{-G_F M_H^2}{4\pi\sqrt{2}} \cdot \begin{bmatrix} 1 & 1/\sqrt{8} & 1/\sqrt{8} & 0 \\ 1/\sqrt{8} & 3/4 & 1/4 & 0 \\ 1/\sqrt{8} & 1/4 & 3/4 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}$$

Require that largest eigenvalue respect the partial-wave unitarity condition $|a_0| \leq 1$

$$\Rightarrow M_H \leq \left(\frac{8\pi\sqrt{2}}{3G_F} \right)^{1/2} = 1 \text{ TeV}/c^2$$

condition for perturbative unitarity

^aConvenient to calculate using *Goldstone-boson equivalence theorem*, which reduces dynamics of longitudinally polarized gauge bosons to scalar field theory with interaction Lagrangian given by $\mathcal{L}_{\text{int}} = -\lambda v h(2w^+w^- + z^2 + h^2) - (\lambda/4)(2w^+w^- + z^2 + h^2)^2$, with $1/v^2 = G_F\sqrt{2}$ and $\lambda = G_F M_H^2/\sqrt{2}$.

▷ If the bound is respected

- ★ weak interactions remain weak at all energies
- ★ perturbation theory is everywhere reliable

▷ If the bound is violated

- ★ perturbation theory breaks down
- ★ weak interactions among W^\pm , Z , and H become strong on the 1-TeV scale

⇒ features of *strong* interactions at GeV energies will characterize *electroweak* gauge boson interactions at TeV energies

Threshold behavior of the pw amplitudes a_{IJ} follows from chiral symmetry

$$a_{00} \approx G_F s / 8\pi\sqrt{2} \quad \text{attractive}$$

$$a_{11} \approx G_F s / 48\pi\sqrt{2} \quad \text{attractive}$$

$$a_{20} \approx -G_F s / 16\pi\sqrt{2} \quad \text{repulsive}$$

New phenomena are to be found in the EW interactions at energies not much larger than 1 TeV

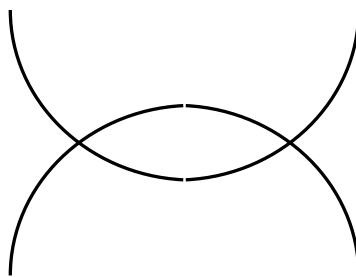
▷ Triviality of scalar field theory

Only *noninteracting* scalar field theories make sense on all energy scales

Quantum field theory vacuum is a dielectric medium that screens charge \Rightarrow *effective charge* is a function of the distance or, equivalently, of the energy scale

running coupling constant

In $\lambda\phi^4$ theory, it is easy to calculate the variation of the coupling constant λ in perturbation theory by summing bubble graphs



$\lambda(\mu)$ is related to a higher scale Λ by

$$\frac{1}{\lambda(\mu)} = \frac{1}{\lambda(\Lambda)} + \frac{3}{2\pi^2} \log(\Lambda/\mu)$$

(Perturbation theory reliable only when λ is small, lattice field theory treats strong-coupling regime)

For stable Higgs potential (*i.e.*, for vacuum energy not to race off to $-\infty$), require $\lambda(\Lambda) \geq 0$

Rewrite RGE as an inequality

$$\frac{1}{\lambda(\mu)} \geq \frac{3}{2\pi^2} \log(\Lambda/\mu) .$$

implies an *upper bound*

$$\lambda(\mu) \leq 2\pi^2/3 \log(\Lambda/\mu)$$

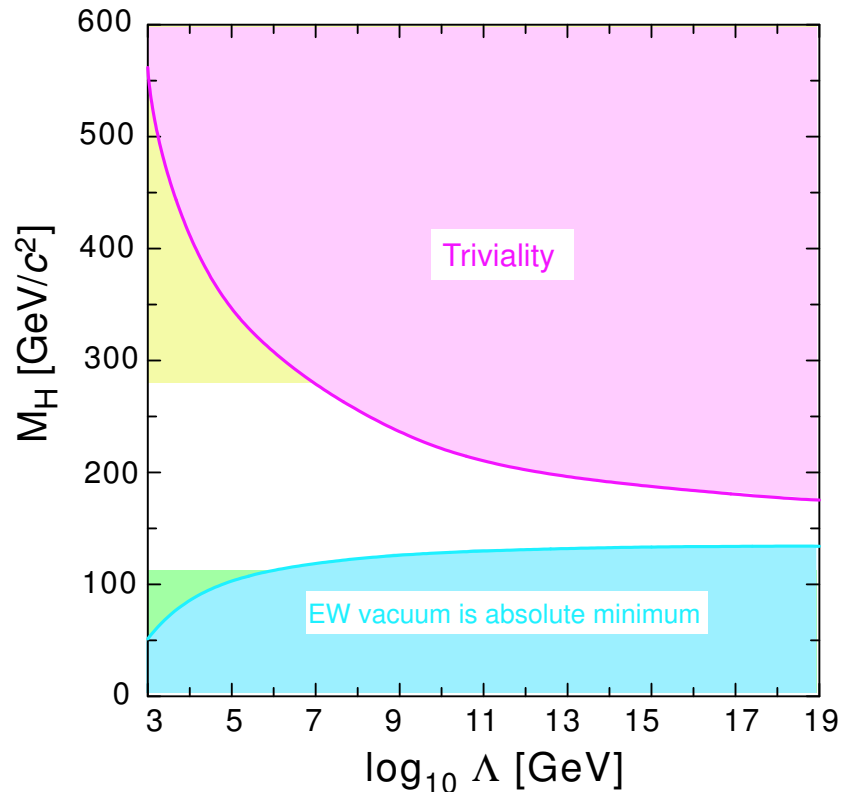
If we require the theory to make sense to arbitrarily high energies—or short distances—then we must take the limit $\Lambda \rightarrow \infty$ while holding μ fixed at some reasonable physical scale. In this limit, the **bound** forces $\lambda(\mu)$ to zero. \rightarrow free field theory “trivial”

Rewrite as bound on M_H :

$$\Lambda \leq \mu \exp\left(\frac{2\pi^2}{3\lambda(\mu)}\right)$$

Choose $\mu = M_H$, and recall $M_H^2 = 2\lambda(M_H)v^2$

$$\Lambda \leq M_H \exp\left(4\pi^2 v^2 / 3M_H^2\right)$$



Moral: For any M_H , there is a *maximum energy scale* Λ^* at which the theory ceases to make sense. The description of the Higgs boson as an elementary scalar is at best an effective theory, valid over a finite range of energies

Perturbative analysis breaks down when $M_H \rightarrow 1 \text{ TeV}/c^2$ and interactions become strong

Lattice analyses $\implies M_H \lesssim 710 \pm 60 \text{ GeV}/c^2$ if theory describes physics to a few percent up to a few TeV

If $M_H \rightarrow 1 \text{ TeV}$ EW theory lives on brink of instability

▷ Lower bound by requiring EWSB vacuum

$$V(v) < V(0)$$

Requiring that $\langle\phi\rangle_0 \neq 0$ be an absolute minimum of the one-loop potential up to a scale Λ yields the vacuum-stability condition

$$M_H^2 > \frac{3G_F\sqrt{2}}{8\pi^2} (2M_W^4 + M_Z^4 - 4m_t^4) \log(\Lambda^2/v^2)$$

... for $m_t \lesssim M_W$

(No illuminating analytic form for heavy m_t)

If the Higgs boson is relatively light—which would itself require explanation—then the theory can be self-consistent up to very high energies

If EW theory is to make sense all the way up to a unification scale $\Lambda^* = 10^{16}$ GeV, then

$$134 \text{ GeV}/c^2 \lesssim M_H \lesssim 177 \text{ GeV}/c^2$$

Higgs-Boson Properties

$$\Gamma(H \rightarrow f\bar{f}) = \frac{G_F m_f^2 M_H}{4\pi\sqrt{2}} \cdot N_c \cdot \left(1 - \frac{4m_f^2}{M_H^2}\right)^{3/2}$$

$\propto M_H$ in the limit of large Higgs mass

$$\Gamma(H \rightarrow W^+W^-) = \frac{G_F M_H^3}{32\pi\sqrt{2}} (1-x)^{1/2} (4-4x+3x^2)$$

$$x \equiv 4M_W^2/M_H^2$$

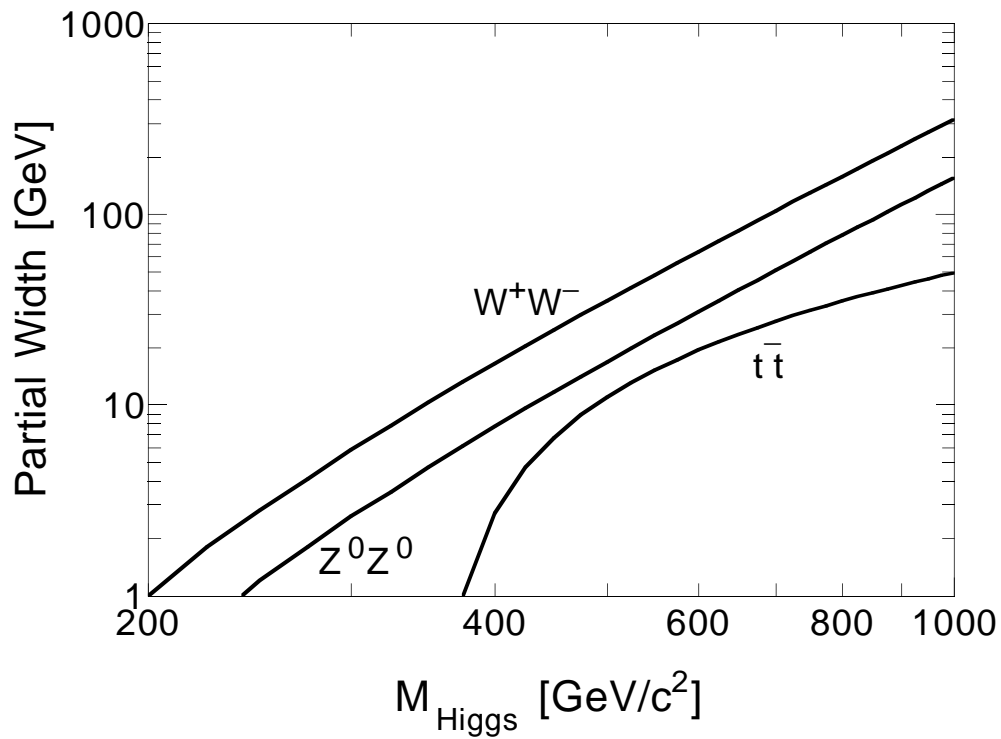
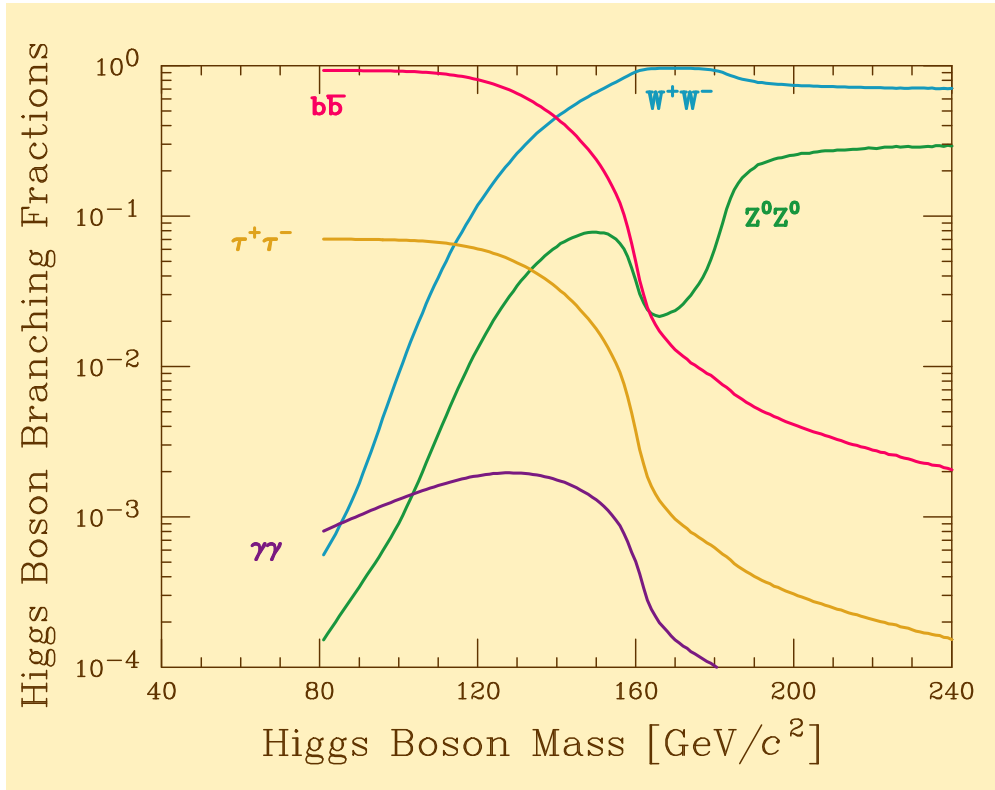
$$\Gamma(H \rightarrow Z^0Z^0) = \frac{G_F M_H^3}{64\pi\sqrt{2}} (1-x')^{1/2} (4-4x'+3x'^2)$$

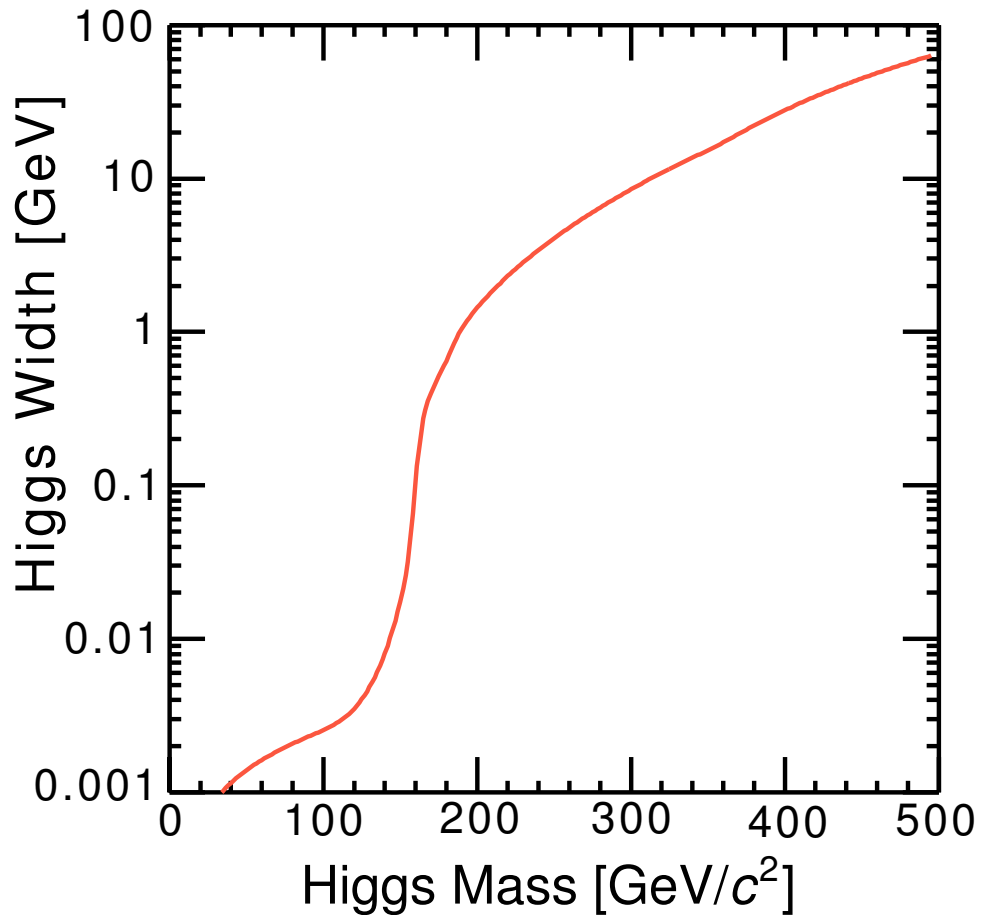
$$x' \equiv 4M_Z^2/M_H^2$$

asymptotically $\propto M_H^3$ and $\frac{1}{2}M_H^3$, respectively
($\frac{1}{2}$ from weak isospin)

$2x^2$ and $2x'^2$ terms \Leftrightarrow decays into transversely polarized gauge bosons

Dominant decays for large M_H into pairs of longitudinally polarized weak bosons





Below W^+W^- threshold, $\Gamma_H \lesssim 1$ GeV

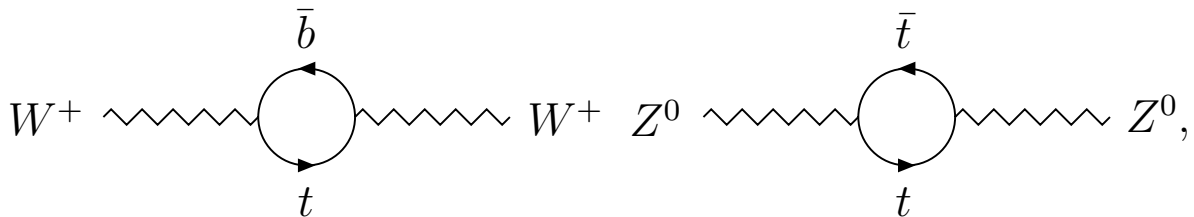
Far above W^+W^- threshold, $\Gamma_H \propto M_H^3$

For $M_H \rightarrow 1$ TeV/c², Higgs boson is an *ephemeron*, with a perturbative width approaching its mass.

Clues to the Higgs-boson mass

Sensitivity of EW observables to m_t gave early indications for massive top

quantum corrections to SM predictions for M_W and M_Z arise from different quark loops



...alter link between the M_W and M_Z :

$$M_W^2 = M_Z^2 (1 - \sin^2 \theta_W) (1 + \Delta\rho)$$

$$\text{where } \Delta\rho \approx \Delta\rho^{(\text{quarks})} = 3G_F m_t^2 / 8\pi^2 \sqrt{2}$$

strong dependence on m_t^2 accounts for precision of m_t estimates derived from EW observables

m_t known to $\pm 3\%$ from Tevatron ...

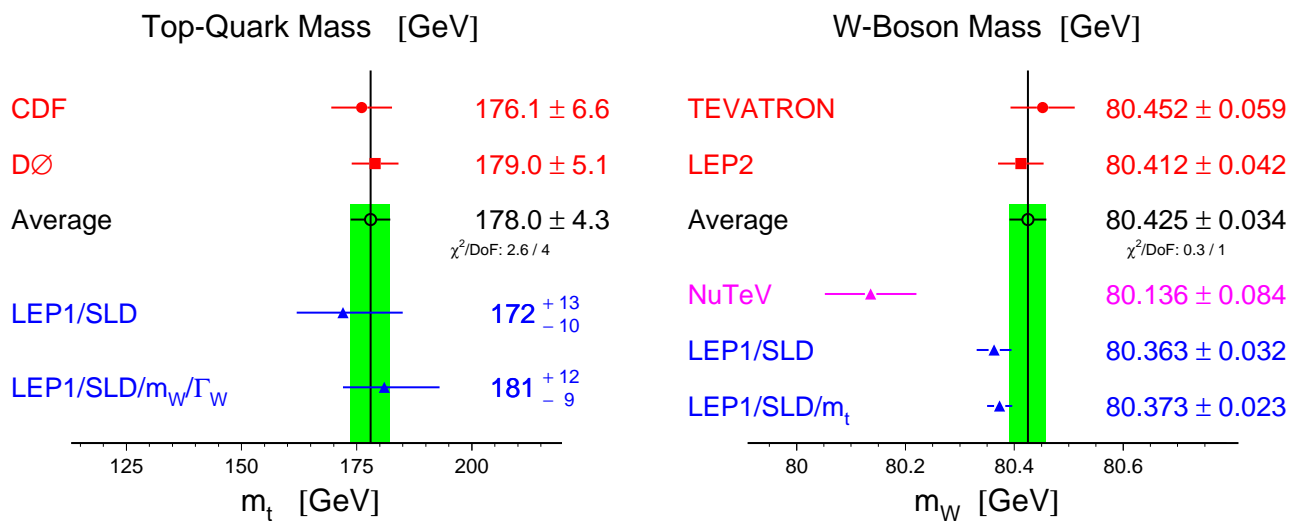
\implies look beyond the quark loops to next most important quantum corrections:

Higgs-boson effects

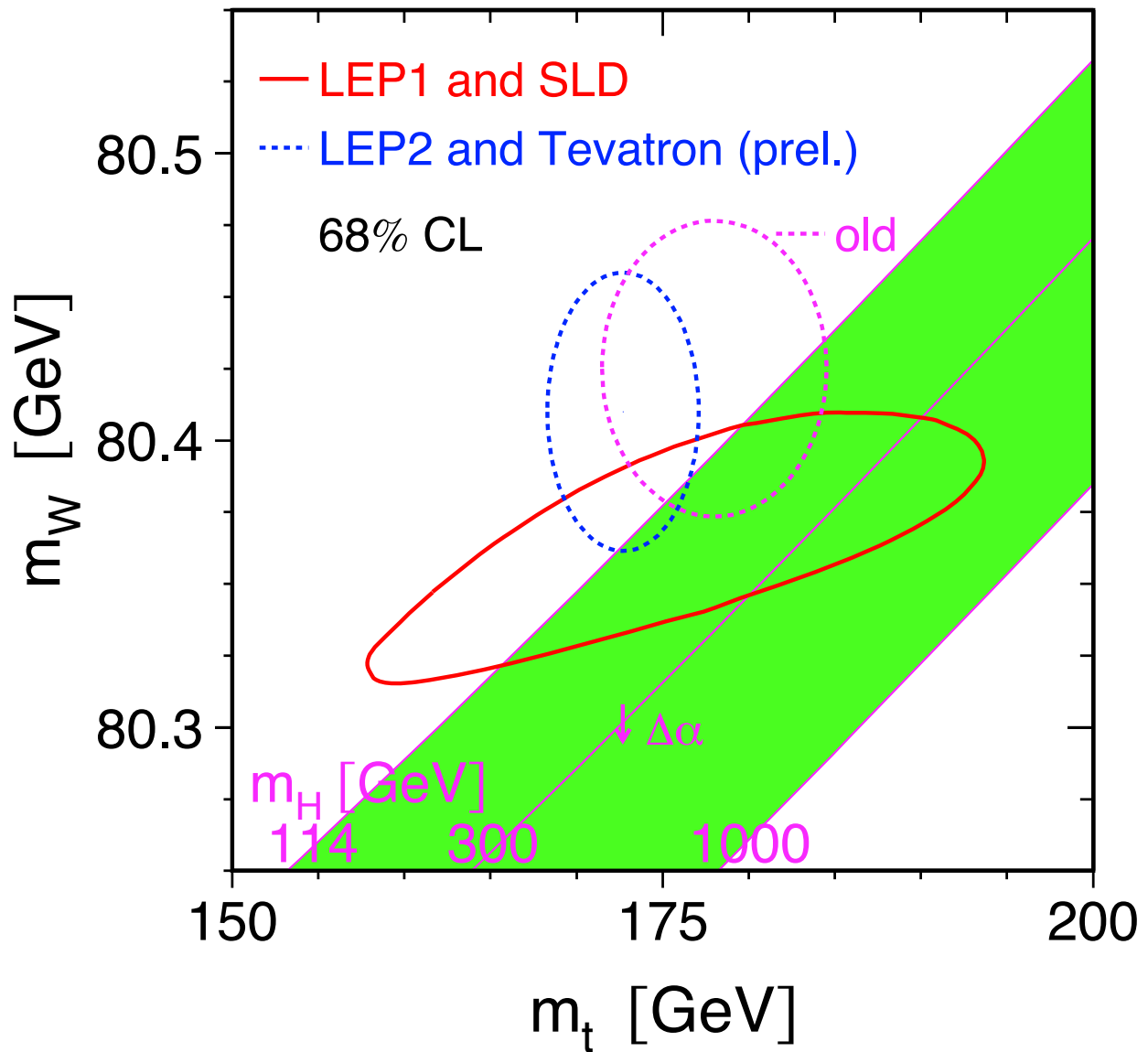
H quantum corrections smaller than t corrections, exhibit more subtle dependence on M_H than the m_t^2 dependence of the top-quark corrections

$$\Delta\rho^{(\text{Higgs})} = C \cdot \ln\left(\frac{M_H}{v}\right)$$

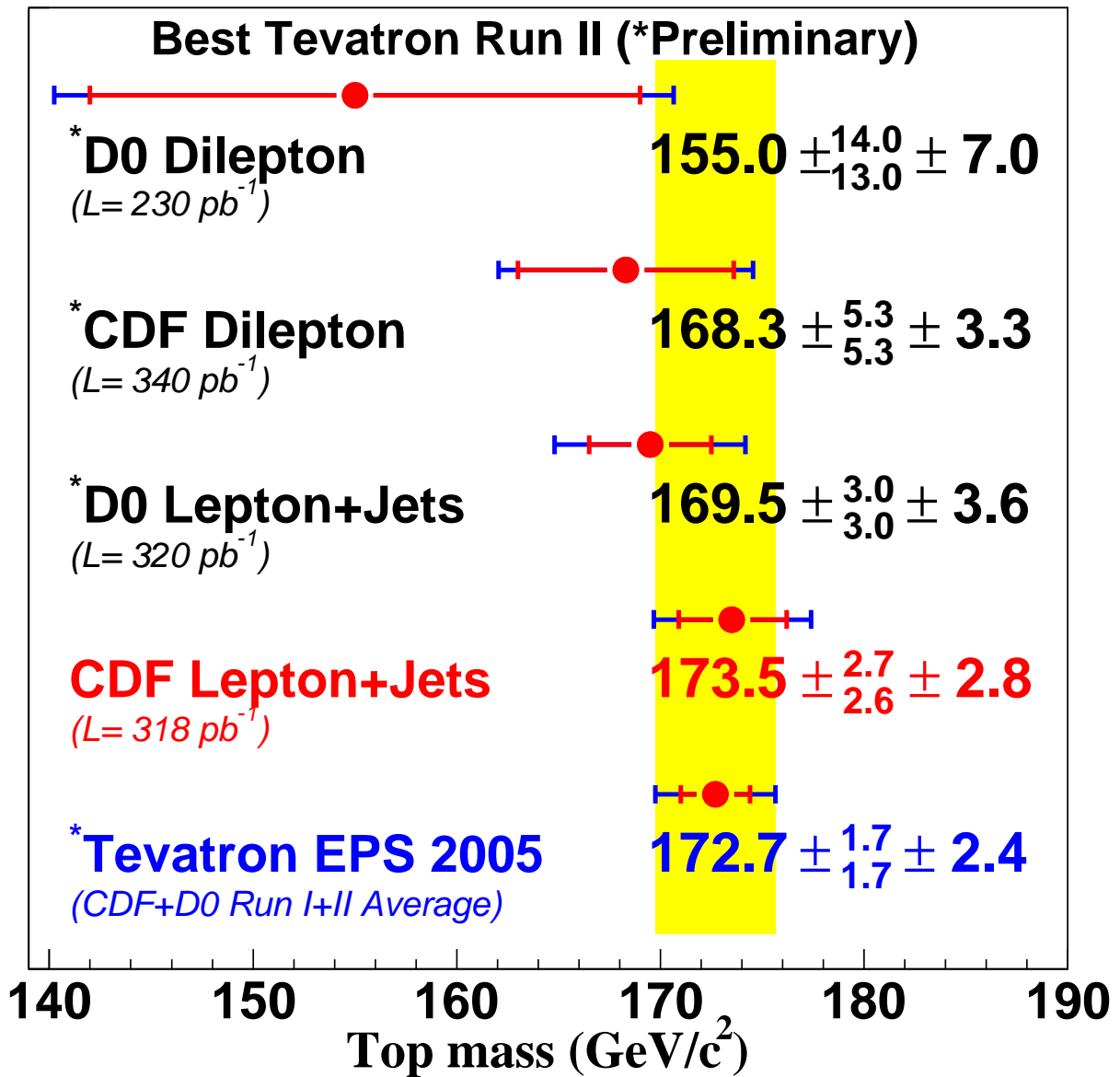
M_Z known to 23 ppm, m_t and M_W well measured



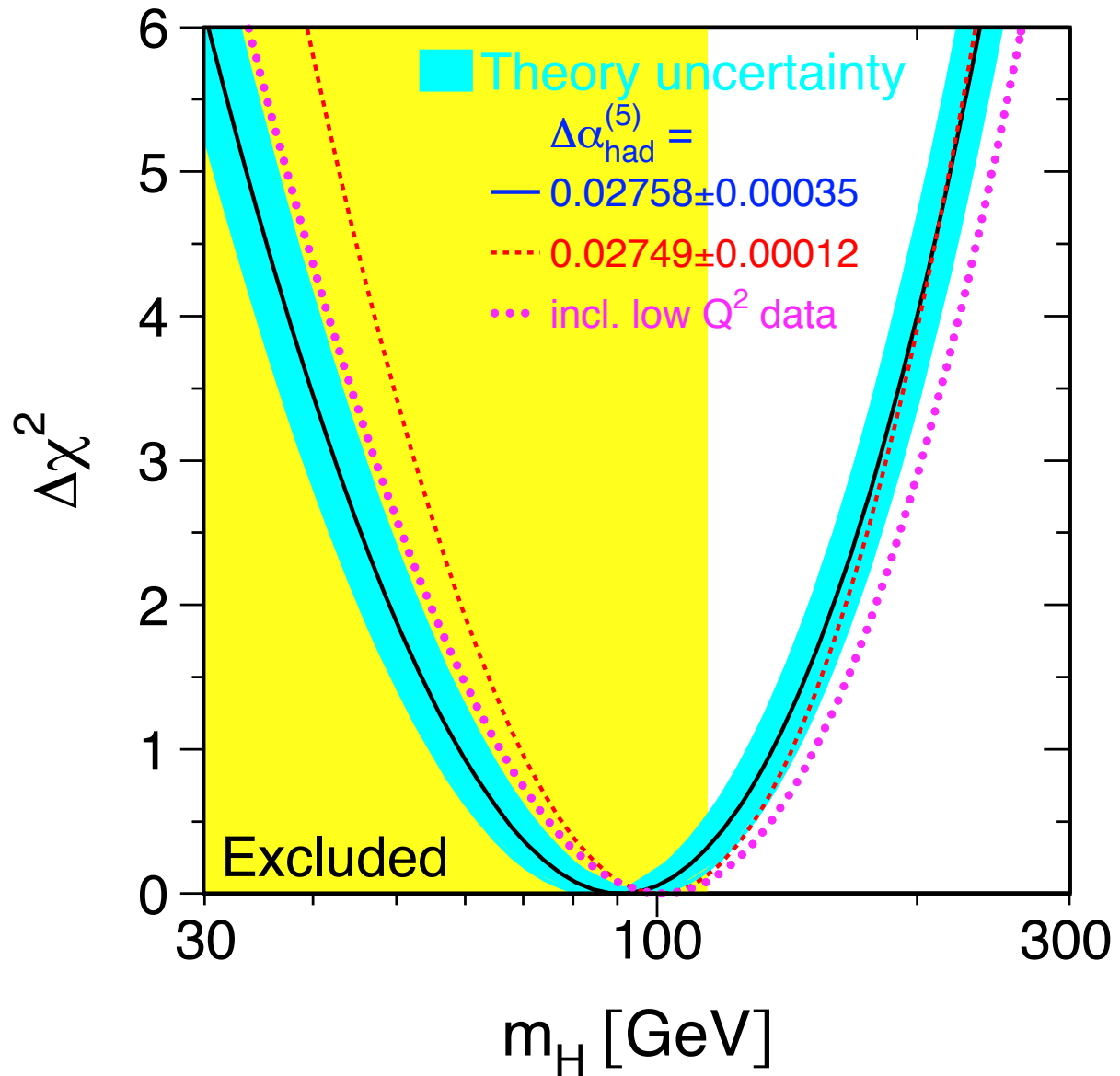
so examine dependence of M_W upon m_t and M_H



Direct, indirect determinations agree reasonably
 Both favor a light Higgs boson,
within framework of SM analysis.



Fit to a universe of data



Within SM, LEPWWG deduce a 95% CL upper limit, $M_H \lesssim 219 \text{ GeV}/c^2$.

Direct searches at LEP $\Rightarrow M_H > 114.4 \text{ GeV}/c^2$,
excluding much of the favored region

either the Higgs boson is just around the corner, or
SM analysis is misleading

Things will soon be popping!

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Things will soon be popping!

Expect progress from M_W - m_t - M_H correlation

- ▷ Tevatron and LHC measurements will determine m_t within 1 or 2 GeV/c^2
- ▷ ...and improve δM_W to about 15 MeV/c^2
- ▷ As the Tevatron's integrated luminosity approaches 10 fb^{-1} , CDF and DØ will begin to explore the region of M_H not excluded by LEP
- ▷ ATLAS and CMS will carry on the exploration of the Higgs sector at the LHC

Assessment

25 YEARS OF CONFIRMATIONS OF

$$SU(2)_L \otimes U(1)_Y$$

★ neutral currents

★ W^\pm, Z^0

★ charm

(+ experimental guidance)

★ τ, ν_τ

★ b, t

+ experimental surprises

★ narrowness of ψ, ψ'

★ long B lifetime

★ large $B^0-\bar{B}^0$ mixing

★ heavy top

★ neutrino oscillations

10 YEARS OF PRECISION MEASUREMENTS...
... FIND NO SIGNIFICANT DEVIATIONS

QUANTUM CORRECTIONS TESTED AT $\pm 10^{-3}$

NO "NEW" PHYSICS ... YET!

Theory tested at distances
from 10^{-17} cm
to $\sim 10^{22}$ cm

origin Coulomb's law (tabletop experiments)

smaller $\left\{ \begin{array}{l} \text{Atomic physics} \rightarrow \text{QED} \\ \text{high-energy experiments} \rightarrow \text{EW theory} \end{array} \right.$

larger $M_\gamma \approx 0$ in planetary ... measurements

IS EW THEORY TRUE ?
COMPLETE ??

EWSB: another path?

Modeled EWSB on Ginzburg–Landau description of SC phase transition

had to introduce new, elementary scalars

GL is not the last word on superconductivity:
dynamical Bardeen–Cooper–Schrieffer theory

The elementary fermions—**electrons**—and gauge interactions—**QED**—needed to generate the scalar bound states are already present in the case of superconductivity. **Could a scheme of similar economy account for EWSB?**

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y + \text{massless } u \text{ and } d$$

Treat $SU(2)_L \otimes U(1)_Y$ as perturbation

$m_u = m_d = 0$: QCD has exact $SU(2)_L \otimes SU(2)_R$ chiral symmetry. At an energy scale $\sim \Lambda_{\text{QCD}}$, strong interactions become strong, fermion condensates appear, and $SU(2)_L \otimes SU(2)_R \rightarrow SU(2)_V$

\implies 3 Goldstone bosons, one for each broken generator: 3 massless pions (Nambu)

Broken generators: 3 axial currents; couplings to π measured by pion decay constant f_π

Turn on $SU(2)_L \otimes U(1)_Y$: EW gauge bosons couple to axial currents, acquire masses of order $\sim g f_\pi$

$$\mathcal{M}^2 = \begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & g^2 & gg' \\ 0 & 0 & gg' & g'^2 \end{pmatrix} \frac{f_\pi^2}{4},$$

$(W^+, W^-, W_3, \mathcal{A})$

same structure as standard EW theory. Diagonalize:

$M_W^2 = g^2 f_\pi^2/4$, $M_Z^2 = (g^2 + g'^2) f_\pi^2/4$, $M_A^2 = 0$, so

$$\frac{M_Z^2}{M_W^2} = \frac{(g^2 + g'^2)}{g^2} = \frac{1}{\cos^2 \theta_W}$$

Massless pions disappear from physical spectrum, to become longitudinal components of weak bosons

$$M_W \approx 30 \text{ MeV}/c^2$$