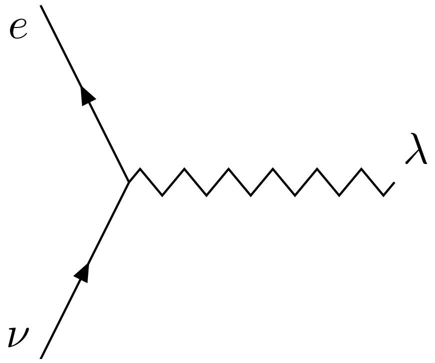
Interactions . . .

$$\mathcal{L}_{W\text{-}\ell} = -\frac{g}{2\sqrt{2}}[\bar{\nu}\gamma^{\mu}(1-\gamma_5)eW_{\mu}^{+} + \bar{e}\gamma^{\mu}(1-\gamma_5)\nu W_{\mu}^{-}]$$

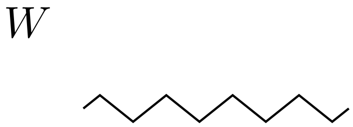
$+$ similar terms for μ and τ

Feynman rule:



$$\frac{-ig}{2\sqrt{2}}\gamma_{\lambda}(1-\gamma_5)$$

gauge-boson propagator:



$$= \frac{-i(g_{\mu\nu} - k_{\mu}k_{\nu}/M_W^2)}{k^2 - M_W^2} \, .$$

Compute $\nu_\mu e \to \mu\nu_e$

$$\sigma(\nu_\mu e \to \mu\nu_e) = \frac{g^4 m_e E_\nu}{16\pi M_W^4} \frac{[1-(m_\mu^2 - m_e^2)/2m_e E_\nu]^2}{(1+2m_e E_\nu/M_W^2)}$$

Reproduces 4-fermion result at low energies if

$$\begin{aligned} \frac{g^4}{16 M_W^4} &= 2G_F^2 \\ \Rightarrow g^4 &= 32(G_F M_W^2)^2 = 64\left(\frac{G_F M_W^2}{\sqrt{2}}\right)^2 \\ \Rightarrow \frac{g}{2\sqrt{2}} &= \left(\frac{G_F M_W^2}{\sqrt{2}}\right)^{\frac{1}{2}} \end{aligned}$$

Using $M_W = gv/2$, determine

$$\boxed{v = (G_F\sqrt{2})^{-\frac{1}{2}} \approx 246 \text{ GeV}}$$

the electroweak scale

$$\Rightarrow \langle\phi^0\rangle_0 = (G_F\sqrt{8})^{-\frac{1}{2}} \approx 174 \text{ GeV}$$

W-propagator modifies HE behavior

$$\sigma(\nu_\mu e \to \mu\nu_e) = \frac{g^4 m_e E_\nu}{16\pi M_W^4} \frac{[1 - (m_\mu^2 - m_e^2)/2m_e E_\nu]^2}{(1 + 2m_e E_\nu/M_W^2)}$$

$$\lim_{E_\nu \to \infty} \sigma(\nu_\mu e \to \mu\nu_e) = \frac{g^4}{32\pi M_W^2} = \frac{G_F^2 M_W^2}{\sqrt{2}}$$

independent of energy!

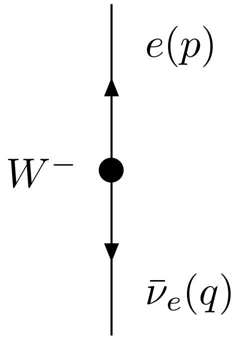
partial-wave unitarity respected for

$$s < M_W^2[\exp(\pi\sqrt{2}/G_F M_W^2) - 1]$$

W-boson properties

No prediction yet for M_W (haven't determined g)

Leptonic decay $W^- \rightarrow\ e^-\nu_e$



$$p \approx \left(\frac{M_W}{2}; \frac{M_W \sin\theta}{2}, 0, \frac{M_W \cos\theta}{2}\right)$$

$$q \approx \left(\frac{M_W}{2}; -\frac{M_W \sin\theta}{2}, 0, -\frac{M_W \cos\theta}{2}\right)$$

$$\mathcal{M} = -i\left(\frac{G_F M_W^2}{\sqrt{2}}\right)^{\frac{1}{2}} \bar{u}(e,p)\gamma_\mu(1-\gamma_5)v(\nu,q)\,\varepsilon^\mu$$

$\varepsilon^\mu = (0;\hat{\varepsilon})$: W polarization vector in its rest frame

$$\begin{aligned}|\mathcal{M}|^2 &= \frac{G_F M_W^2}{\sqrt{2}}\mathrm{tr}\left[\not{\varepsilon}(1-\gamma_5)\not{q}(1+\gamma_5)\not{\varepsilon}^*\not{p}\right]\ ; \\ \mathrm{tr}[\cdots] &= [\varepsilon\cdot q\,\varepsilon^*\cdot p - \varepsilon\cdot\varepsilon^*\,q\cdot p + \varepsilon\cdot p\,\varepsilon^*\cdot q + i\epsilon_{\mu\nu\rho\sigma}\varepsilon^\mu q^\nu \varepsilon^{*\rho}p^\sigma]\end{aligned}$$

decay rate is independent of W polarization; look first at longitudinal pol. $\varepsilon^\mu = (0;0,0,1) = \varepsilon^{*\mu}$, eliminate $\epsilon_{\mu\nu\rho\sigma}$

$$|\mathcal{M}|^2 = \frac{4G_F M_W^4}{\sqrt{2}}\sin^2\theta$$

$$\frac{d\Gamma_0}{d\Omega} = \frac{|\mathcal{M}|^2}{64\pi^2}\,\frac{\mathcal{S}_{12}}{M_W^3}$$

$$\mathcal{S}_{12} = \sqrt{[M_W^2 - (m_e + m_\nu)^2][M_W^2 - (m_e - m_\nu)^2]} = M_W^2$$

$$\frac{d\Gamma_0}{d\Omega} = \frac{G_F M_W^3}{16\pi^2\sqrt{2}}\sin^2\theta$$

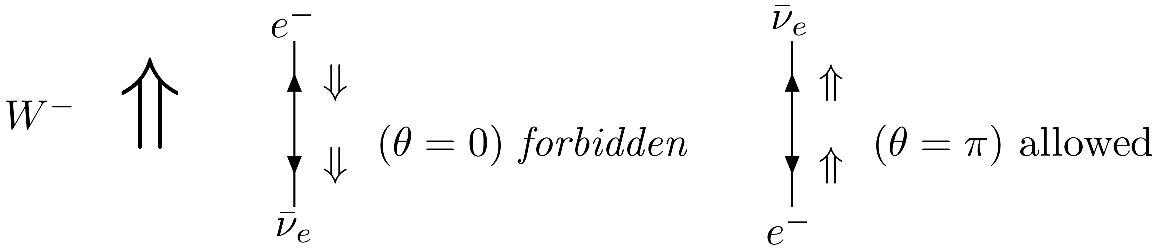
and

$$\boxed{\Gamma(W \to e\nu) = \frac{G_F M_W^3}{6\pi\sqrt{2}}}$$

Other helicities: $\varepsilon^\mu_{\pm 1} = (0; -1, \mp i, 0)/\sqrt{2}$

$$\frac{d\Gamma_{\pm 1}}{d\Omega} = \frac{G_F M_W^3}{32\pi^2\sqrt{2}}(1 \mp \cos\theta)^2$$

Extinctions at $\cos\theta = \pm 1$ are consequences of angular momentum conservation:



(situation reversed for $W^+ \to e^+\nu_e$)

e^+ follows polarization direction of W^+

e^- avoids polarization direction of W^-

important for discovery of $W^\pm$ in $\bar{p}p$ $(\bar{q}q)$ C violation

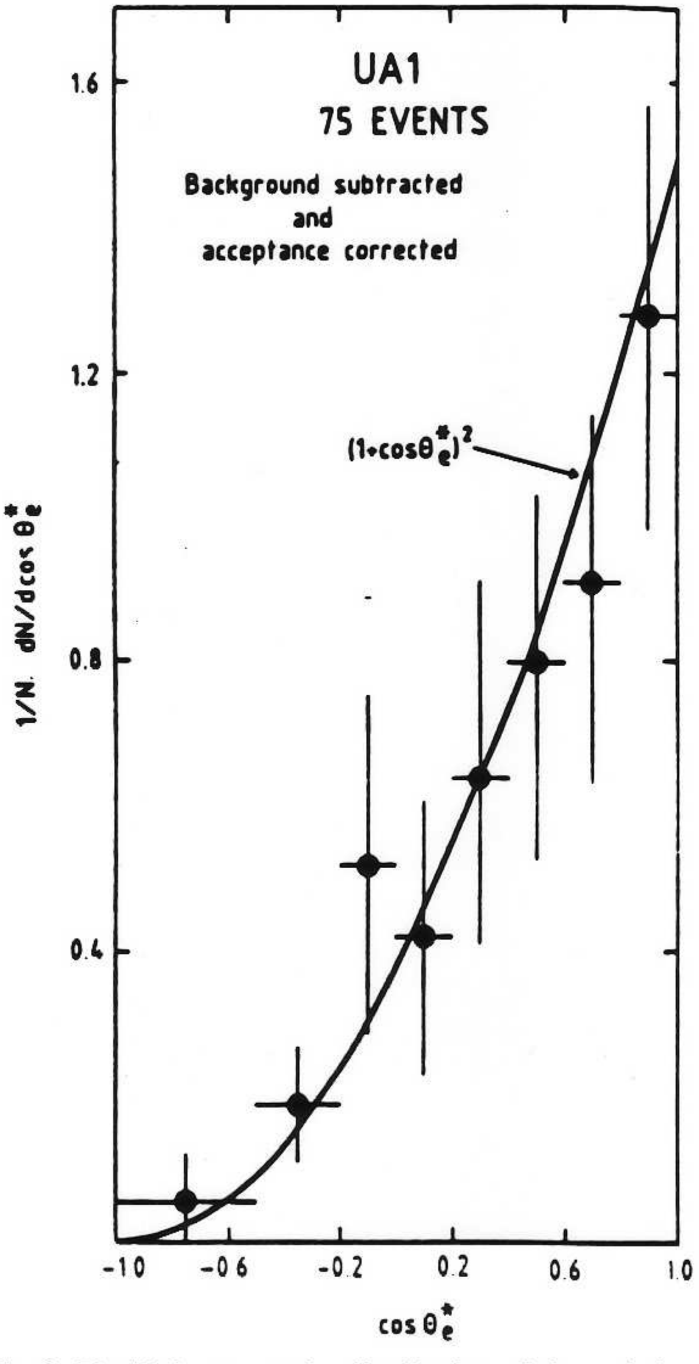


Fig. 2. The W decay angular distribution of the emission angle θ^* of the electron (positron) with respect to the proton (antiproton) direction in the rest frame of the W. Only those events for which the lepton charge and the decay kinematics are well determined have been used. The curve shows the $(V-A)$ expectation of $(1+\cos\theta^*)^2$.

Interactions . . .

$$\mathcal{L}_{A\text{-}\ell} = \frac{gg'}{\sqrt{g^2 + g'^2}} \bar{e}\gamma^\mu e A_\mu$$

. . . vector interaction; ⇒ A_μ as γ, provided

$$\boxed{gg'/\sqrt{g^2 + g'^2} \equiv e}$$

Define $g' = g \tan\theta_W$ θ_W: weak mixing angle

$$\begin{aligned} g &= e/\sin\theta_W \geq e \\ g' &= e/\cos\theta_W \geq e \end{aligned}$$

$Z_\mu = b^3_\mu \cos\theta_W - \mathcal{A}_\mu \sin\theta_W$ $A_\mu = \mathcal{A}_\mu \cos\theta_W + b^3_\mu \sin\theta_W$

$$\mathcal{L}_{Z-\nu} = \frac{-g}{4\cos\theta_W} \bar{\nu}\gamma^\mu(1 - \gamma_5)\nu\, Z_\mu$$

$$\mathcal{L}_{Z-e} = \frac{-g}{4\cos\theta_W} \bar{e}\left[L_e\gamma^\mu(1 - \gamma_5) + R_e\gamma^\mu(1 + \gamma_5)\right] e Z_\mu$$

$$\begin{aligned} L_e &= 2\sin^2\theta_W - 1 = 2x_W + \tau_3 \\ R_e &= 2\sin^2\theta_W = 2x_W \end{aligned}$$

Z-boson properties

Decay calculation analogous to $W^{\pm}$

$$\begin{aligned}\Gamma(Z \to \nu\bar{\nu}) &= \frac{G_F M_Z^3}{12\pi\sqrt{2}} \\ \Gamma(Z \to e^+e^-) &= \Gamma(Z \to \nu\bar{\nu})\left[L_e^2 + R_e^2\right]\end{aligned}$$

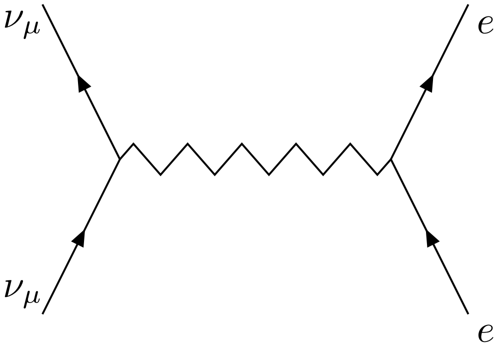
Z-boson properties

Decay calculation analogous to $W^{\pm}$

$$\begin{aligned}\Gamma(Z \to \nu\bar{\nu}) &= \frac{G_F M_Z^3}{12\pi\sqrt{2}} \\ \Gamma(Z \to e^+e^-) &= \Gamma(Z \to \nu\bar{\nu})\left[L_e^2 + R_e^2\right]\end{aligned}$$

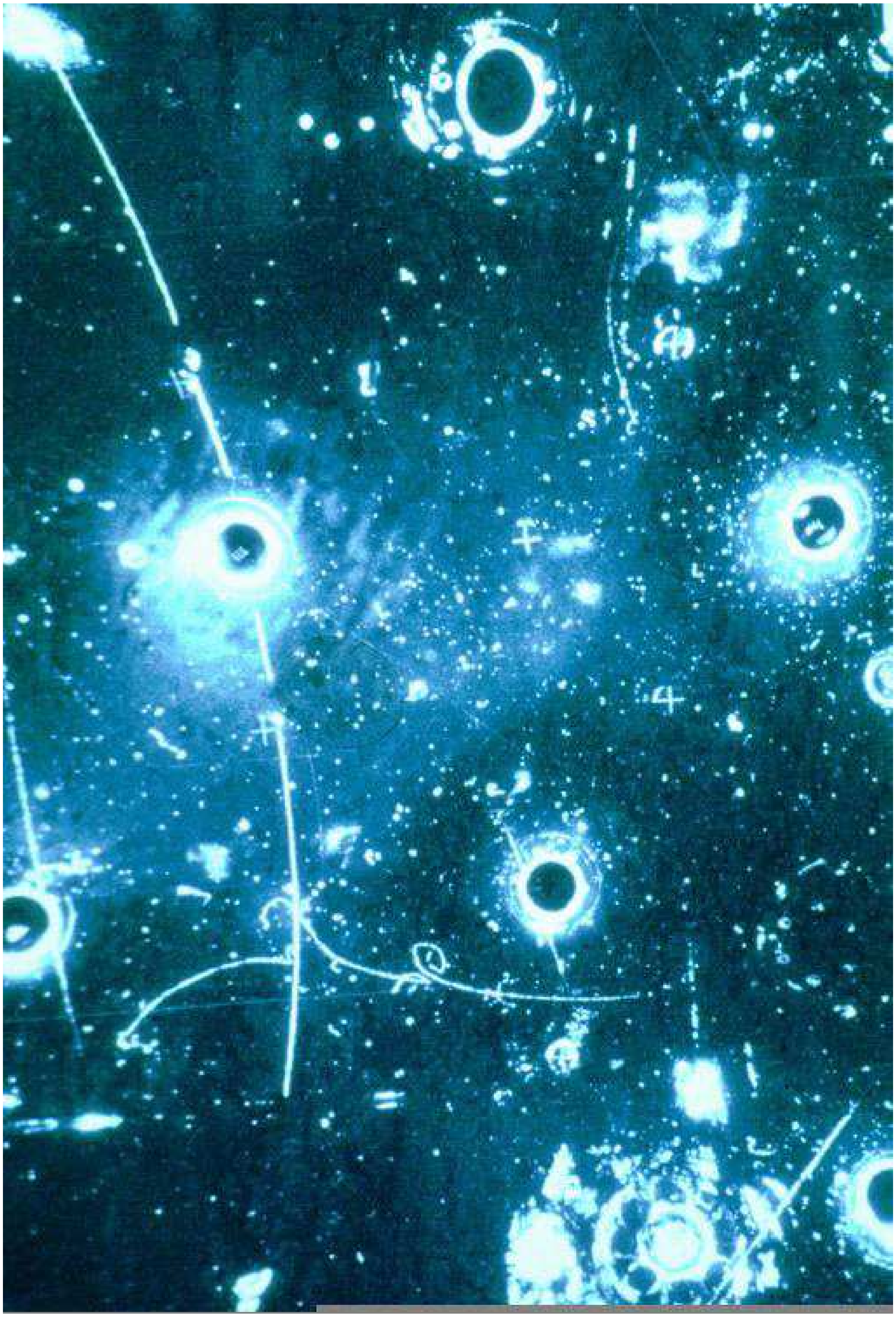
Neutral-current interactions

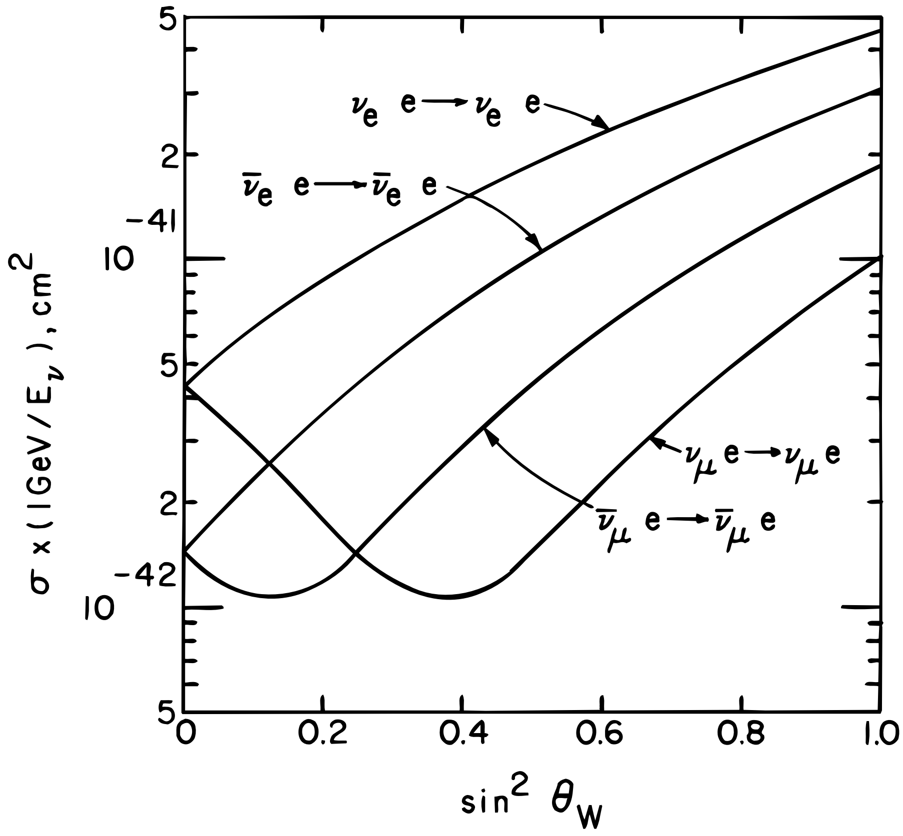
New νe reaction, not present in $V - A$



$$\begin{aligned}\sigma(\nu_\mu e \to \nu_\mu e) &= \frac{G_F^2 m_e E_\nu}{2\pi}\left[L_e^2 + R_e^2/3\right] \\ \sigma(\bar{\nu}_\mu e \to \bar{\nu}_\mu e) &= \frac{G_F^2 m_e E_\nu}{2\pi}\left[L_e^2/3 + R_e^2\right] \\ \sigma(\nu_e e \to \nu_e e) &= \frac{G_F^2 m_e E_\nu}{2\pi}\left[(L_e + 2)^2 + R_e^2/3\right] \\ \sigma(\bar{\nu}_e e \to \bar{\nu}_e e) &= \frac{G_F^2 m_e E_\nu}{2\pi}\left[(L_e + 2)^2/3 + R_e^2\right]\end{aligned}$$

Gargamelle $\nu_\mu e$ Event





“Model-independent” analysis

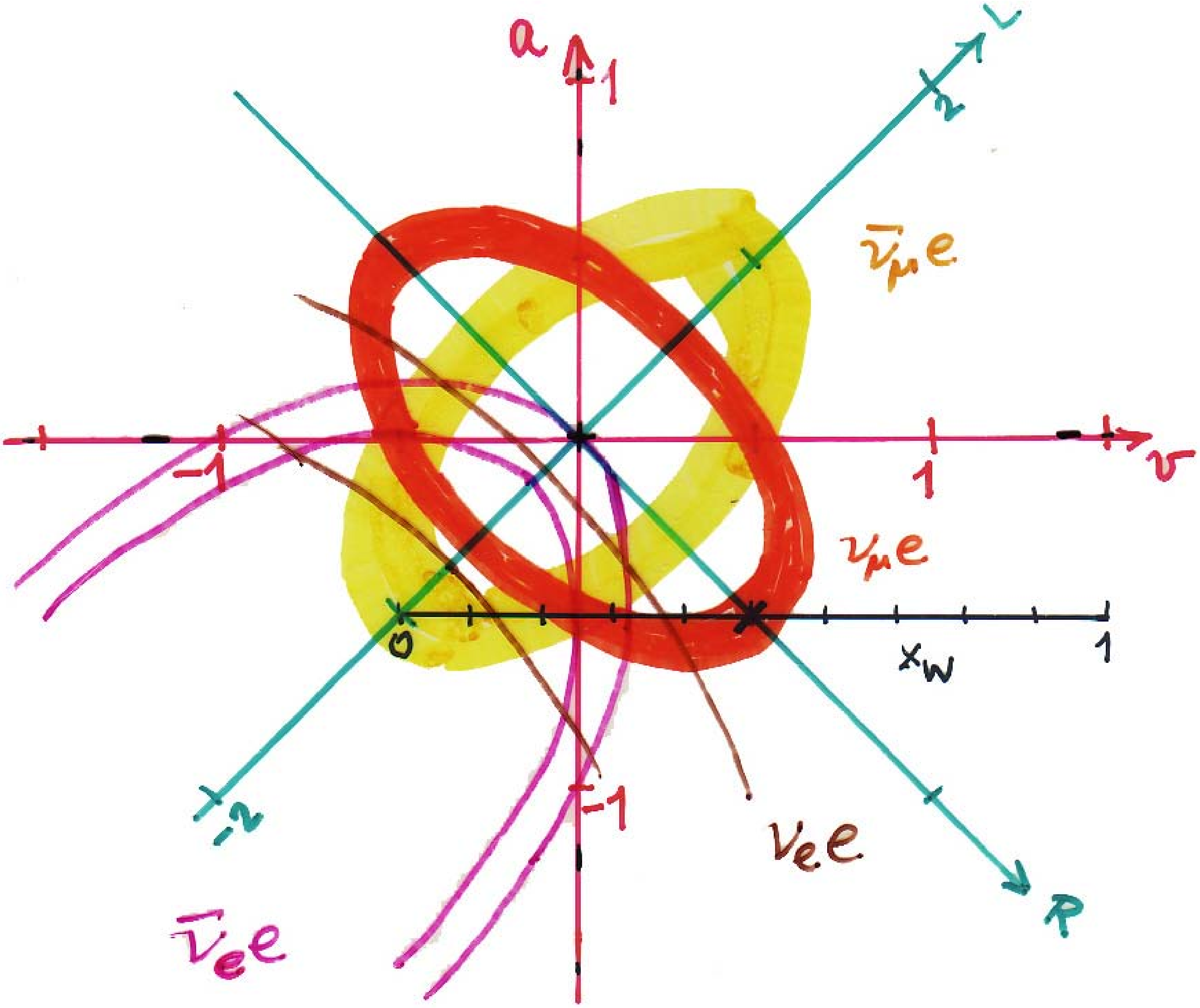
Measure all cross sections to determine chiral couplings L_e and R_e or traditional vector and axial couplings v and a

$$a = \tfrac{1}{2}(L_e - R_e) \qquad v = \tfrac{1}{2}(L_e - R_e)$$

$$L_e = v + a \qquad R_e = v - a$$

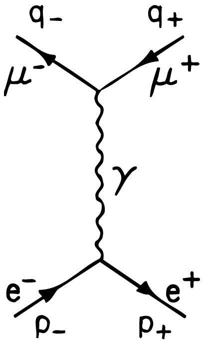
model-independent in V, A framework

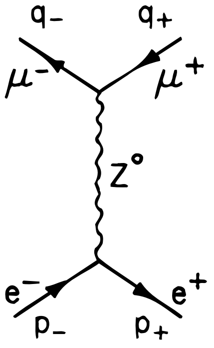
Neutrino–electron scattering



Twofold ambiguity remains even after measuring all four cross sections: same cross sections result if we interchange $R_e \leftrightarrow -R_e$ $(v \leftrightarrow a)$

Consider $e^+e^- \to \mu^+\mu^-$





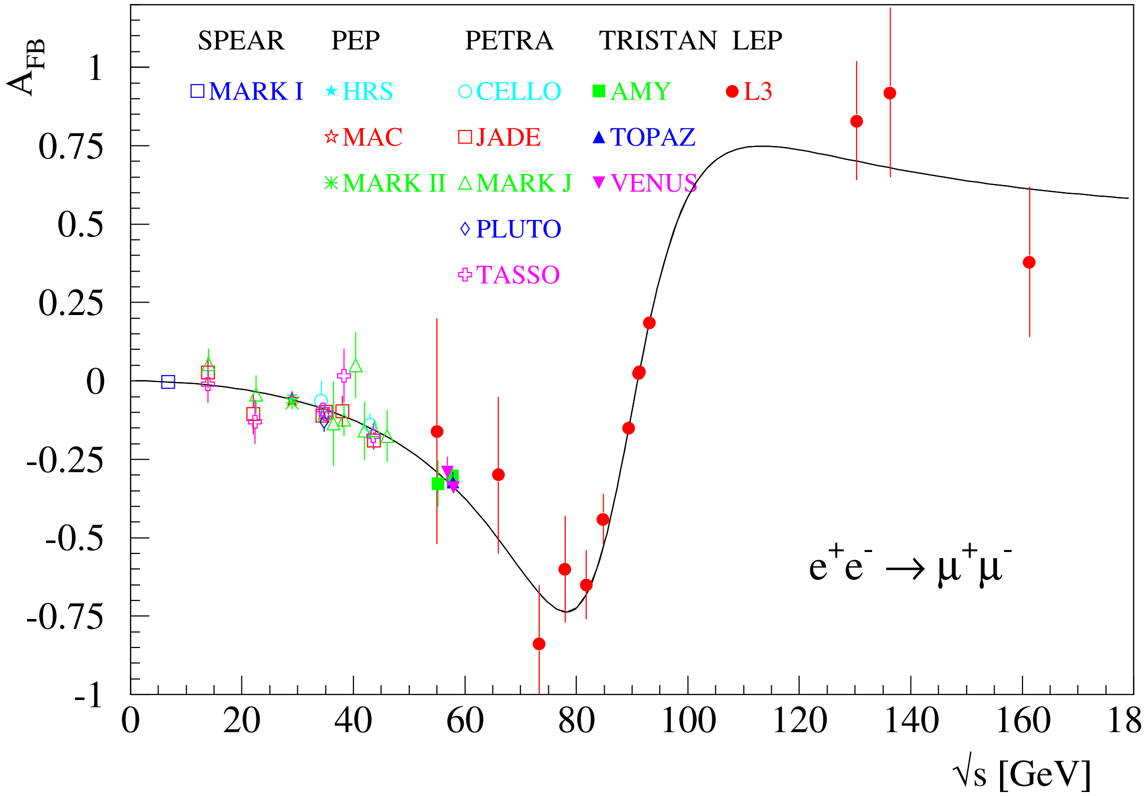
$$\begin{aligned}
\mathcal{M} \quad &= \quad -ie^2\bar{u}(\mu,q_-)\gamma_\lambda Q_\mu v(\mu,q_+)\frac{g^{\lambda\nu}}{s}\bar{v}(e,p_+)\gamma_\nu u(e.p_-) \\
&+\frac{i}{2}\left(\frac{G_F M_Z^2}{\sqrt{2}}\right)\bar{u}(\mu,q_-)\gamma_\lambda[R_\mu(1+\gamma_5)+L_\mu(1-\gamma_5)]v(\mu,q_+) \\
&\times\frac{g^{\lambda\nu}}{s-M_Z^2}\bar{v}(e,p_+)\gamma_\nu[R_e(1+\gamma_5)+L_e(1-\gamma_5)]u(e,p_-)
\end{aligned}$$

muon charge $Q_\mu = -1$

$$\begin{aligned}
\frac{d\sigma}{dz} \quad &= \quad \frac{\pi\alpha^2 Q_\mu^2}{2s}(1+z^2) \\
&-\frac{\alpha Q_\mu G_F M_Z^2(s-M_Z^2)}{8\sqrt{2}[(s-M_Z^2)^2+M_Z^2\Gamma^2]} \\
&\times[(R_e+L_e)(R_\mu+L_\mu)(1+z^2)+2(R_e-L_e)(R_\mu-L_\mu)z] \\
&+\frac{G_F^2 M_Z^4 s}{64\pi[(s-M_Z^2)^2+M_Z^2\Gamma^2]} \\
&\times[(R_e^2+L_e^2)(R_\mu^2+L_\mu^2)(1+z^2)+2(R_e^2-L_e^2)(R_\mu^2-L_\mu^2)z]
\end{aligned}$$

$$\text{F-B asymmetry } A \equiv \frac{\int_0^1 \, dz \, d\sigma/dz - \int_{-1}^0 \, dz \, d\sigma/dz}{\int_{-1}^1 \, dz \, d\sigma/dz}$$

$$\begin{aligned}\lim_{s/M_Z^2 \ll 1} A &= \frac{3G_F s}{16\pi\alpha Q_\mu \sqrt{2}}(R_e - L_e)(R_\mu - L_\mu) \\ &\approx -6.7 \times 10^{-5}\left(\frac{s}{1\ \text{GeV}^2}\right)(R_e - L_e)(R_\mu - L_\mu) \\ &= -3G_F s a^2/4\pi\alpha\sqrt{2}\end{aligned}$$

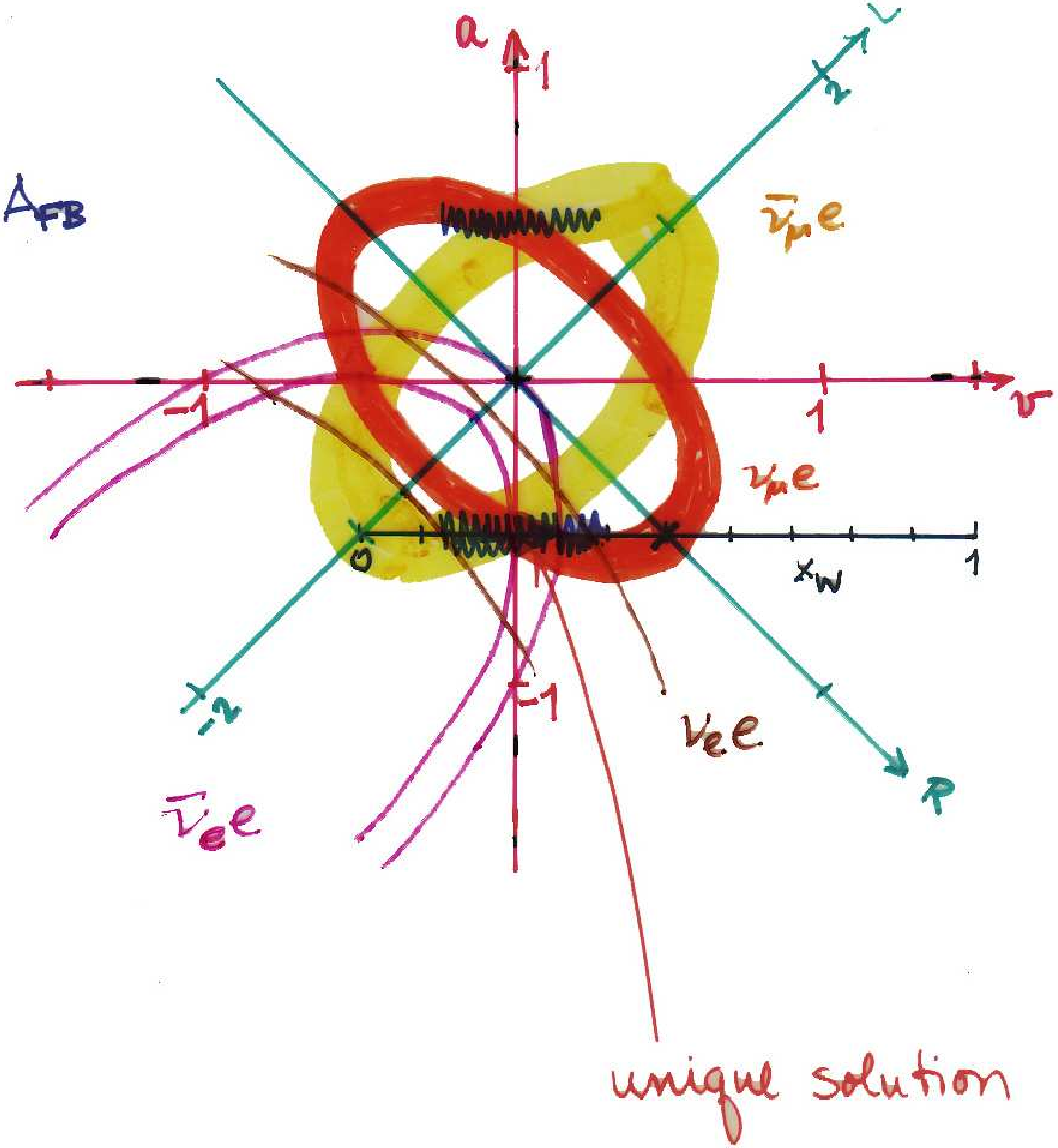


J. Mnich *Phys. Rep.* **271**, 181-266 (1996)

Measuring A resolves ambiguity

Validate EW theory, measure $\sin^2\theta_W$

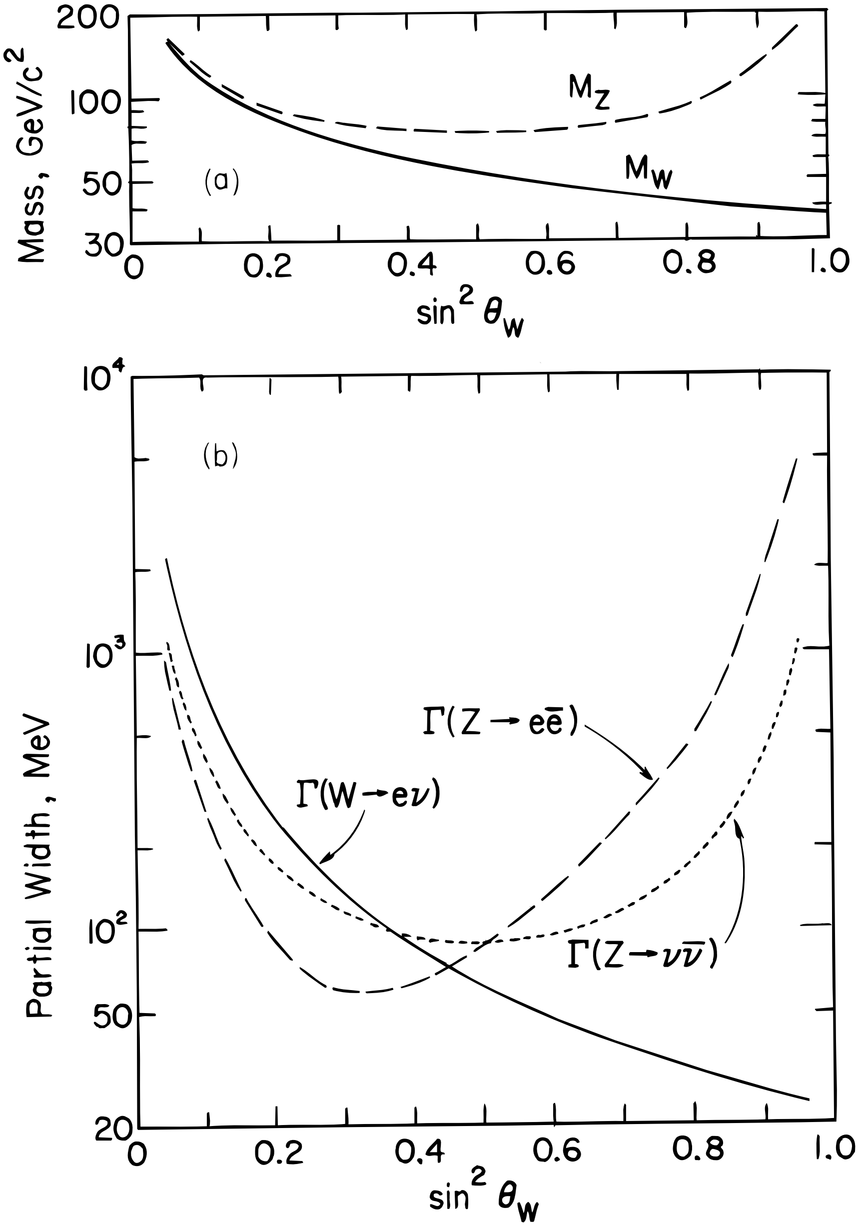
Neutrino–electron scattering



With a measurement of $\sin^2\theta_W$, predict

$$M_W^2 = g^2v^2/4 = e^2/4G_F\sqrt{2}\sin^2\theta_W \approx (37.3\ \text{GeV}/c^2)^2/\sin^2\theta_W$$

$$M_Z^2 = M_W^2/\cos^2\theta_W$$



568 Intermediate Vector Bosons W^+, W^-, and Z^0

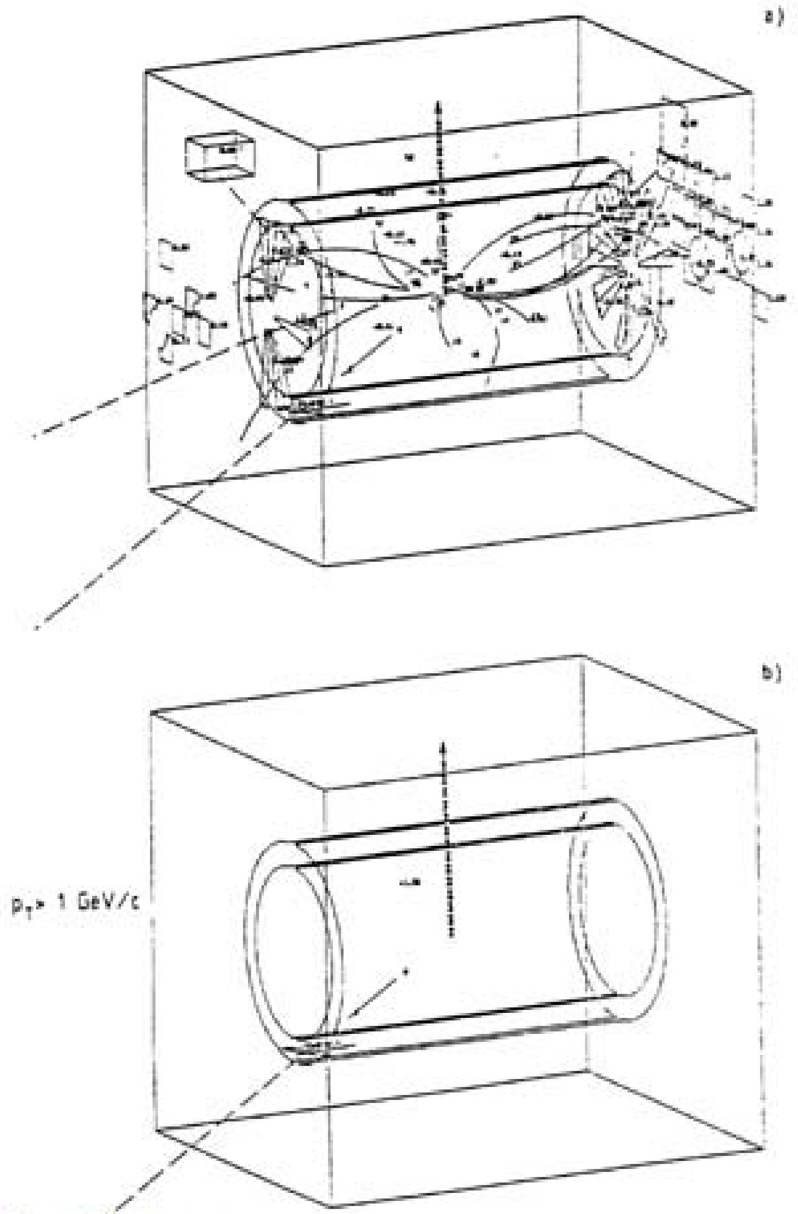
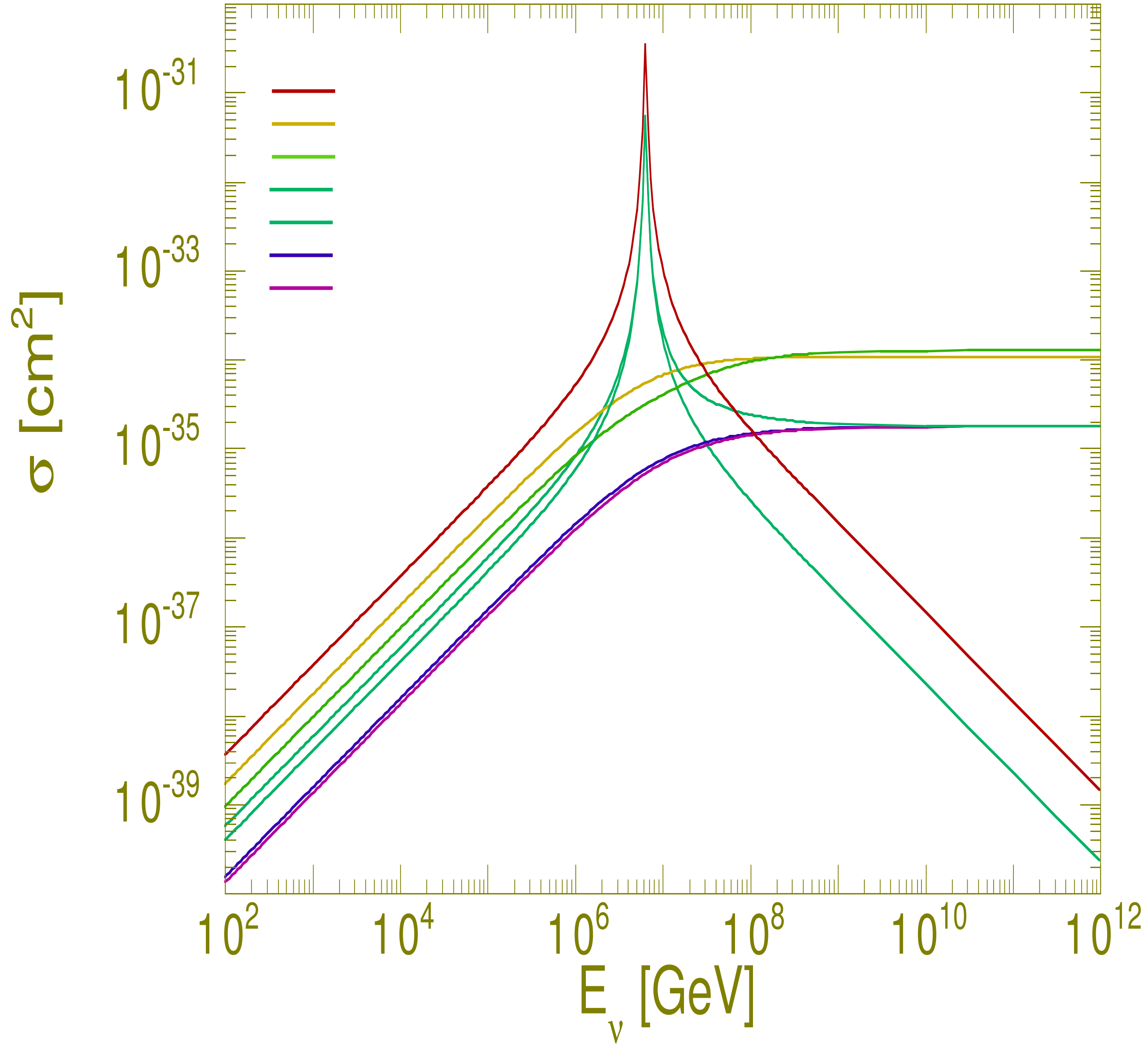


Fig. 16a,b

UA1



At low energies: $\sigma(\bar{\nu}_e e \rightarrow \text{ hadrons}) > \sigma(\nu_\mu e \rightarrow \mu\nu_e) >$
$\sigma(\nu_e e \rightarrow \nu_e e) > \sigma(\bar{(}\nu_e e \rightarrow \bar{\nu}_\mu \mu) > \sigma(\bar{\nu}_e e \rightarrow \bar{\nu}_e e) >$
$\sigma(\nu_\mu e \rightarrow \nu_\mu e) > \sigma(\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e)$

EW interactions of quarks

- ▷ Left-handed doublet

$$\begin{array}{cccc} & I_3 & Q & Y = 2(Q - I_3) \\ \mathsf{L}_q = \begin{pmatrix} u \\ d \end{pmatrix}_L & \begin{array}{c} \frac{1}{2} \\ -\frac{1}{2} \end{array} & \begin{array}{c} +\frac{2}{3} \\ -\frac{1}{3} \end{array} & \frac{1}{3} \end{array}$$

- ▷ two right-handed singlets

	I_3	Q	$Y = 2(Q - I_3)$
$\mathsf{R}_u = u_R$	0	$+\frac{2}{3}$	$+\frac{4}{3}$
$\mathsf{R}_d = d_R$	0	$-\frac{1}{3}$	$-\frac{2}{3}$

- ▷ CC interaction

$$\mathcal{L}_{W\text{-}q} = \frac{-g}{2\sqrt{2}} \left[\bar{u}_e \gamma^\mu (1 - \gamma_5) d \, W_\mu^+ + \bar{d} \gamma^\mu (1 - \gamma_5) u \, W_\mu^- \right]$$

identical in form to $\mathcal{L}_{W\text{-}\ell}$: universality ⇔ weak isospin

- ▷ NC interaction

$$\mathcal{L}_{Z\text{-}q} = \frac{-g}{4 \cos\theta_W} \sum_{i=u,d} \bar{q}_i \gamma^\mu \left[L_i (1 - \gamma_5) + R_i (1 + \gamma_5) \right] q_i Z_\mu$$

$$L_i = \tau_3 - 2Q_i \sin^2\theta_W \qquad R_i = -2Q_i \sin^2\theta_W$$

equivalent in form (not numbers) to $\mathcal{L}_{Z\text{-}\ell}$

Trouble in Paradise

Universal $u \leftrightarrow d$, $\nu_e \leftrightarrow e$ *not quite right*

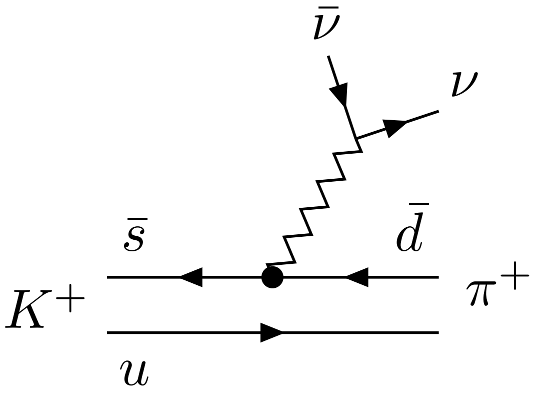
$$\text{Good:} \quad \left(\begin{array}{c} u \\ d \end{array}\right)_L \longrightarrow \text{Better:} \quad \left(\begin{array}{c} u \\ d_\theta \end{array}\right)_L$$

$$d_\theta \equiv d\,\cos\theta_C + s\,\sin\theta_C \qquad \cos\theta_C = 0.9736 \pm 0.0010$$

"Cabibbo-rotated" doublet perfects CC interaction (up to small third-generation effects) but ⇒ serious trouble for NC

$$\begin{aligned} \mathcal{L}_{Z\text{-}q} \quad = \quad & \frac{-g}{4\cos\theta_W} Z_\mu \left\{ \bar{u}\gamma^\mu \left[L_u(1-\gamma_5) + R_u(1+\gamma_5) \right] u \right. \\ & + \bar{d}\gamma^\mu \left[L_d(1-\gamma_5) + R_d(1+\gamma_5) \right] d\,\cos^2\theta_C \\ & + \bar{s}\gamma^\mu \left[L_d(1-\gamma_5) + R_d(1+\gamma_5) \right] s\,\sin^2\theta_C \\ & + \bar{d}\gamma^\mu \left[L_d(1-\gamma_5) + R_d(1+\gamma_5) \right] s\,\sin\theta_C\cos\theta_C \\ & \left. + \,\bar{s}\gamma^\mu \left[L_d(1-\gamma_5) + R_d(1+\gamma_5) \right] d\,\sin\theta_C\cos\theta_C \right\} \end{aligned}$$

Strangeness-changing NC interactions highly suppressed!



BNL E-787/E-949 has three $K^+ \to \pi^+\nu\bar{\nu}$ candidates, with $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu}) = 1.47^{+1.30}_{-0.89} \times 10^{-10}$

(SM: 0.8 ± 0.3)

Phys. Rev. Lett. **93**, 031801 (2004)

Glashow–Iliopoulos–Maiani

two left-handed doublets

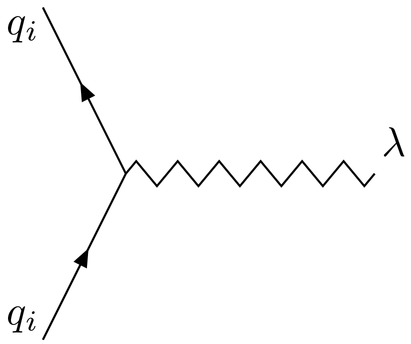
$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \qquad \begin{pmatrix} u \\ d_\theta \end{pmatrix}_L \quad \begin{pmatrix} c \\ s_\theta \end{pmatrix}_L$$

$$(s_\theta = s\cos\theta_C - d\sin\theta_C)$$

$+$ right-handed singlets, e_R, μ_R, u_R, d_R, c_R, s_R

Required new charmed quark, c

Cross terms vanish in $\mathcal{L}_{Z\text{-}q}$,



$$\frac{-ig}{4\cos\theta_W}\gamma_\lambda[(1-\gamma_5)L_i + (1+\gamma_5)R_i] \quad ,$$

$$L_i = \tau_3 - 2Q_i\sin^2\theta_W \qquad R_i = -2Q_i\sin^2\theta_W$$

flavor-diagonal interaction!

Straightforward generalization to n quark doublets

$$\mathcal{L}_{W\text{-}q} = \frac{-g}{2\sqrt{2}} \left[\bar{\Psi}\gamma^{\mu}(1-\gamma_5)\mathcal{O}\Psi\, W_{\mu}^{+} + \text{h.c.}\right]$$

composite $\Psi = \begin{pmatrix} u \\ c \\ \vdots \\ \\ d \\ s \\ \vdots \end{pmatrix}$

flavor structure $\mathcal{O} = \begin{pmatrix} 0 & U \\ 0 & 0 \end{pmatrix}$

U: unitary quark mixing matrix

Weak-isospin part: $\mathcal{L}_{Z\text{-}q}^{\text{iso}} = \dfrac{-g}{4\cos\theta_W}\bar{\Psi}\gamma^{\mu}(1-\gamma_5)\left[\mathcal{O},\mathcal{O}^{\dagger}\right]\Psi$

Since $\left[\mathcal{O},\mathcal{O}^{\dagger}\right] = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \propto \tau_3$

$\boxed{\Rightarrow \text{NC interaction is flavor-diagonal}}$

General $n \times n$ quark-mixing matrix U:

$n(n-1)/2$ real ∠, $(n-1)(n-2)/2$ complex phases

3×3 (Cabibbo–Kobayashi-Maskawa): 3 ∠ + 1 phase

⇒ CP violation

Qualitative successes of $SU(2)_L \otimes U(1)_Y$ theory:

- ▷ neutral-current interactions
- ▷ necessity of charm
- ▷ existence and properties of $W^{\pm}$ and Z^0

Decade of precision tests EW (one-per-mille)

M_Z	$91\,187.6 \pm 2.1$ MeV/c^2
Γ_Z	2495.2 ± 2.3 MeV
$\sigma^0_{\text{hadronic}}$	41.541 ± 0.037 nb
Γ_{hadronic}	1744.4 ± 2.0 MeV
Γ_{leptonic}	83.984 ± 0.086 MeV
$\Gamma_{\text{invisible}}$	499.0 ± 1.5 MeV

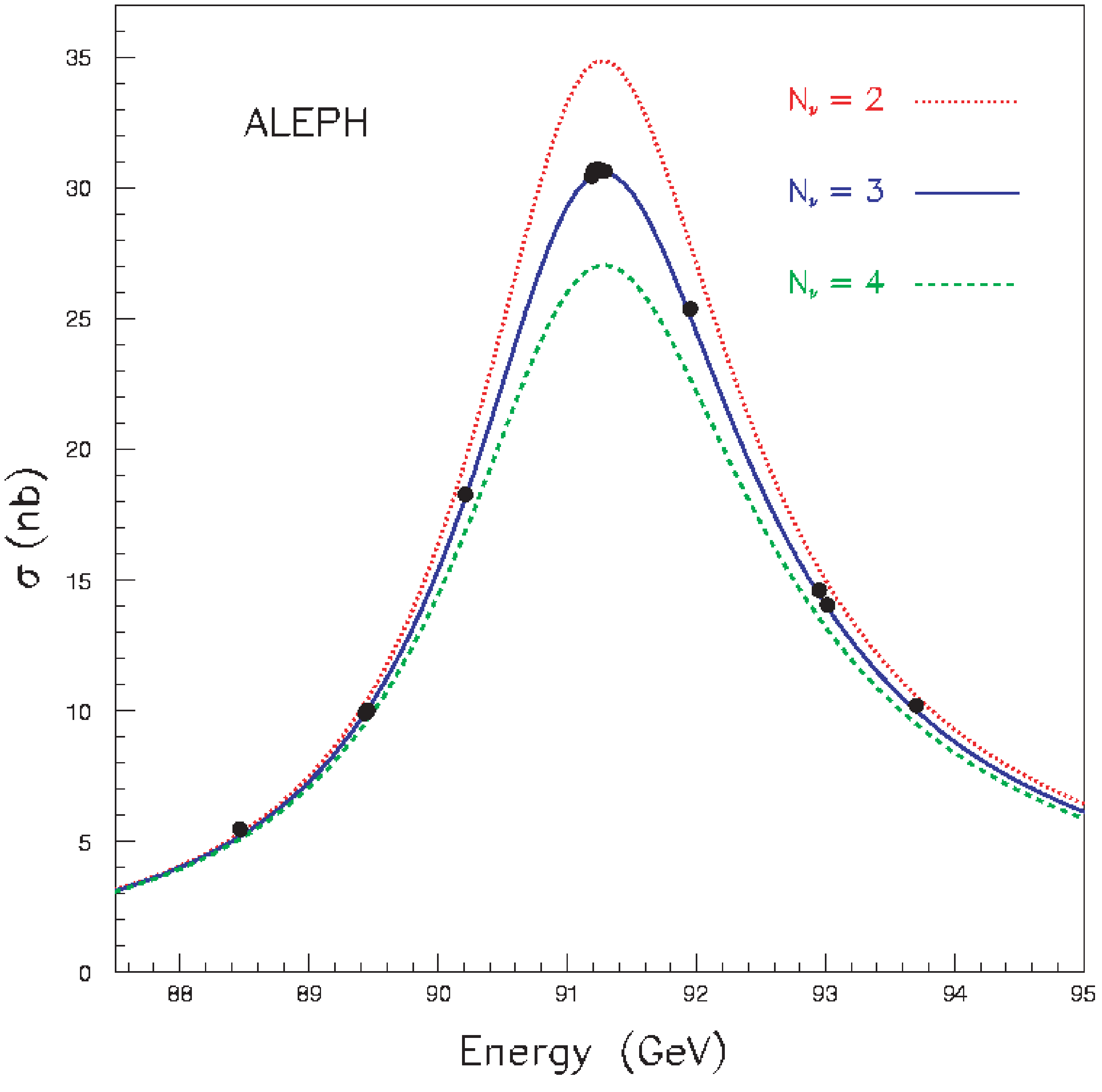
where $\Gamma_{\text{invisible}} \equiv \Gamma_Z - \Gamma_{\text{hadronic}} - 3\Gamma_{\text{leptonic}}$

light neutrinos $N_\nu = \Gamma_{\text{invisible}}/\Gamma^{\text{SM}}(Z \to \nu_i \bar{\nu}_i)$

Current value: $\boxed{N_\nu = 2.994 \pm 0.012}$

. . . excellent agreement with ν_e, ν_μ, and ν_τ

Three light neutrinos



The top quark must exist

- ▷ Two families

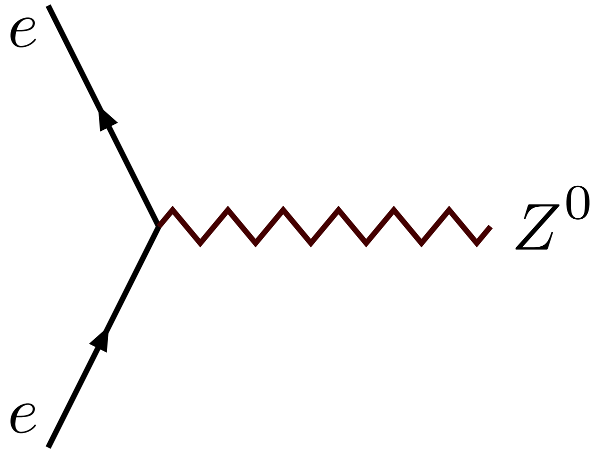
$$\begin{pmatrix} u \\ d \end{pmatrix}_L \qquad \begin{pmatrix} c \\ s \end{pmatrix}_L$$

don't account for CP violation. Need a third family . . . or another answer.

Given the existence of b, (τ)

- ▷ top is needed for an anomaly-free EW theory
- ▷ absence of FCNC in b decay ($b \not\to s\ell^+\ell^-$, etc.)
- ▷ b has weak isospin $I_{3L} = -\frac{1}{2}$; needs partner

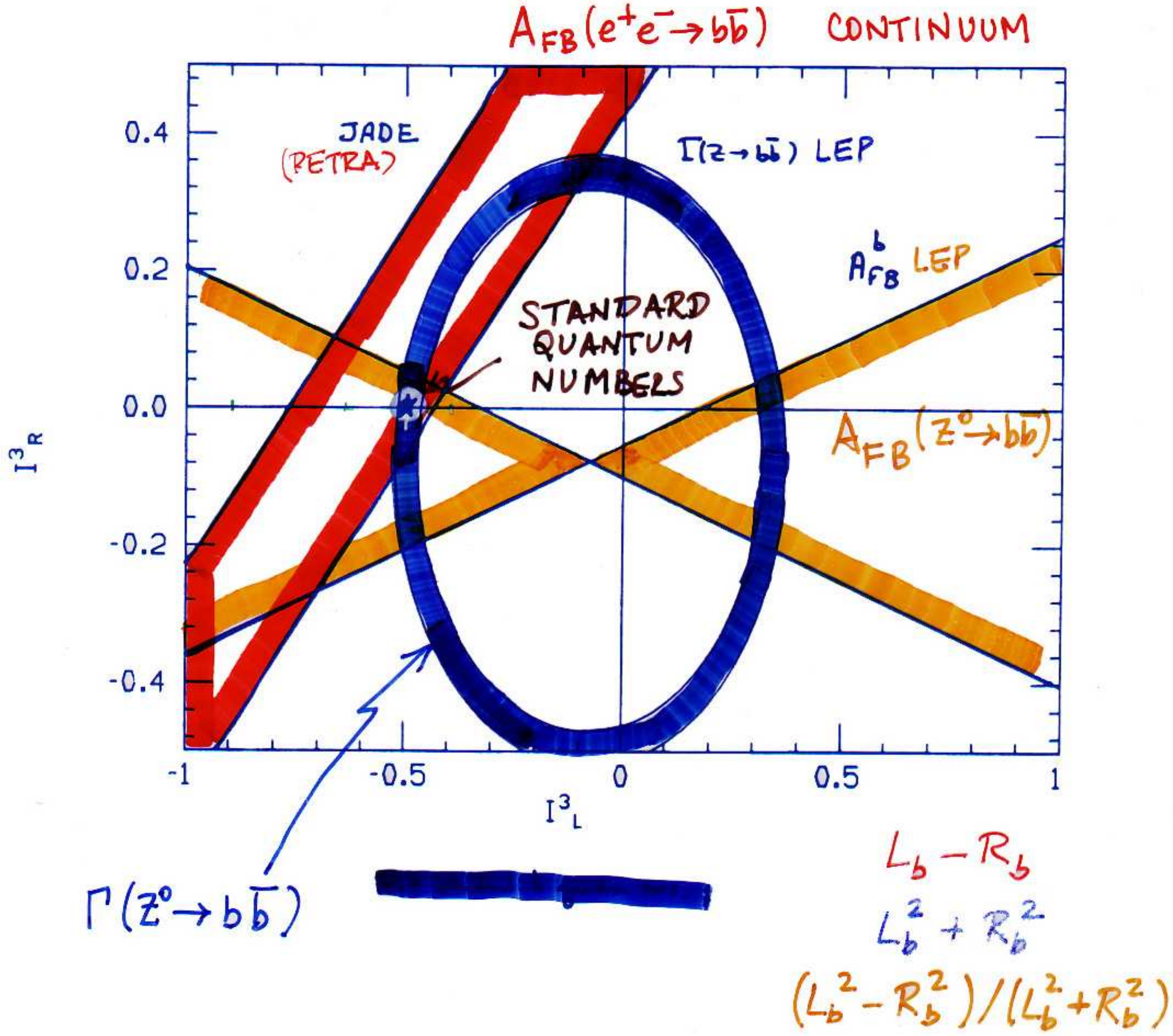
$$\begin{pmatrix} t \\ b \end{pmatrix}_L$$



$$L_b = I_{3L} - Q_b \sin^2\theta_W$$

$$R_b = I_{3R} - Q_b \sin^2\theta_W$$

Measure $I_{3L}^{(b)} = -0.490^{+0.015}_{-0.012}$ $I_{3R}^{(b)} = -0.028 \pm 0.056$



Needed: top with $I_{3L} = +\frac{1}{2}$

D. Schaile & P. Zerwas, *Phys. Rev.* D**45**, 3262 (1992)