pp physics, RWTH, WS 2003/04, T.Hebbeker

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2.2.4 Collider cross sections and branching fractions

2.2.4.1 Rough estimate of cross sections

To get a rough idea about the order of magnitude of SM and other cross sections we consider as 'reference process' $e^+e^- \longrightarrow \mu^+\mu^-$ via s channel photon exchange, with the well known total cross section

$$\sigma_{ref}^{elw} = \frac{4\pi}{3}\,\frac{1}{s'}\,\alpha^2 \approx 0.1\,\mathrm{pb}\cdot\frac{(1\,\mathrm{TeV})^2}{s'} \tag{1}$$

s' is the center of mass energy of the colliding elementary fermions. Of course, this expression is not valid for quarks, since they have smaller charges. But these factors are of the order of 1 and will be neglected here.

Electroweak processes are interesting for us only for effective collision energies $> M_W, M_Z$, since we are looking for heavy particles. Furthermore the weak couplings are of the same order as the electromagnetic coupling (electrowak unification!), so that formula (1) can be used also for W and Z exchange.

Strong processes differ mainly due to the higher coupling constant, so the reference cross section must be multiplied by $(\alpha_s/\alpha)^2$. Additional vertex factors are of $\mathcal{O}(1)$ and will be ignored.

$$\sigma_{ref}^{QCD} \approx \sigma_{ref}\,\frac{\alpha_s^2}{\alpha^2} \approx 20\,\mathrm{pb}\cdot\frac{(1\,\mathrm{TeV})^2}{s'} \tag{2}$$

When will our fourmulae fail ? Certainly for resonance production (e.g. Z'), for t-channel processes with small energy transfer ...

So far we have ignored the kinematical suppression due to final state masses m_i not negligible with respect to $\sqrt{s'}$. Here we approximate this effect by a step function:

$$\sigma = \begin{cases} 0 & \sqrt{s'} < 2\sum m_i \equiv s_{thr} \\ \sigma_{ref} & \sqrt{s'} \geq 2\sum m_i \equiv s_{thr} \end{cases} \tag{3}$$

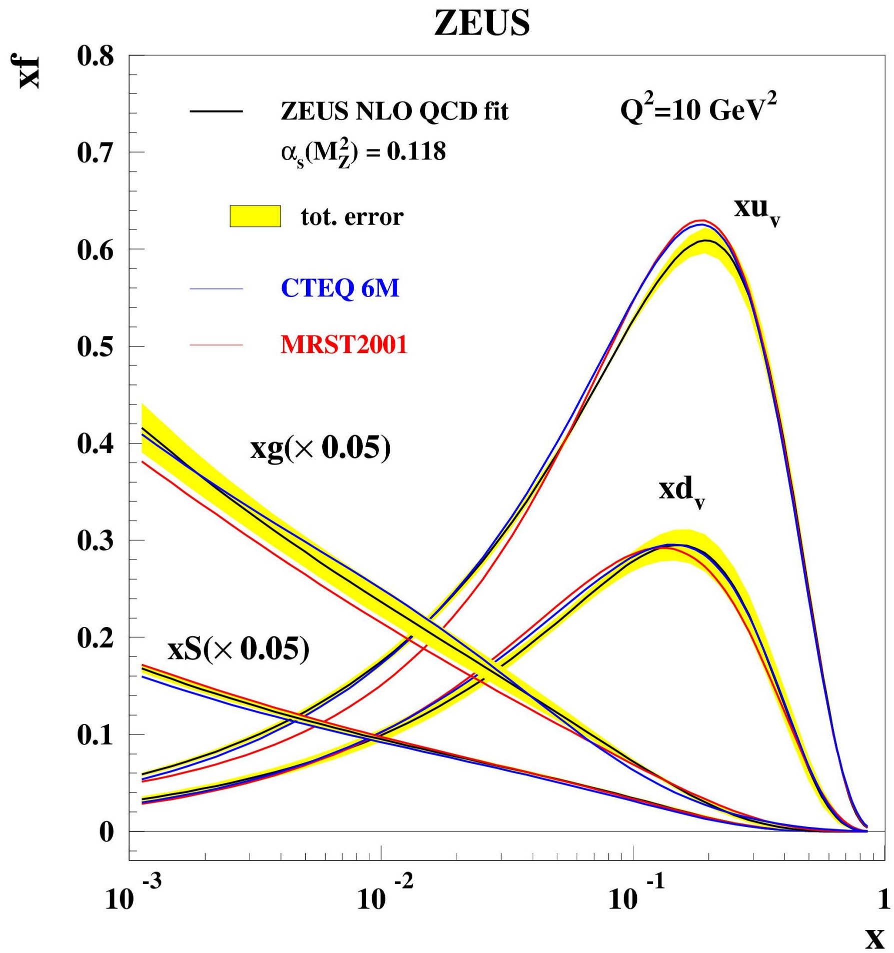
Finally we have to know the relation between s and s', which is given by the structure functions:

$$s' = s \cdot x_1\, x_2 \tag{4}$$

We assume that the process is only possible if both x_i fulfill

$$x_i > x_{thr} \equiv \sqrt{\frac{s_{thr}}{s}} \tag{5}$$

Since the dominant contributions come from s' values near s_{thr} we insert s_{thr} into formulae (1) or (2). The corresponding probabilities are given by the structure functions,



which we approximate (see figure) in the following way, in the range $x_i = 0.01 - 0.5$:

$$f_q(x) = 10 \cdot (-0.045 - 0.15 \log_{10} x)/x \tag{6}$$

$$f_g(x) = 20 \cdot (-0.045 - 0.15 \log_{10} x)/x \tag{7}$$

and ignore the Q^2 dependence. f_q denotes the sum of all sea quarks, which dominate at low x relative to the valence quarks.

Altogether, we expect that cross section estimates based on these approximations will be ok within a factor of 10 or so.

Example: Top pair cross section at LHC. Literature tells us that gg annihilation is the dominant contribution:

$$\sigma = 20\,\mathrm{pb} \cdot \left(\frac{1000\,\mathrm{GeV}}{700\,\mathrm{GeV}}\right)^2 \cdot 3.2^2 = 400\,\mathrm{pb} \tag{8}$$

The last factor

$$P_{struct} \equiv \left(\int_{x_{thr}}^{1} f_g(x)\,\mathrm{d}x\right)^2 = 3.2^2 \qquad x_{thr} = 0.05 \tag{9}$$

contains the probability to find partons (gluons) with a sufficient momentum inside the protons. A full calculation yields $800\,\mathrm{pb}$ (LHC).

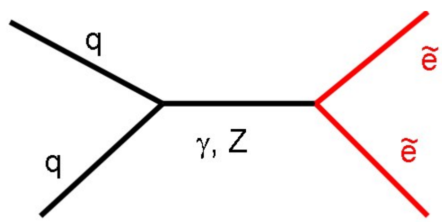
2.2.4.2 SUSY cross sections

The following table shows selected cross sections calculated with Pythia for the Tevatron conditions and for the LHC.

	Tevatron		LHC	
	B'	L'	B'	L'
$\tilde{\chi}_1\,\tilde{\chi}_1$	0.001 pb	$<$ 0.001 pb	0.001 pb	0.0014 pb
$\tilde{\chi}_1\,\tilde{\chi}_1^{\pm}$	0.002 pb	$<$ 0.001 pb	0.014 pb	0.002 pb
$\tilde{\chi}_2\,\tilde{\chi}_1^{\pm}$	0.07 pb	0.001 pb	1.3 pb	0.090 pb
$\tilde{t}\,\bar{\tilde{t}}$	0.001 pb	-	1.7 pb	0.053 pb
$\tilde{e}^+\,\tilde{e}^-$	0.009 pb	0.0006 pb	0.12 pb	0.004 pb
$\tilde{g}\,\tilde{g}$	-	-	4.0 pb	0.090 pb

This table shows that at Tevatron it will be difficult to test these scenarios, while at LHC there will be no problem to find or exclude SUSY for both parameter sets.

Can we understand these xsections (absolutely) ?
We estimate the selectron cross section (sum of $\tilde{e}_L\,\overline{\tilde{e}_L}$ and $\tilde{e}_R\,\overline{\tilde{e}_R}$) at the LHC in scenario L':



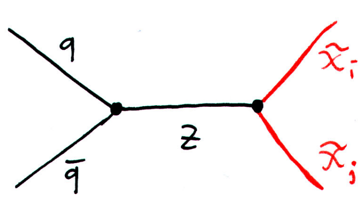
With a mass of $\sim$ $400\,\mathrm{GeV}$ we get

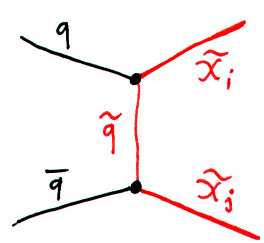
$$\sigma = 0.1\,\mathrm{pb}\cdot\left(\frac{1000\,\mathrm{GeV}}{1600\,\mathrm{GeV}}\right)^2\cdot 0.74^2 = 0.02\,\mathrm{pb} \tag{10}$$

$$P_{struct} \equiv \left(\int_{x_{thr}}^{1} f_q(x)\,\mathrm{d}x\right)^2 = 0.74^2 \qquad x_{thr} = 0.11 \tag{11}$$

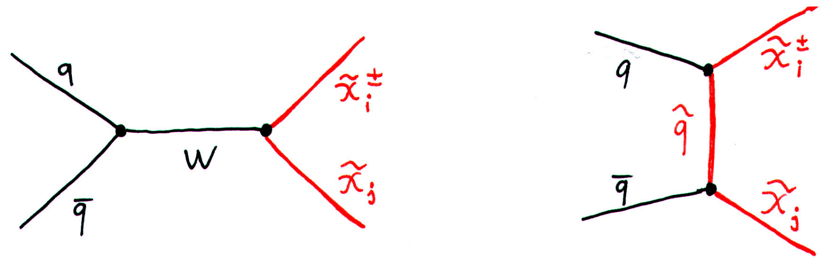
is smaller than for the top production, since there are less sea quarks than gluons and since the selectrons are heavier than top quarks. This is the right order of magnitude!

Can we understand the relative size of the different cross sections ?
Most intriguing is the very small value for $\tilde{\chi}_1\,\tilde{\chi}_1$ compared to $\tilde{\chi}_2\,\tilde{\chi}_1^{\pm}$[1], in spite of the mass differences. The following feynman diagrams contribute (to lowest order):





[1] $\sigma(\tilde{\chi}_1\,\tilde{\chi}_1)$ and $\sigma(\tilde{\chi}_1\,\tilde{\chi}_1^{\pm})$ are small, too!



The t (and u) channel processes with squark exchange are suppressed for our scenarios, since the squarks are heavy: $m(\tilde{q}) \gg M_Z$. In the SM the $Z-ZZ$ and $Z-\gamma\gamma$ vertices do not exist, thus also the couplings $Z-\tilde{B}\tilde{B}$ and $Z-\tilde{W}^3\tilde{W}^3$ vanish. But the lightest neutralino is dominantly 'gaugino', as we have seen in the example in section 2.2.3.2. (scenario L'), while the higgsino contribution is only $\sim 10\%$, leading to a suppression factor of 0.01.

$$\tilde{\chi}_1 = 0.995\,\tilde{B} - 0.014\,\tilde{W}^3 + 0.090\,\tilde{H}_u^0 - 0.031\,\tilde{H}_d^0 \tag{12}$$

Therefore also the s channel Z exchange is strongly suppressed.

To understand why the xsection for $\tilde{\chi}_2\,\tilde{\chi}_1^\pm$ is particularly large, we must investigate the composition of $\tilde{\chi}_2$ and $\tilde{\chi}_1^\pm$, we choose as an example again model L':

$$\tilde{\chi}_2 = -0.039\,\tilde{B} - 0.962\,\tilde{W}^3 + 0.226\,\tilde{H}_u^0 - 0.146\,\tilde{H}_d^0 \tag{13}$$

With formulae similar to those given in section 2.2.3.2. we can obtain

$$\tilde{\chi}_1^\pm = 0.98\,\tilde{W}^\pm + 0.20\,\tilde{H}^\pm \tag{14}$$

In principle a $W\tilde{\chi}^\pm\chi$ coupling can occur via higgsino components or via gaugino components. Here both the lightest chargino and the two first neutralinos are dominantly gaugino, so we can neglect the higgsino part. The gaugino coupling is determined by the wino components in the neutralino, since the bino $\tilde{B}$ does not couple to W and $\tilde{W}$. Therefore $\tilde{\chi}_1$ is suppressed with respect to $\tilde{\chi}_2$ by $(0.014/0.96)^2 \sim 2\cdot 10^{-4}$. The cross sections do not differ by such a large factor, since kinematically the lighter $\tilde{\chi}_1$ is preferred.

2.2.4.3 SUSY signatures

A) Tevatron

Per experiment we expect a total luminosity of $5000/\text{pb}$. To see a new particle in this environment, a rule of thumb says that at least 100 must be produced (acceptance, efficiency, background!). So, the cross section table tells us that scenario L' is very difficult, and for case B' the best channel is:

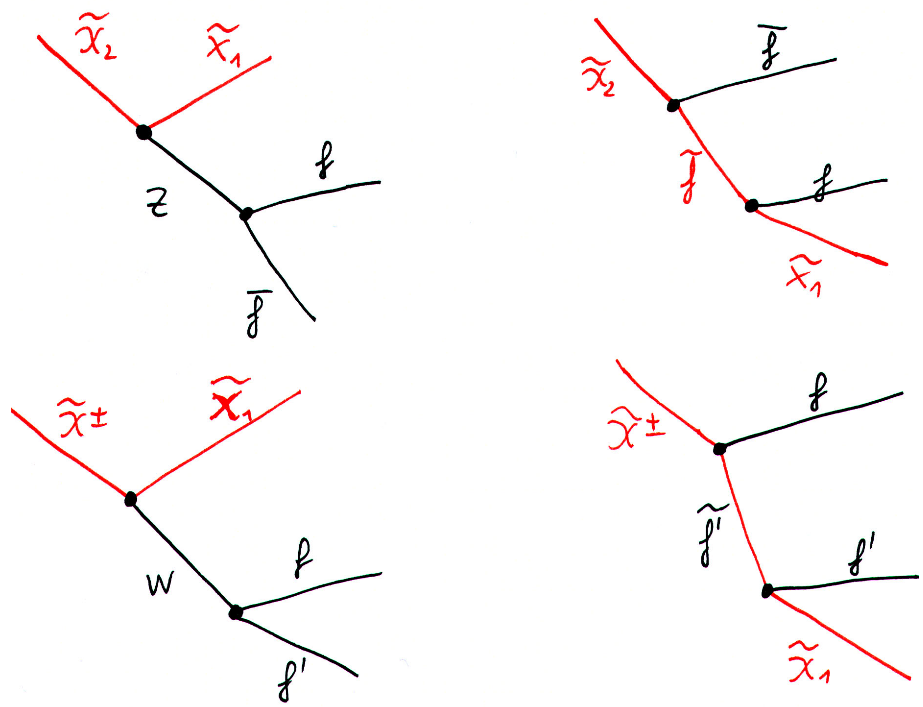
$\tilde{\chi}_2\,\tilde{\chi}_1^\pm$

The dominant decay modes for these two SUSY particles are[2] (scenario B'):

[2] calculated with Isajet

initial	final	branching fraction
$\tilde{\chi}_2$	$e^+ e^- \tilde{\chi}_1$	5%
$\tilde{\chi}_2$	$\mu^+ \mu^- \tilde{\chi}_1$	5%
$\tilde{\chi}_2$	$\tau^+ \tau^- \tilde{\chi}_1$	41%
$\tilde{\chi}_2$	$\nu \bar{\nu} \tilde{\chi}_1$	49%
$\tilde{\chi}_2$	$q \bar{q} \tilde{\chi}_1$	0.04%
$\tilde{\chi}_1^\pm$	$e \nu \tilde{\chi}_1$	18%
$\tilde{\chi}_1^\pm$	$\mu \nu \tilde{\chi}_1$	18%
$\tilde{\chi}_1^\pm$	$\tau \nu \tilde{\chi}_1$	63%
$\tilde{\chi}_1^\pm$	$q \bar{q} \tilde{\chi}_1$	$< 0.01\%$

with the corresponding feynman diagrams:



In scenario B' the second-lightest neutralino decays preferentially into a real $\tilde{f}_R$ plus f, which is kinematically allowed for leptons, but not for quarks. The decay mode via virtual Z is suppressed (off shell). Neutrinos are preferred, since their NC couplings are bigger than for charged leptons (which have vector couplings ~ 0).

The τ final state is dominant due to the relatively large value of $\tan\beta$; this leads to a significant $\tilde{\tau}_R - \tilde{\tau}_L$ mixing, lowering the mass of $\tilde{\tau}_1$ with respect to $\tilde{e}_R$. The neutralino $\tilde{\chi}_2$ decays into $\tilde{\tau}_1$, which subsequently decays into τ and the LSP.

Since the cross section and the leptonic branching fractions are large, one can hope to find an excess of events with at least three leptons plus missing energy. In scenario B' one expects $\sim 350\ \tilde{\chi}_2 \tilde{\chi}_1^\pm$ events. Half of them are invisible (3 or more neutrinos). Of the τ decays only 36% yield either e or μ, thus about 15% of all events give at least three leptons; counting only electrons and muons, this leaves about 50 events.

Background reactions are $W\,Z, Z\,Z, t\,\bar{t}$. Many of those events can be rejected by requiring that all dilepton masses are **outside** the Z peak. With suitable cuts the signal efficiency is around 30%, while the background fraction can be kept as low as 20%. Thus one might see $15+3$ events for an expected background of 3 events - just enough to claim an excess!

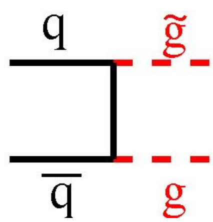
B) LHC

With luminosities well above $100/\mathrm{fb}$ all channels listed in the above table seem detectable - except LSP pair production (invisible!). However, for benchmark L' the selectron cross section is small and background ($WW \rightarrow ee\nu\nu$) important, so that it will probably not be possible to establish this process.

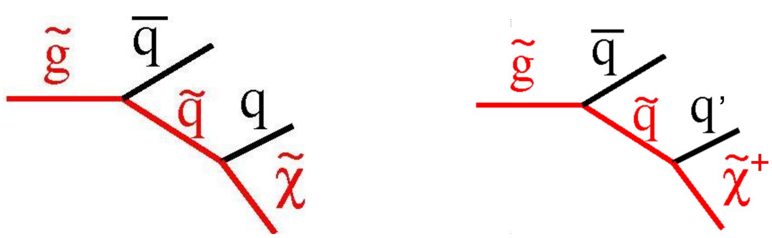
As an example we discuss here briefly gluino pair production:

<u>$\tilde{g}\,\tilde{g}$</u>

The production is decribed by this feynman diagram:



Gluinos decay dominantly as shown in the following figure:



These decay channels are unsuppressed only if (some) squarks are lighter than the gluino; this is in general not the case, as can be seen from the corresponding mass formulae. However, in scenarios B' and L' the squarks are sufficiently light. Let's look in detail[3] at model B' (Isajet). The neutralino/chargino composition in terms of binos/winos/higgsinos is similar to scenario L', discussed above. The gluino decay modes are:

[3]here we abbreviate $u\,d = u\,\bar{d} + \bar{u}\,d$ etc

	branching fraction
$u\,u\,\tilde{\chi}_1$	11%
$u\,u\,\tilde{\chi}_2$	2%
$d\,d\,\tilde{\chi}_1$	10%
$d\,d\,\tilde{\chi}_2$	2%
$u\,d\,\tilde{\chi}_1^{\pm}$	7%
$t\,t\,\tilde{\chi}_1$	1%
$t\,t\,\tilde{\chi}_2$	1%
$b\,b\,\tilde{\chi}_1$	4%
$b\,b\,\tilde{\chi}_2$	3%
$t\,b\,\tilde{\chi}_1^{\pm}$	13%

Comments:

- The intermediate state with a $\tilde{q}_R$ is preferred since the 'right' squarks are lighter.
- While the $\tilde{q}_R$ decays dominantly into $\tilde{\chi}_1\,q$, the squarks $\tilde{q}_L$ strongly prefer $\tilde{\chi}_2\,q$ or $\tilde{\chi}_1^{\pm}\,q'$ as final state. Reason: The coupling constant of 'left' squarks is:

$$g_L \sim g_V + g_A = (I_3 - 2Q\sin^2\theta_W) + (I_3) = 2I_3 - 2Q\sin^2\theta_W \tag{15}$$

  So they couple via electroweak isospin I_3 to winos ($\tilde{\chi}_2, \tilde{\chi}_1^{\pm}$!), while the 'right' coupling

$$g_R \sim g_V - g_A = (I_3 - 2Q\sin^2\theta_W) - (I_3) = -2Q\sin^2\theta_W \tag{16}$$

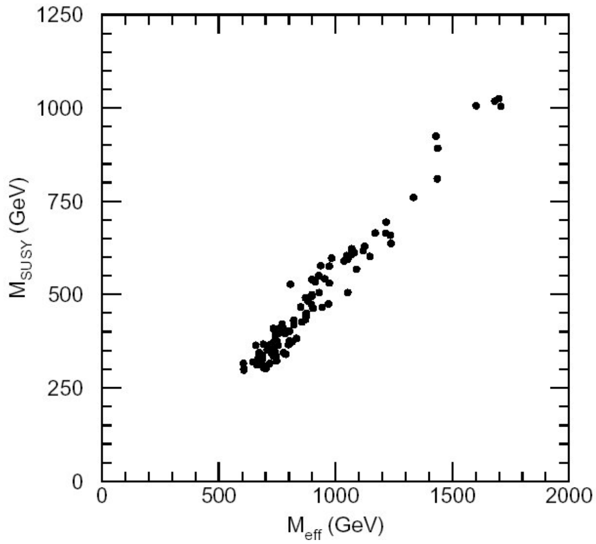
  to winos vanishes. The $\tilde{q}_R$ couples to bino components, so that the lightest $\tilde{\chi}_1$ is preferred.
- The stop $\tilde{t}_1$ is quite light, but this advantage is eaten up by the heavy top quark!
- Since sbottom and stop states are mixtures, there is no preferred coupling to either $\tilde{\chi}_1$ or $\tilde{\chi}_2/\tilde{\chi}_1^{\pm}$.
- The channels not shown in the table (s, c) contribute as much as u, d.
- The neutralino $\tilde{\chi}_2$ decays dominantly into $\tilde{\chi}_1$ plus a $\tau^+\tau^-$ pair (41%), thanks to the low $\tilde{\tau}_1$ mass, other decay modes lead to electron, muon or neutrino pairs, see above.
- The chargino will decay dominantly into leptons and $\tilde{\chi}_1$, see above.

Thus, also gluino decays yield leptons. However, since we have discussed similar signatures before, we concentrate here on the decay of the gluino into hadrons plus LSP (about half of all decays); the signature is: jets plus missing energy. Of course, apart from gluino pair production also squark production contributes to this final state!

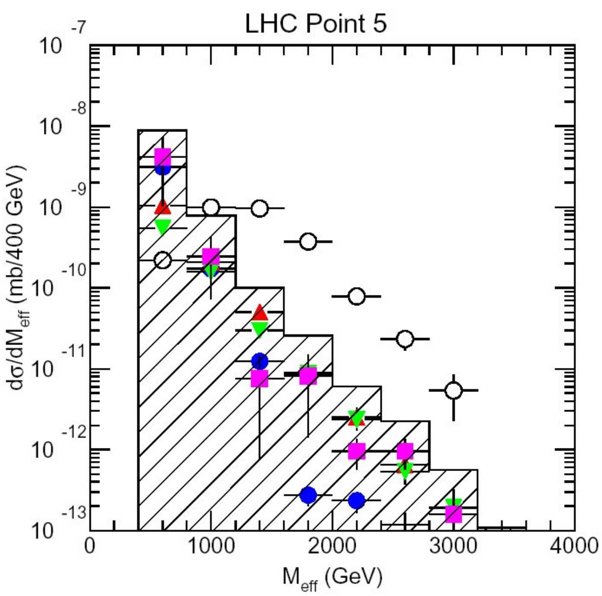
It turns out that the masses M_{SUSY} of the SUSY particles (gluinos, squarks) can be estimated by the following observable:

$$M_{eff} = \not{E}_T + \sum_{jets\,j} p_T^j \tag{17}$$

Monte Carlo studies show that in the MSSM-4 there is a strong correlation:



Here $M_{SUSY} = \min(m_{\tilde{g}}, m_{\tilde{q}})$. M_{eff} is bigger than M_{SUSY} since per event two primary SUSY particles are created! The next figure shows the expected M_{eff} distribution [4]



compared to background expectations, in the ATLAS experiment. The following selection cuts were applied:

$$\not{E}_T > 100\,\text{GeV} \qquad p_T^j > 50\,\text{GeV}, j = 1\ldots4 \quad (100\,\text{GeV} \ \text{for} \ j = 1) \qquad (18)$$

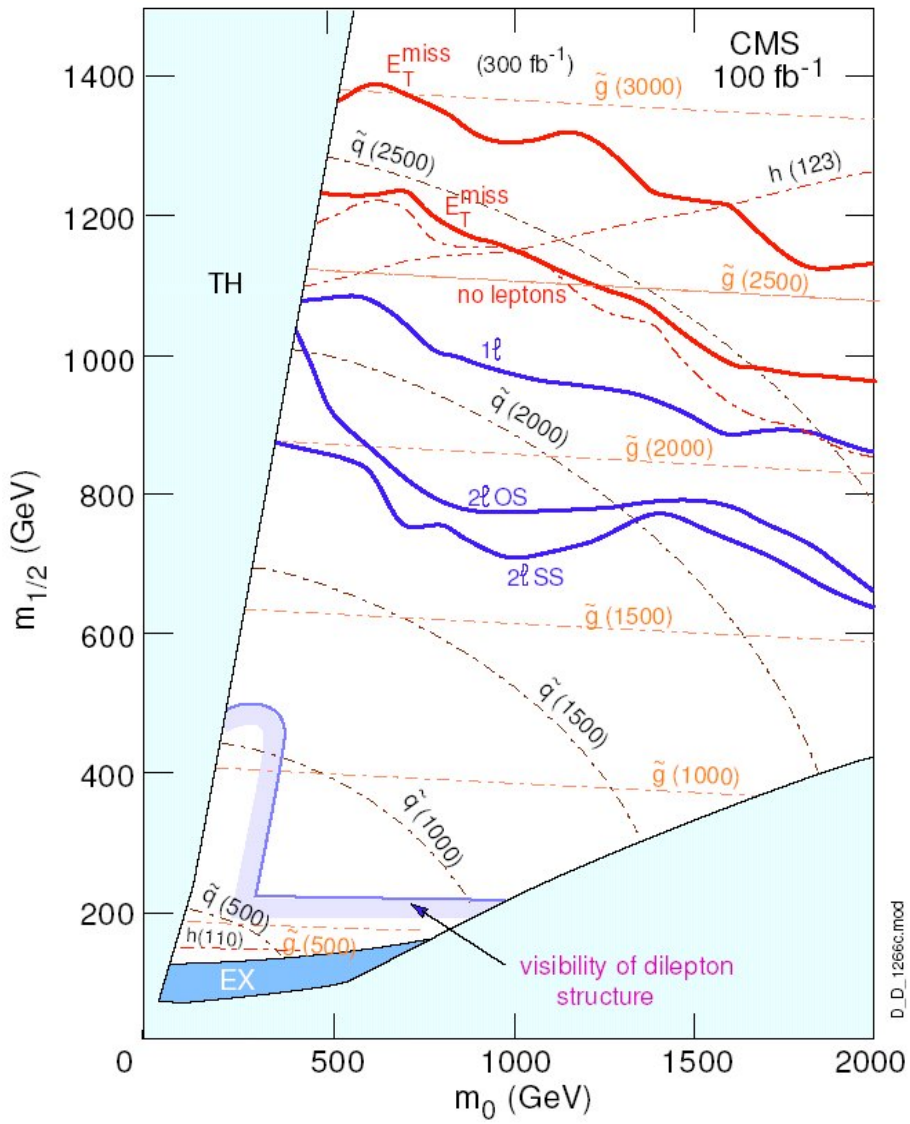
The SUSY cross section is large, sinnce gluinos and squarks are produced via strong interactions. As can be seen form the figure the backround[5] is small compared to the signal at large M_{eff} values, thus a discovery should not be a problem.

Finally we compare the overall SUSY reach at the LHC comparing different signatures[6]:

[4]for LHCC benchmark point 5: $m_0 = 100\,\text{GeV}$, $m_{1/2} = 300\,\text{GeV}$, $A_0 = 300\,\text{GeV}$, $\tan\beta = 2.1$, $\mu > 0$. The gluino mass is $767\,\text{GeV}$.

[5]top = filled circles, W+jets = triangles, Z + jets = inverted triangles, QCD = squares, SUSY = open circles

[6]$\tan\beta = 35$, $A_0 = 0$



Overall the channel 'missing E_T plus jets' gives the best coverage of the $m_0 - m_{1/2}$ plane! These SUSY parameters correspond to gluino and squark masses beyond $2\ \mathrm{TeV}$.