pp physics, RWTH, WS 2003/04, T.Hebbeker

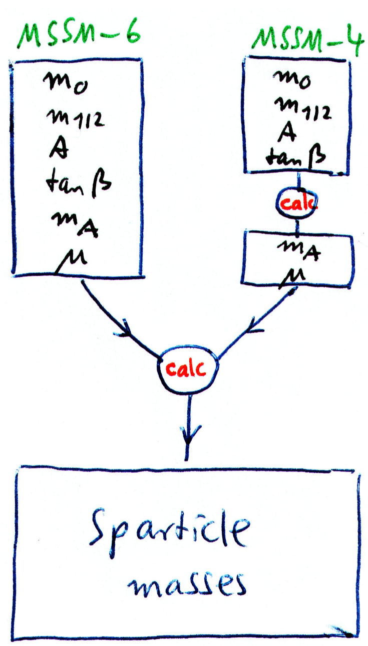
2004-01-29

2.2. Supersymmetry

References:
- H.E. Haber and G.L. Kane, 'The Search for Supersymmetry: Probing Physics Beyond the Standard Model', Phys. Rep. 117 (1985) 75.
- Stephen P. Martin, 'A Supersymmetry primer', hep-ph/9709356
- W. de Boer, 'Grand Unified Theories and Supersymmetry in Particle Physics and Cosmology', hep-ph/9402266
- Review of Particle Physics, Phys. Rev. D (2002) 1
- L. Roszkowski et al, 'New Cosmological and Experimental Constraints on the MSSM', hep-ph/0106334
- M. Battaglia et al, 'Updated Post-WMAP Benchmarks for Supersymmetry', hep-ph/0306219

Web form to calculate SUSY parameters:
http://www.phys.ufl.edu/∼jblender/isajet/isajet.html

I consider only models with 'gravity mediated supersymmetry breaking' without R parity violation. There are two variants, which are often discussed in the literature and for which many experimental and cosmological constraints have been evaluated. The more general one, here called MSSM-6, depends on 6 free parameters, the other one, MSSM-4, on 4 (plus a sign); we will discuss the corresponding parameters below[1]. MSSM stands for 'Minimal Supersymmetric Extension of the Standard Model'; 'minimal' refers to the particle content.



[1]The nomenclature is not very uniform in the literature: the names CMSSM and MSUGRA are often used.

Additional constraints from cosmology are often imposed: dark matter = LSP = $\tilde{\chi}_1^0 \equiv \tilde{\chi}_1$.

Even in the MSSM-4 (to be explored here) it takes a beginner some time to 'understand' the parameter space. In order to move in this direction we will in the following:
a) start from two 'benchmark scenarios' (= two points in the MSSM-4 parameter space),
b) calculate the corresponding SUSY masses, and
c) explore the associated cross sections and branching fractions (as far as they are relevant for the Tevatron/LHC).

But first of all we list the particle content of the minimal SUSY model[2]:

Particle	Spin	Susy-Partner	Spin
ν_e	1/2	$\tilde{\nu}_e^L$	0
e^-	1/2	$\tilde{\mathrm{e}}_L^-, \tilde{\mathrm{e}}_R^-$	0
u	1/2	$\tilde{\mathrm{u}}_L, \tilde{\mathrm{u}}_R$	0
d	1/2	$\tilde{\mathrm{d}}_L, \tilde{\mathrm{d}}_R$	0
$\gamma, \mathrm{Z}, h, H, A$	1, 0	$\tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0, \tilde{\chi}_4^0$	1/2
$\mathrm{W}^\pm, H^\pm$	1,0	$\tilde{\chi}_1^\pm, \tilde{\chi}_2^\pm$	1/2
g	1	$\tilde{\mathrm{g}}$	1/2

The number of degrees of freedom is the same on the left and on the right, line by line. Note that (at least) five physical higgs fields, $h, A, H, H^\pm$ are required. While the lightest higgs h plays a central role in SUSY searches, due to its theoretical mass constraint of $\leq 130\,\mathrm{GeV}$, we will focus here on the particles with the quantum number R parity $= -1$, while 'normal' particles and all higgs bosons have $R = 1$.

2.2.1 MSSM-parameters

In the MSSM-6 there are six parameters beyond the Standard Model parameters (however, the higgs mass is now predicted as a function of the SUSY parameters). The parameters are:

- common scalar mass m_0 for all sfermions at GUT scale
- common gaugino mass $m_{1/2}$ for all gauginos at GUT scale
- higgs mass m_A
- higgsino mass parameter μ (at elw. scale)
- universal trilinear coupling A_0 at GUT scale
- ratio of higgs vacuum expectation values $\tan\beta$ (at elw. scale)

It is very important to realize at which energy scale these numbers are defined, see below.

[2]Probably right handed neutrinos need to be added. . .

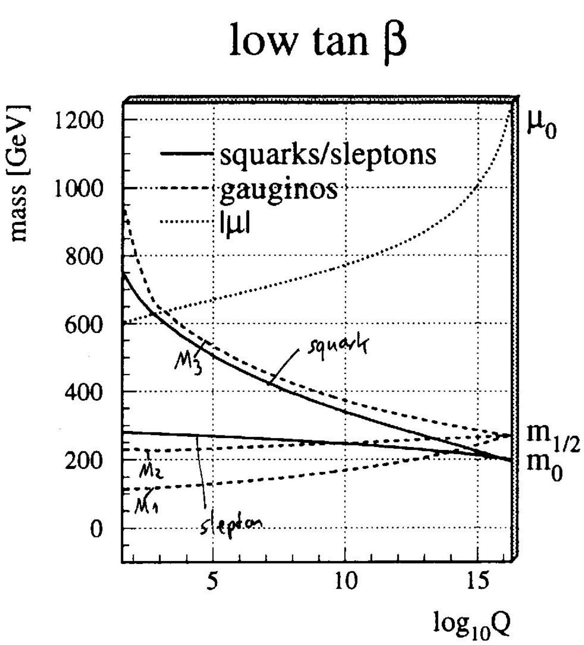
Remarks:

- The masses and A are 'soft supersymmetry breaking parameters' - they are arbitrary constants in an effective Lagrangian compatible with SUSY breaking - since we dont know more about it!
- In SUSY at least two higgs doublets are required, the corresponding vacuum expectation values are v_u and v_d, where, as in the Standard Model, $v_u^2 + v_d^2 \equiv v^2 = 1/(\sqrt{2}G_F) = (246\,\mathrm{GeV})^2$ is fixed while their ratio $(v_u = v\,\sin\beta)/(v_d = v\,\cos\beta) = \tan\beta$ is a free parameter. β appears in many formulae, since the higgs couplings determine masses and radiative corrections as well as cross sections and decay rates.
- all higgs masses are fixed by m_A and $\tan\beta$. Example (lowest order): $m_{H^\pm}^2 = m_W^2 + m_A^2$
- the higgsino mass parameter μ determines the higgsino masses - however the situation is more complicated due to the mixing with the gauginos.
- the 'trilinear coupling' A describes the strength of the sfermion-higgs couplings, similar to the Yukawa couplings in the fermion-higgs term in the SM. Thus A influences the sfermion masses, see below.

The underlying assumptions in the MSSM-6 are:

- no CPV (beyond SM)
- grand unification of couplings at $\sim 10^{16}\,\mathrm{GeV}$
- universal mass for sfermions and universal trilinear coupling at the GUT scale
- universal gaugino mass at the GUT scale $\longrightarrow$ GUT inspired relation between the gaugino masses at elw. scale

The energy scale dependence of SUSY mass parameters is illustrated here (from R. Ehret):



Here masses denoted by a capital M are gaugino masses at the elw. scale. Running implies quantitatively for the SU(2) gaugino:

$$M_2 = 0.82\, m_{1/2} \tag{1}$$

The beforementioned gaugino mass relations are:

$$M_1 = \frac{5}{3} \tan^2\theta_W \cdot M_2 \approx 0.4 \cdot m_{1/2} \tag{2}$$
$$M_3 = \frac{\alpha_s}{\alpha} \sin^2\theta_W \cdot M_2 \approx 3 \cdot m_{1/2} \tag{3}$$

where M_1 and $M_3 = m_{\tilde{g}}$ are the U(1) and SU(3) gaugino masses.

Furthermore we constrain the parameter space by excluding all points where the lightest SUSY Particle (LSP) is **not** neutral (charge and color), so that dark matter can be explained by SUSY. Since sneutrinos tend to be relatively heavy, this implies LSP = $\tilde{\chi}_1$.

What are the allowed values for the six (four) parameters ? There is no exact answer! In order to avoid the higgs mass hierarchy problem, one must have SUSY masses below $1\ \mathrm{TeV}$, thus the dimensionful parameters like m_0 cannot be much larger. For $\tan\beta$ one chooses values betwen 1 and $\sim 35 = m_t/m_b$, since in models with a universal Yukawa coupling the quark masses are proportional to v_u and v_d. So:

m_0	$0 - 2\,\mathrm{TeV}$
$m_{1/2}$	$0 - 2\,\mathrm{TeV}$
m_A	$0 - 2\,\mathrm{TeV}$
$\lvert A\rvert$	$0 - 2\,\mathrm{TeV}$
$\lvert\mu\rvert$	$0 - 2\,\mathrm{TeV}$
$\tan\beta$	$1 - 50$

Low values of the mass parameters as well as the regime $\tan\beta < 3$ are already excluded experimentally.

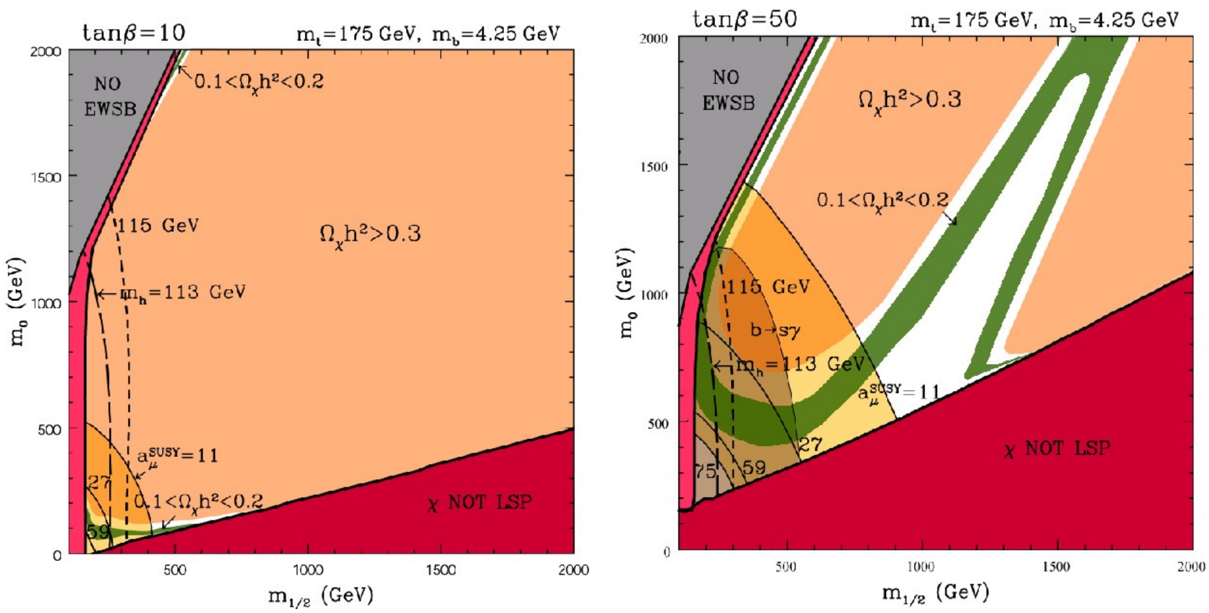
So far we discussed the MSSM-6 model. Two additional constraints allow to calculate m_A and $|\mu|$ from the other parameters, thus bringing us to the MSSM-4:
a) also the higgs masses (scalar particles!) are determined by m_0
b) electroweak (!) symmetry breaking occurs automatically via radiative corrections influencing the higgs potential. This implies a relation fixing μ^2.

Therefore, in the MSSM-4 there are four independent parameters (see figure above) and the sign of μ[3].

2.2.2 MSSM benchmark scenarios

Several groups have analysed the existing experimental (LEP, $b \longrightarrow s\,\gamma$, $g_\mu - 2\ldots$) and cosmological constraints (WMAP: $\Omega_{dark}\, h^2 \sim 0.1$) and have calculated the remaining 'allowed' regions in the MSSM-4 parameter space. Example (L. Roszkowski et al):

[3]Since mass terms in the Lagrangian are $\sim m^2$ the value of m may be negative. It turns out that only a relative sign between $m_{1/2}$ and μ matters; by convention $m_{1/2} > 0$



In both cases $\mu > 0$ was assumed. We will restrict ourselves to this case, since $\mu < 0$ is disfavored by measurements of $g_\mu - 2$. The limits shown here are not very sensitive to A, so this parameter was set to 0.

It must be stressed, that different authors arrive at different contours (calculations of Ω_{dark} are difficult!) and that variations of standard model parameters (e.g. $m_{top} = (175 \pm 5)\,\mathrm{GeV}$) can have dramatic effects on the contours in the $m_0 - m_{1/2}$ plane, in particular at high values of $\tan\beta$.

In some of those papers analyzing the MSSM-4 parameter space some 'representative' benchmark scenarios have been proposed, typically 10, which are scattered over the *allowed* parameter region. They are further investigated in view of the experimental possibilities at existing and future colliders.

We will pick up one proposal (M. Battaglia et al, table 2, reproduced in the appendix of this lecture note) and have a closer look at the following two points in parameter space (terminology taken from that table):

	B'	L'
m_0/GeV	57	303
$m_{1/2}/\mathrm{GeV}$	250	450
$\tan\beta$	10	47
A/GeV	0	0

Note that low values of $\tan\beta$ are not considered, since they are already excluded.

2.2.3 SUSY masses

If we want to calculate masses, we need to compute μ and m_A first. Since both are related to Higgs masses, which 'suffer' heavily from radiative corrections, a simple tree level formula is not sufficient. Instead one can use the following crude ($\pm 10\%$) approximations:

$$\mu^2 = \left(\frac{2100}{\tan^2\beta} - 1.0\right) \cdot m_0^2 + \left(\frac{-60}{\tan^2\beta} + 1.2\right) \cdot m_{1/2}^2 + 49000\,\mathrm{GeV}^2 \qquad (4)$$

$$m_A^2 = \left(\frac{5700}{\tan^2\beta} - 3.5\right) \cdot m_0^2 + \left(\frac{-160}{\tan^2\beta} + 1.3\right) \cdot m_{1/2}^2 + 18000\,\mathrm{GeV}^2 \tag{5}$$

yielding

	B'	L'
μ/GeV	389	501
m_A/GeV	416	427

Alternatively one can simply use the values from the appendix, as we will do for the following computations.

2.2.3.1 Fermion masses

Sparticle masses at the electroweak scale are given by the general formula:

$$m^2(\tilde{f}) - m^2(f) = m_0^2 + \Delta_{gauge} \tag{6}$$

Δ_{gauge} decribes the running from GUT to ELW scales, determined by the coupling to the different gauge fields:

$$\Delta_{gauge} = \Delta_{SU(3)} + \Delta_{SU(2)} + \Delta_{U(1)} \tag{7}$$

The individual terms are proportional to the corresponding gaugino mass squared and the sfermion-gaugino coupling^2. With GUT assumptions:

$$\begin{aligned} \Delta_{SU(3)} &= N_C^2 \cdot 0.91\, M_2^2 && (8)\\ \Delta_{SU(2)} &= T^2 \cdot 2.96\, M_2^2 && (9)\\ \Delta_{U(1)} &= Y^2 \cdot 0.22\, M_2^2 && (10) \end{aligned}$$

Note the dependence on weak isospin T and weak hypercharge $Y = Q - T_3$. Since left- and right handed fermions differ in these quantum numbers, also the corresponding SUSY masses are different from each other:

$$\begin{aligned} m^2(\tilde{u}_L) - m^2(u) &= m_0^2 + 8.95\, M_2^2 && (11)\\ m^2(\tilde{d}_L) - m^2(d) &= m_0^2 + 8.95\, M_2^2 && (12)\\ m^2(\tilde{\nu}) &= m_0^2 + 0.80\, M_2^2 && (13)\\ m^2(\tilde{e}_L) - m^2(e) &= m_0^2 + 0.80\, M_2^2 && (14)\\ m^2(\tilde{u}_R) - m^2(u) &= m_0^2 + 8.30\, M_2^2 && (15)\\ m^2(\tilde{d}_R) - m^2(d) &= m_0^2 + 8.22\, M_2^2 && (16)\\ m^2(\tilde{e}_R) - m^2(e) &= m_0^2 + 0.22\, M_2^2 && (17) \end{aligned}$$

However, the situation is more complicated; the mass matrix contains also non diagonal terms. For 'up' type fermions:

$$\begin{pmatrix} m_L^2(\tilde{f}) & m_f \cdot (A - \mu\cot\beta) \\ m_f \cdot (A - \mu\cot\beta) & m_R^2(\tilde{f}) \end{pmatrix} \tag{18}$$

Mass matrix for 'down' type fermions:

$$\begin{pmatrix} m_L^2(\tilde{f}) & m_f \cdot (A - \mu \tan\beta) \\ m_f \cdot (A - \mu \tan\beta) & m_R^2(\tilde{f}) \end{pmatrix} \tag{19}$$

The sfermion fields $\tilde{f}_R$ and $\tilde{f}_L$ mix into observable mass eigenstates $\tilde{f}_1 < \tilde{f}_2$:

$$\tilde{f}_1 = \tilde{f}_L \cos\theta + \tilde{f}_R \sin\theta \tag{20}$$

$$\tilde{f}_2 = -\tilde{f}_L \sin\theta + \tilde{f}_R \cos\theta \tag{21}$$

Note: This 'mixing' is relevant only if m_f is large, i.e. for $\tilde{\tau}, \tilde{b}, \tilde{t}$. The masses m_1 and m_2 and the mixing angle θ can be calculated by diagonalizing the mass matrix:

$$m_{1,2}^2 = \frac{1}{2}(m_R^2 + m_L^2) \mp \sqrt{(m_L^2 - m_R^2)^2/4 + (m_f(A - \mu \cot\beta))^2} \tag{22}$$

for 'up' and similarly for 'down'.

Applying the sfermion mass formulae to our benchmark scenario, we get

	B'	L'
$m(\tilde{\nu})/\mathrm{GeV}$	191	447
$m(\tilde{e}_L)/\mathrm{GeV}$	191	447
$m(\tilde{e}_R)/\mathrm{GeV}$	112	349
$m(\tilde{d}_L)/\mathrm{GeV}$	616	1145
$m(\tilde{d}_R)/\mathrm{GeV}$	591	1101
$m(\tilde{u}_L)/\mathrm{GeV}$	616	1145
$m(\tilde{u}_R)/\mathrm{GeV}$	593	1105
$m(\tilde{\tau}_1)/\mathrm{GeV}$	105	316
$m(\tilde{\tau}_2)/\mathrm{GeV}$	195	471
$m(\tilde{b}_1)/\mathrm{GeV}$	586	1066
$m(\tilde{b}_2)/\mathrm{GeV}$	621	1177
$m(\tilde{t}_1)/\mathrm{GeV}$	618	1119
$m(\tilde{t}_2)/\mathrm{GeV}$	641	1158

These numbers are in reasonable agreement with the expert's results as given in the appendix.

In these examples the mixing between sfermions f_L and f_R is small; for other parameters it can be large and the lightest stop $\tilde{t}_1$ might be the lightest squark!

2.2.3.2 Gaugino masses

Here 'mixing' has to be taken into account.

To determine the neutralino masses, we start with the fields

$$(\tilde{B},\ \tilde{W}^3,\ \tilde{H}_u^0,\ \tilde{H}_d^0) \tag{23}$$

the corresponding mass matrix reads

$$\begin{pmatrix} M_1 & 0 & -M_Z \cos\beta\ \sin\theta_W & M_Z \sin\beta\ \sin\theta_W \\ 0 & M_2 & M_Z \cos\beta\ \cos\theta_W & -M_Z \sin\beta\ \cos\theta_W \\ -M_Z \cos\beta\ \sin\theta_W & M_Z \cos\beta\ \cos\theta_W & 0 & -\mu \\ M_Z \sin\beta\ \sin\theta_W & -M_Z \sin\beta\ \cos\theta_W & -\mu & 0 \end{pmatrix} \tag{24}$$

The off diagonal elements are due to gaugino-higgs-higgsino couplings. With the GUT gaugino mass relations and $\sin^2\theta_W = 0.23$:

$$\begin{pmatrix} 0.4\, m_{1/2} & 0 & -0.48\ \cos\beta\, M_Z & 0.48 \sin\beta\, M_Z \\ 0 & 0.8\, m_{1/2} & 0.88\ \cos\beta\, M_Z & -0.88 \sin\beta\, M_Z \\ -0.48\ \cos\beta\, M_Z & 0.88 \cos\beta\, M_Z & 0 & -\mu \\ 0.48\ \sin\beta\, M_Z & -0.88 \sin\beta, M_Z & -\mu & 0 \end{pmatrix} \tag{25}$$

After diagonalization one obtains the mass eigenstates

$$\tilde\chi^0_1, \tilde\chi^0_2, \tilde\chi^0_3, \tilde\chi^0_4 \tag{26}$$

and the rotation matrices, which tell us the composition of the neutralinos in terms of bino, wino and higgsino fields. This is quite important, since the coupling (neutralino-particle-sparticle and gaugeboson-neutralino-neutralino) is quite different for these fields. For example: a higgsino couples preferrentially to heavy particles, the photino (mixture of $\tilde B$ and $\tilde W$, as in SM) does not couple to the Z.

In the limit $M_Z \ll |\mu \pm M_1|, |\mu \pm M_2|$ the neutralino mass matrix is considerably simplified, since the terms $\sim M_Z$ can be neglected. In that case the LSP mass $m(\tilde\chi_1)$ is $0.4\, m_{1/2}$ if μ if $m_{1/2} \gg |\mu|$. If $m_{1/2} \ll |\mu|$ we have also the following nice approximate relation: $m(\tilde\chi_2) \approx m(\tilde\chi^\pm_1) = 2\, m(\tilde\chi_1)$.

Similarly, to compute chargino masses and composition, the matrix

$$\begin{pmatrix} M_2 & \sqrt{2} M_W \sin\beta \\ \sqrt{2} M_W \cos\beta & \mu \end{pmatrix} \tag{27}$$

must be diagonalized, giving[4]

$$m^2(\tilde\chi^\pm_1, \tilde\chi^\pm_2) = 0.5 \cdot \left[M_2^2 + \mu^2 + 2M_W^2 \right. \tag{28}$$

$$\left. \mp \sqrt{(M_2^2 - \mu^2)^2 + 4M_W^4 \cos^2(2\beta) + 4M_W^2 (M_2^2 + \mu^2 + 2M_2\, \mu\ \sin(2\beta))} \right] \tag{29}$$

For the benchmark points we find with these formulae:

	B'	L'
$m(\tilde\chi_1)/\mathrm{GeV}$	99	184
$m(\tilde\chi_2)/\mathrm{GeV}$	186	356
$m(\tilde\chi_3)/\mathrm{GeV}$	345	568
$m(\tilde\chi_4)/\mathrm{GeV}$	367	581
$m(\tilde\chi^\pm_1)/\mathrm{GeV}$	185	501
$m(\tilde\chi^\pm_2)/\mathrm{GeV}$	386	501

Example for composition: In model L' the LSP is

$$\tilde\chi_1 = 0.995\, \tilde B - 0.014\, \tilde W^3 + 0.090\, \tilde H^0_u - 0.031\, \tilde H^0_d \tag{30}$$

It is dominantly 'gaugino' ('bino'), and not 'higgsino'.

[4] The chargino mixing is a bit more involved, see article by S.P. Martin, section 7.3

APPENDIX

from M. Battaglia et al, hep-ph/0306219

Supersymmetric spectra in post-WMAP benchmarks calculated with ISASUGRA 7.67

Model	A'	B'	C'	D'	E'	F'	G'	H'	I'	J'	K'	L'	M'
$m_{1/2}$	600	250	400	525	300	1000	375	935	350	750	1300	450	1840
m_0	107	57	80	101	1532	3440	113	244	181	299	1001	303	1125
$\tan\beta$	5	10	10	10	10	10	20	20	35	35	46	47	51
sign(μ)	+	+	+	−	+	+	+	+	+	+	−	+	+
m_t	175	175	175	175	171	171	175	175	175	175	175	175	175
Masses													
$\|\mu(m_Z)\|$	773	339	519	663	217	606	485	1092	452	891	1420	563	1940
h	116	113	117	117	114	118	117	122	117	121	123	118	124
H	896	376	584	750	1544	3525	525	1214	444	888	1161	480	1623
A	889	373	580	745	1534	3502	522	1206	441	882	1153	477	1613
$H^{\pm}$	899	384	589	754	1546	3524	532	1217	453	892	1164	490	1627
χ	242	95	158	212	112	421	148	388	138	309	554	181	794
χ_2	471	180	305	415	184	610	286	750	266	598	1064	351	1513
χ_3	778	345	525	671	229	622	492	1100	459	899	1430	568	1952
χ_4	792	366	540	678	302	858	507	1109	475	908	1437	582	1959
$\chi_1^{\pm}$	469	178	304	415	175	613	285	750	265	598	1064	350	1514
$\chi_2^{\pm}$	791	366	541	679	304	846	507	1108	475	908	1435	582	1956
$\tilde{g}$	1367	611	940	1208	800	2364	887	2061	835	1680	2820	1055	3884
e_L, μ_L	425	188	290	376	1543	3499	285	679	304	591	1324	434	1660
e_R, μ_R	251	117	174	224	1534	3454	185	426	227	410	1109	348	1312
ν_e, ν_μ	412	167	274	362	1539	3492	270	665	290	579	1315	423	1648
τ_1	249	109	167	217	1521	3427	157	391	150	312	896	194	796
τ_2	425	191	291	376	1534	3485	290	674	312	579	1251	420	1504
ν_τ	411	167	273	360	1532	3478	266	657	278	558	1239	387	1492
u_L, c_L	1248	558	859	1103	1639	3923	814	1885	778	1554	2722	1001	3670
u_R, c_R	1202	542	830	1064	1637	3897	787	1812	754	1497	2627	969	3528
d_L, s_L	1250	564	863	1107	1641	3924	818	1887	783	1556	2723	1004	3671
d_R, s_R	1197	541	828	1059	1638	3894	786	1804	752	1491	2615	967	3509
t_1	958	411	653	860	1052	2647	617	1477	584	1207	2095	753	2857
t_2	1184	576	837	1048	1387	3373	792	1753	748	1428	2366	920	3231
b_1	1147	514	789	1015	1375	3356	737	1719	677	1377	2297	844	3149
b_2	1181	535	816	1043	1602	3816	770	1761	725	1423	2349	904	3217

Table 2: *Proposed post-WMAP CMSSM benchmark points and mass spectra (in GeV), as calculated using* ISASUGRA 7.67 *[20] and adapting the values of m_0 and $\tan\beta$ (when it is large) to give the best fit to the* SSARD *spectra shown in Table 1, as described in the text.*