pp physics, RWTH, WS 2003/04, T.Hebbeker

2004-11-19

2.1.4.2 CKM matrix and meson oscillations

The neutral K and B mesons are certainly interesting quantum mechanical systems; their particular importance for high energy physics lies in the connection to CKM matrix elements.

Let's first discuss the lifetime difference alias y. The large value in the kaon system can be understood easily: The decay

$$K_s \to \pi^0\pi^0 \text{ oder } \pi^+\pi^- \qquad (\mathrm{CP-eigenvalue} = +1): \tag{1}$$

is favored by phase space with respect to

$$K_l \to \pi^0\pi^0\pi^0 \text{ oder } \pi^+\pi^-\pi^0 \qquad (\mathrm{CP-eigenvalue} = -1): \tag{2}$$

In B decays the difference between the partial decay widths of the many decay modes are much smaller. They depend on the dominant CKM matrix elements, the particle masses involved and QCD corrections.

The more interesting quantity is x alias Δm.

As can be seen from the B mixing box diagrams (chapter 2.1.4), the mixing probability depends on V_{td} or V_{ts}, since the virtual top exchange dominates (proof not given here). It can be shown that the mass difference is proportional to the mixing strength:

$$\Delta m_q \sim |V_{tq}|^2 \tag{3}$$

The factor $|V_{tb}|^2 \approx 1$ is not displayed. The remaining factors (also not shown) contain QCD effects and the top mass, leading to theoretical uncertainties of $\sim 20\%$. The uncertainties drop out to a large extent when considering the ratio m_s/m_d. We understand now why x_s is expected to be two orders of magnitude larger than x_d!

2.1.4.3 Oscillation measurements at pp colliders

To measure B oscillations, B^0 and $\overline{B^0}$ must be distinguished, that is their flavor must be tagged at the time of production and decay. There are different possibilities, we will discuss here only the lepton tag

$$B^0 \to l^+ X \qquad \overline{B^0} \to l^- X \tag{4}$$

i.e. the sign of the lepton charge is equal to the sign of the bottom quark charge. Note that this works also for charged b mesons and for b baryons.

For an oscillation measurement the following steps must be made:

- identification of B hadron/jet: secondary vertex!

- determination of flavor at decay (t).
- determination of flavor at production ($t = 0$).
  Since in most cases the b ($\bar{b}$) is produced together with a $\bar{b}$ (b), we need to tag the 'other' b hadron, which can be done the same way - via its leptonic decay[1].
- measurement of proper decay time. This can be determined from the decay length L and the B momentum p.
  The momentum measurement is easy only if the B hadron is completely reconstructed; if some decay products are not seen (neutral ones), an appropriate correction factor must be applied.

Therefore one looks for $b\bar{b}$ events (secondary vertices!) with two associated leptons[2], determines the charges, classifies the events as 'same-sign' or 'opposite sign', and plots the asymmetry

$$A_{mix}(t) = \frac{N_{os}(t) - N_{ss}(t)}{N_{os}(t) + N_{ss}(t)} \tag{5}$$

as a function of proper decay time t. In the ideal case (no background, misidentification, resolution effects...) the two classes represent the mixed and unmixed events. In the limit of negligible lifetime differences $y \longrightarrow 0$ the integrated oscillation probability becomes simply

$$P_{\overline{B}^0}(t) \;=\; \frac{1}{2}\, e^{-\Gamma t} \cdot (1 - \cos(\Delta m\; t)\,) \tag{6}$$

This probability is proportional to N_{ss}, the number of same sign dilepton events as a function of decay time:

$$N_{ss}(t) \sim e^{-\Gamma t} \cdot (1 - \cos(\Delta m\; t)\,) \tag{7}$$

Similarly:

$$N_{os}(t) \sim e^{-\Gamma t} \cdot (1 + \cos(\Delta m\; t)\,) \tag{8}$$

If we start with a $\overline{B^0}$ instead of a B^0, the signs of the leptons change, but the formulae for N_{ss} and N_{os} are still valid. Thus:

$$A_{mix}(t) = \cos(\Delta m\, t) \tag{9}$$

In this asymmetry the exponential decay term drops out - however it nevertheless determines the event statistics when performing the measurement!

The first hint for B oscillations was found by the UA1 collaboration in 1986, which saw an excess of same-sign dilepton events, but without measuring the time dependence. [3]

Thus the measured asymmetry should look like the left graph below:

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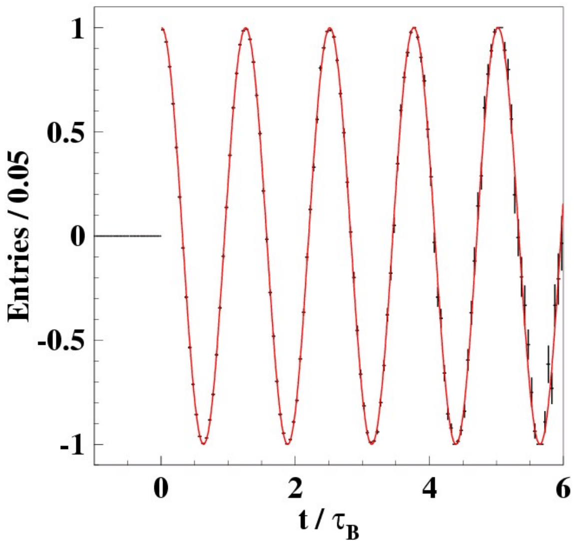
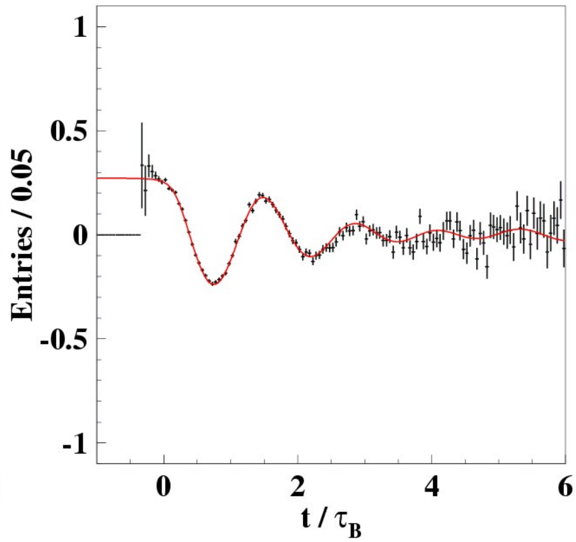
[1]However, we must take into account, that the second meson can also be a neutral one, so it may oscillate, too. This requires a small corretion to the simple formulae we will derive here.

[2]Since the leptonic branching fractions are of the order of 10%, this is a small fraction of all b events!

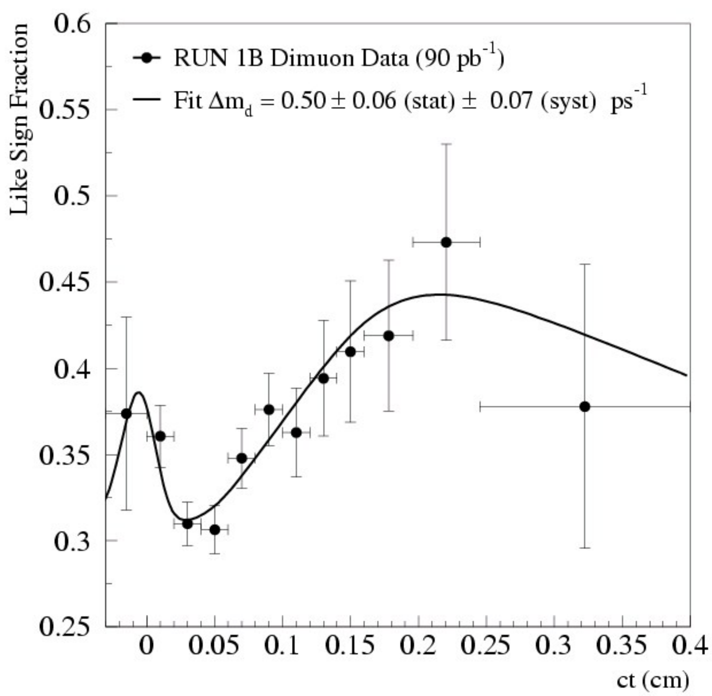
[3]if the time dependence is not measured, often the term 'mixing' is used, else the word 'oscillation' is employed.





In real life several effects deteriorate this cosine curve, in particular the momentum resolution (smears out t, in particular at large values) and the mistag probability (wrong charge, wrong lepton, background…) play a negative role. The resulting Monte Carlo prediction (for CDF, run I) is shown on the right.

CDF has published a measurement based on dimuon events in 1999. The sample contains 2044 like-sign and 3924 opposite-sign muon events. The result:



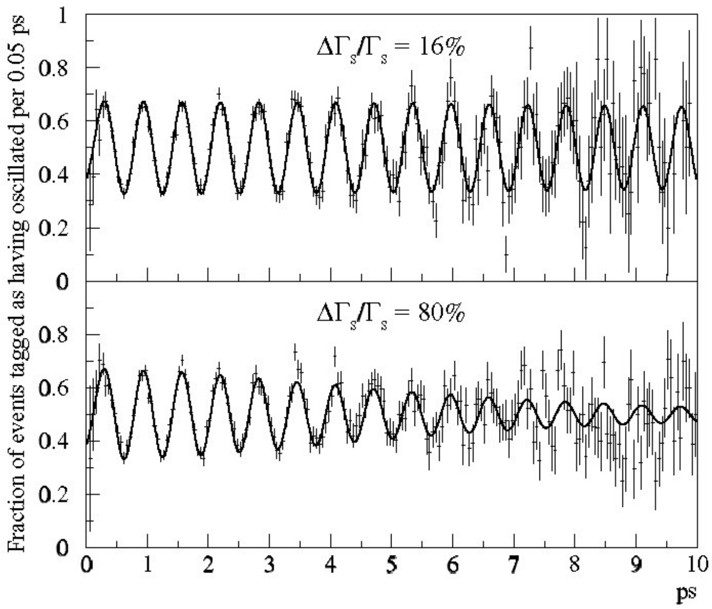
What is plotted here is not the asymmetry A_{mix}, but the related quantity

$$F_{mix}(t) = \frac{N_{ss}(t)}{N_{os}(t) + N_{ss}(t)} = \frac{1}{2} \cdot (1 - \cos(\Delta m\, t)\,) = \sin^2 \frac{\Delta m\, t}{2} \tag{10}$$

$$\Delta m_d = (0.503 \pm 0.064 \pm 0.071)/\text{ps} \tag{11}$$

Note that the 'peak' in the figure near $t = 0$ is due to background and has nothing to do with the oscillation. Details of this analysis: $\longrightarrow$ tutorial. So far the best B_d^0 oscillation measurements were performed with e^+e^- machines.

How the B_s mixing could be measured at the LHC is illustrated by the following curves, obtained in an ATLAS study:



The oscillation amplitude is damped due to backgrounds and wrong sign assignments (second B oscillates, muon from cascade decay $b \rightarrow c \rightarrow \mu^+, \ldots$). Note that once the statistics is sufficient to see the oscillation, the period can be determined quite precisely from a $\cos$-fit. In both cases $\Delta m_s = 10/\mathrm{ps}$ was assumed, but in the upper half of the plot the lifetime difference is small, while it is quite large in the lower half; in this case the simplified formulae can not be used any more and we must go back to formula (12) in section 2.1.4.1:

$$P_{\overline{B}^0}(t) \;=\; \frac{1}{4} \cdot \left(e^{-\Gamma_1 t} + e^{-\Gamma_2 t} - 2e^{-\Gamma t} \cdot \cos(\Delta m\ t)\right)$$

After a while only the $e^{-\Gamma t}$ term with the smaller Γ will survive and the oscillation term plays no role any more.

The implications of the Δm_d measurement on our knowledge of the CKM-Matrix are shown at the end of the next section.

2.1.5. CP violation

2.1.5.1 'The' unitarity triangle

In the Standard Model CP violation is directly related to the non-vanishing complex phase δ in the unitary CKM matrix

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \tag{12}$$

which can be parametrised by three angles and one phase. Here we will use another (frequently used) parametrisation of V, invented by Wolfenstein:

$$V^W = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3\omega e^{-i\delta} \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3\,(1-\omega e^{i\delta}) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4) \qquad \omega\, e^{-i\delta} = \rho - i\eta \quad (13)$$

Advantage: 'Hierarchy' is 'built-in'. Numerically (from experimental data!): $A \approx 0.8$, $\lambda \approx 0.2$. Necessary condition of CPV: $\eta/\rho \neq 0$. Note that V^W is equal to V only up to $\mathcal{O}(\lambda^3)$. ρ and η are not known precisely - yet.

The phase can only be measured through an interference phenomenon, otherwise only the magnitude counts:

$$A \sim V_{xy} \qquad \rightarrow \qquad |A|^2 \sim |V_{xy}|^2 \quad (14)$$

$$A \sim V_{xy} + V_{ab} \qquad \rightarrow \qquad |A|^2 \sim |V_{xy}|^2 + |V_{ab}|^2 + V_{xy}\,V_{ab}^* + V_{xy}^*\,V_{ab} \quad (15)$$

Note that the vertex $q - W - q'$ is described by $V_{qq'}$ while for the antiquark vertex $\bar{q} - W - \bar{q}'$ the amplitude is proportional to $V_{qq'}^*$.

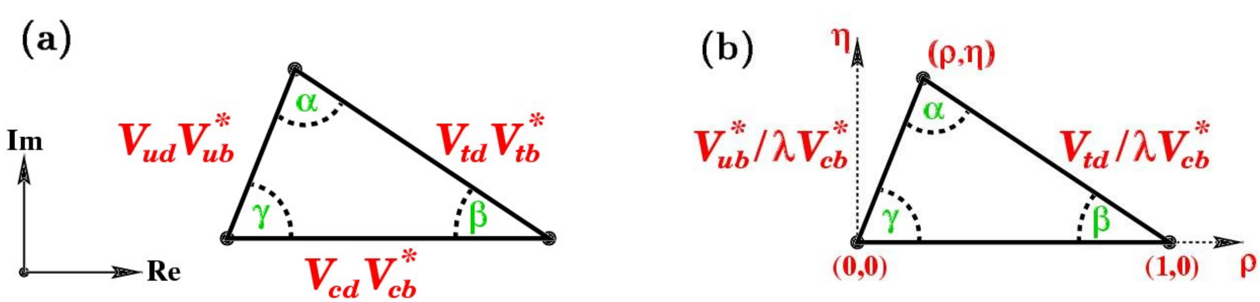
The unitarity condition $V^* \cdot V^T = 1$ implies six normalization relations like

$$1 = V_{cd}\,V_{cd}^* + V_{cs}V_{cs}^* + V_{cb}V_{cb}^* \equiv |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 \quad (16)$$

In these equations the complex phase drops out - thus they are not of interest here. They can be used to determine $\rho^2 + \eta^2$, but not η/ρ. This is different for the six orthogonality relations, for example

$$0 = V_{ud}\,V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* \quad (17)$$

This equation can be represented as a triangle in the complex plane[4]:



When dividing by $V_{cd}\,V_{cb}^*$ we arrive at figure (b). Here we have approximated $V_{ud} = 1$ and used $\lambda = V_{us} = -V_{cd}$:

$$0 \;=\; \frac{V_{ud}\,V_{ub}^*}{V_{cd}V_{cb}^*} \;+\; 1 \;+\; \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} \quad\rightarrow\quad -\frac{V_{ub}^*}{\lambda V_{cb}^*} \;+\; 1 \;-\; \frac{V_{td}}{\lambda V_{cb}^*} \quad (18)$$

In Wolfenstein language, approximating $V_{ud} = 1 - \lambda^2/2 = 1$:

$$0 \;=\; -[\rho + i\eta] \;+\; 1 \;+\; [-(1-\rho-i\eta)] \quad (19)$$

[4]Beware: real part of $V_{cd}V_{cb}^*$ is negative!

Thus, determination of the triangle means measuring ρ, η. Why is such a triangle interesting ?
a) it is related to $\eta/\rho = \tan\delta$.
b) its size and form can be measured through the lengths of their legs, (e.g. $|V_{td}/(\lambda V^*_{cb})|$) - this does not require a CPV process!
c) it can be measured via the angles, in a CP violating process
d) comparing b) and c): important consistency check!

Out of the 6 unitarity triangles two are very similar (identical to $\mathcal{O}(\lambda^3)$) and it is sufficient to discuss one of them, which is (17). While all six triangles have the same area, the other four are 'squashed', that is two of the sides are much longer than the third one and/or all angles are close to 0^0, 90^0 or 180^0, example:

$$0 = V_{ud}V^*_{us} + V_{cd}V^*_{cs} + V_{td}V^*_{ts} \longrightarrow (0.2) + (0.2) + (0.002) \tag{20}$$

But angles θ near 0^0, 90^0 and 180^0 cannot be measured easily (CPV effect $\sim \sin 2\theta$), and it is also difficult to infer the smallest (least known) side from the two big ones.

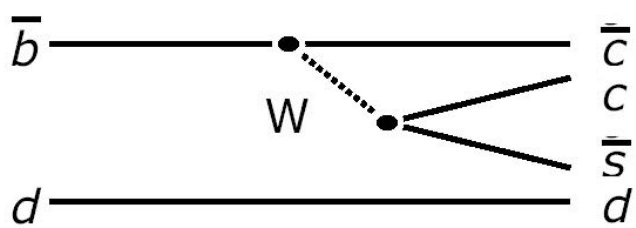
Therefore (17) is **the** unitarity triangle! Note that the three angles are given by

$$\frac{V_{td}V^*_{tb}}{V_{ud}V^*_{ub}} \Big/ \left|\frac{V_{td}V^*_{tb}}{V_{ud}V^*_{ub}}\right| = e^{-i\alpha} \qquad \frac{V_{cd}V^*_{cb}}{V_{td}V^*_{tb}} \Big/ \left|\frac{V_{cd}V^*_{cb}}{V_{td}V^*_{tb}}\right| = e^{-i\beta} \qquad \ldots \tag{21}$$

2.1.5.2 CP violation in neutral B decays - theory

CPV is measurable through decays to CP eigenstates. The best ('golden') channel (relatively high branching fraction, good theoretical understanding) is

$$B^0_d, \overline{B^0_d} \longrightarrow J/\Psi\, K^0_s \longrightarrow l^+l^-\,\pi^+\pi^- \tag{22}$$

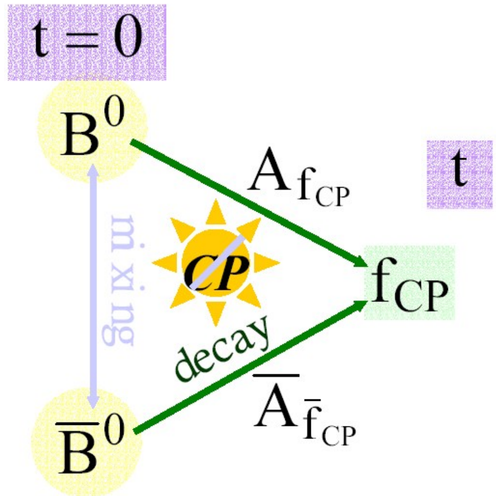


Since

$$CP(B^0) = \overline{B^0} \qquad CP(J/\Psi\, K^0_s) = J/\Psi\, K^0_s \tag{23}$$

CP **conservation** implies **no** difference between B_0 and $\overline{B_0}$ decays into this final state.

In case of CP **violation** there are differences - but they are observable only if an interference phenomenon can be exploited - see above. Here there the two processes that interfere are:
- direct decay of B^0 into CP eigenstate
- oscillation of $B^0 \longrightarrow \overline{B^0}$ and subsequent decay of $\overline{B^0}$.

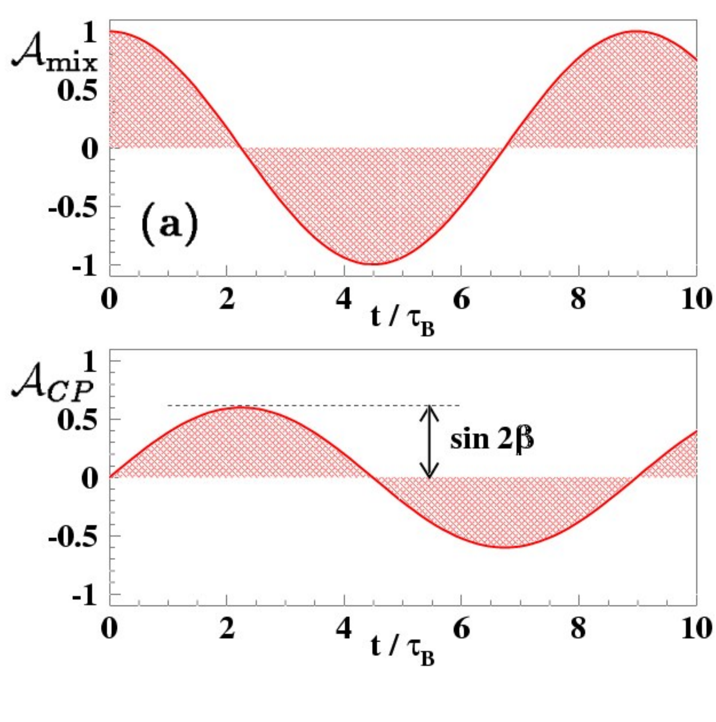


Note that the final state is in both cases the same and there is no way to distingush between the two paths experimentally.

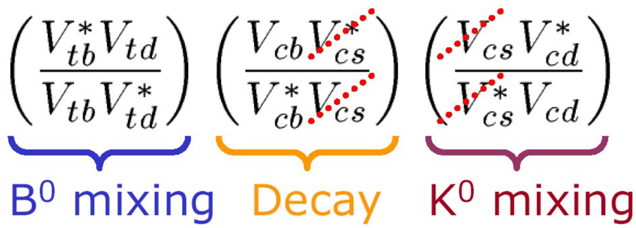
Thus oscillation plays a major role in this context! The CPV effect can be measured through the time dependent CP asymmetry

$$A_{CP}(t) = \frac{\overline{B^0}(t) - B^0(t)}{\overline{B^0}(t) + B^0(t)} = const \cdot \sin \Delta m_d t \tag{24}$$

Here $B^0(t)$ denotes the number of decays at proper time t for a meson that was a pure B^0 state at $t = 0$ (known from the second b), and $\overline{B^0(t)}$ is the probability number of decays at time t for an initially pure $\overline{B^0}$ state. The following graph illustrates on the top the mixing asymmetry and on the bottom the CP asymmetry, which is 'out of phase' by $\pi/2$: at the start we have a pure B^0 state and there is no CP violation; then, when A_{mix} is zero, we have equal amounts of B^0 and $\overline{B^0}$, resulting in a maximum in A_{CP}.



How big is the amplitude $const$ in equation (24) ? Some crude arguments: Three effects play a role, the direct decay $\sim V_{cb}V_{cs}^*$, the B^0 oscillation $\sim V_{tb}^*V_{td}$, and also the K^0 mixing $\sim V_{cs}V_{td}^*$. The ratio r of the amplitudes for $B^0 \to J/\Psi K^0$ and $\overline{B^0} \to J/\Psi K^0$ is given by this expression:



Thus

$$r = \frac{V_{cd}^* V_{cb}}{V_{td}^* V_{tb}} / \frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*} = e^{2i\beta} \tag{25}$$

It is important to note that only CKM matrix elements enter and no QCD corrections - which can be large for other CP eigenstates! Therefore

$$Im(r) = \sin(2\beta) \tag{26}$$

It follows (without proof)

$$const = \sin(2\beta) \tag{27}$$

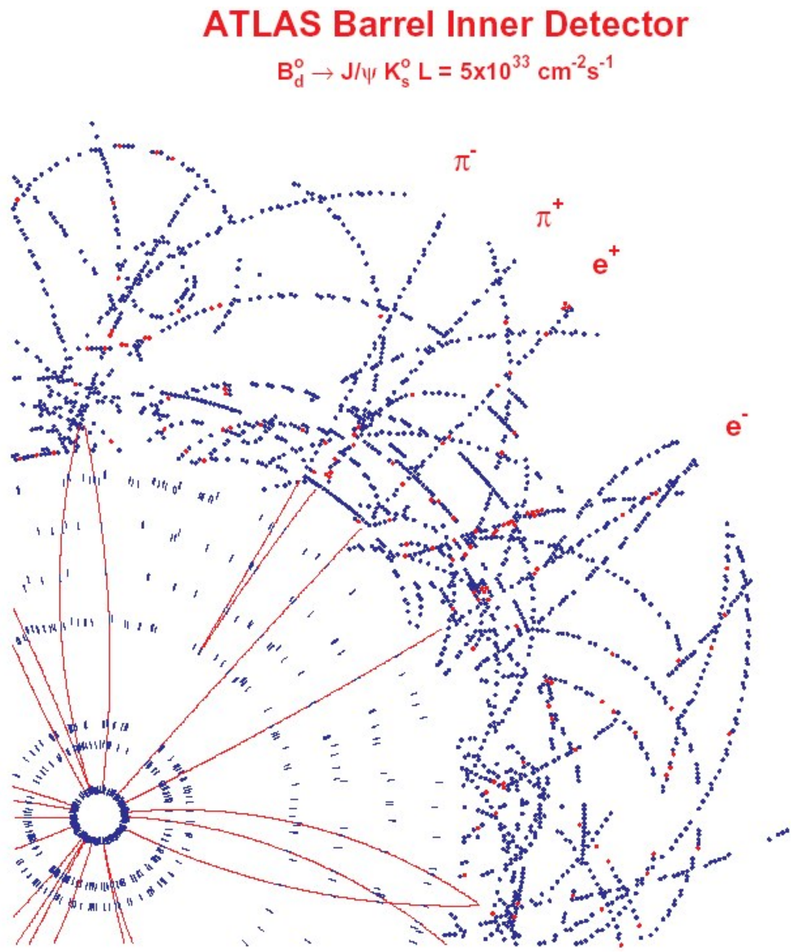
PS: CPV in the B_s meson system is expected to be much smaller than for B_d^0, and is therefore not discussed here. Instead of K_s^0 also K_l^0 could be used, but it is experimentally disfavored (decay into pions rarely observable).

2.1.5.3 CP violation in neutral B decays - experiments

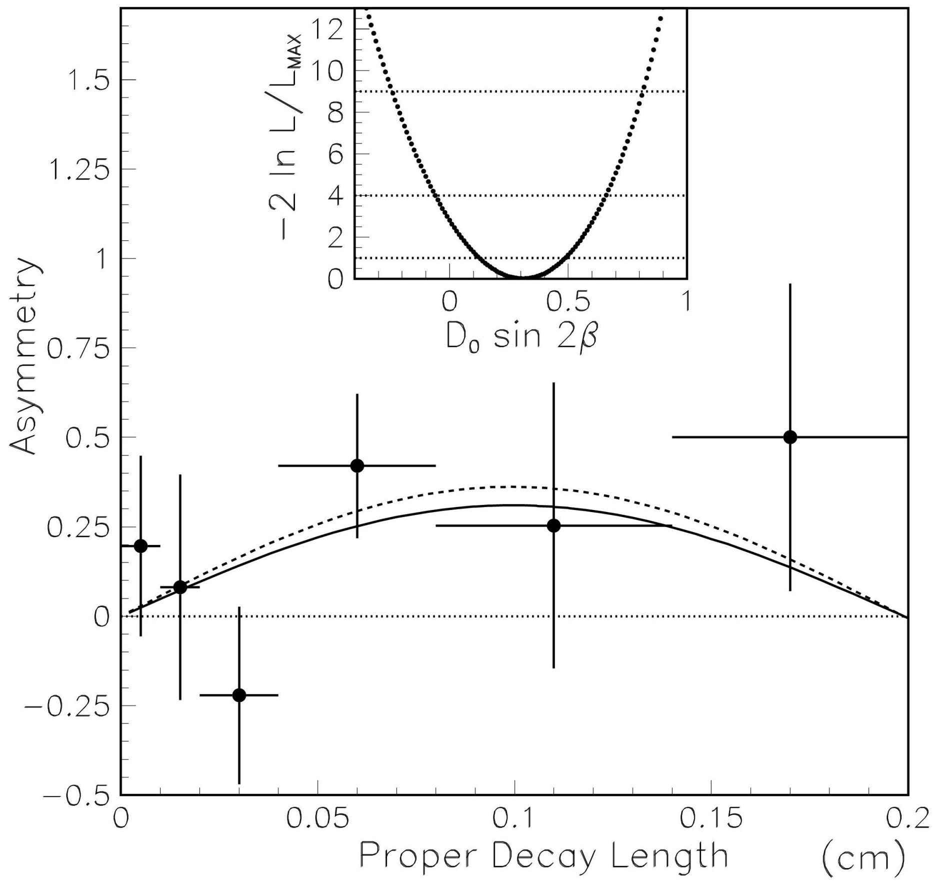
The decay into $J/\Psi\ K^0$ has been measured in many experiments, in spite of the rather small branching fractions

$$BR(B^0 \to J/\Psi K^0) = 0.09\% \qquad BR(J/\Psi \to \mu^+\mu^-) = 6\% \tag{28}$$

The following graph shows a simulated ATLAS event where the J/Ψ decays into an e^+e^- pair.



The CP asymmetry has been measured by several collaborations, including CDF (via $J/\Psi \to \mu^+\mu^-$).



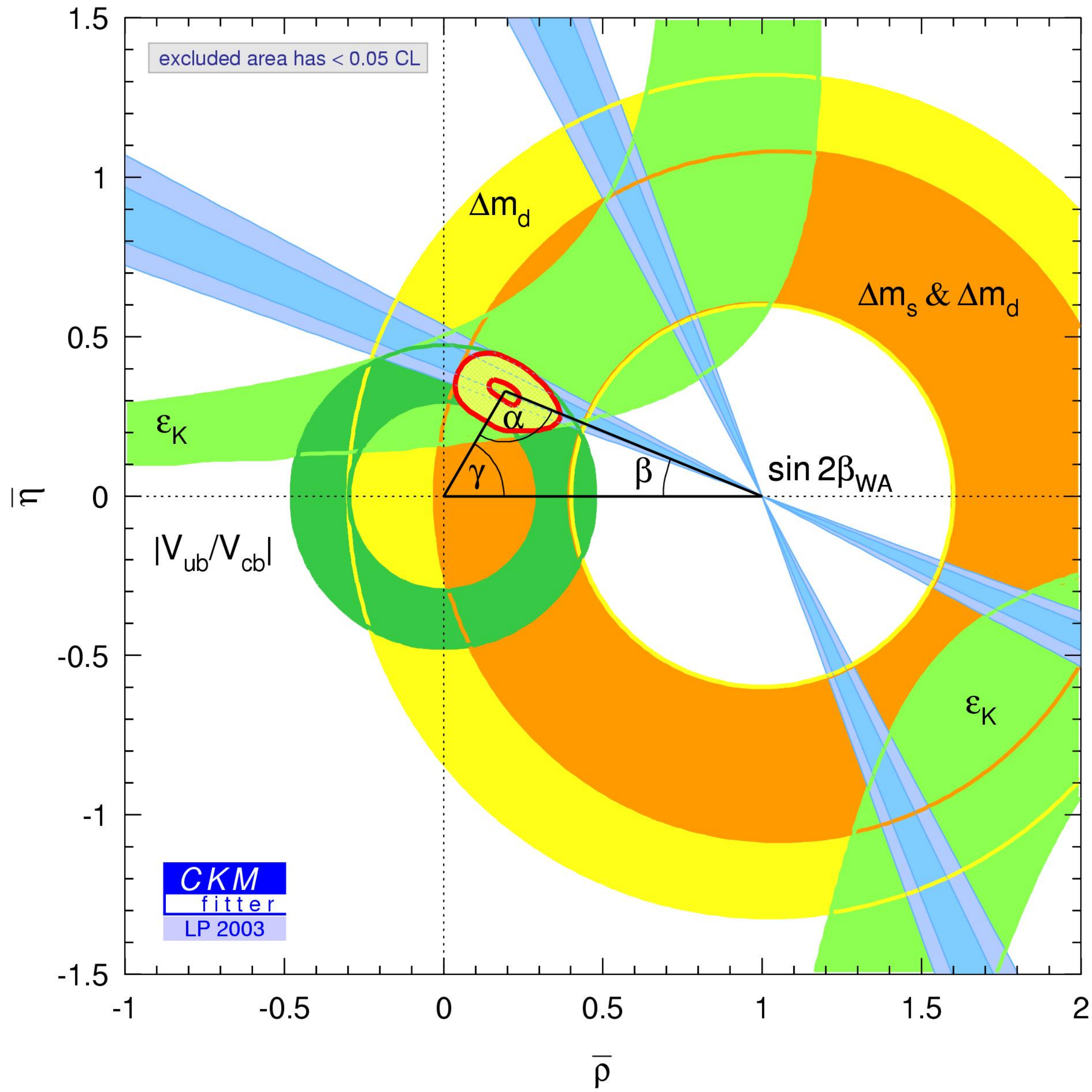
About 200 neutral B events have been used for this analysis. The insert shows the result of the fit, including a dilution factor D_0. The latest CDF number (run I) is $\sin 2\ \beta\ =\ 0.79^{+0.41}_{-0.44}$. At the moment the best results come from the e^+e^- experiments Babar and Belle, dominating the world average of

$$\sin(2\beta) = 0.701 \pm 0.053 \tag{29}$$

Clearly, CP violation is established in the B_d^0 system.

2.1.5.4 The unitarity triangle in 2003

Finally we summarize the experimental knowledge on the unitarity triangle:



Here $\bar{\rho} = \rho\,(1-\lambda^2/2) \sim \rho$ and $\bar{\eta} = \eta\,(1-\lambda^2/2) \sim \eta$. Note that all the different measurements are consistent within this framework, a major success of the Standard Model!