Theoretical physics for the LHC, Lectures II and III

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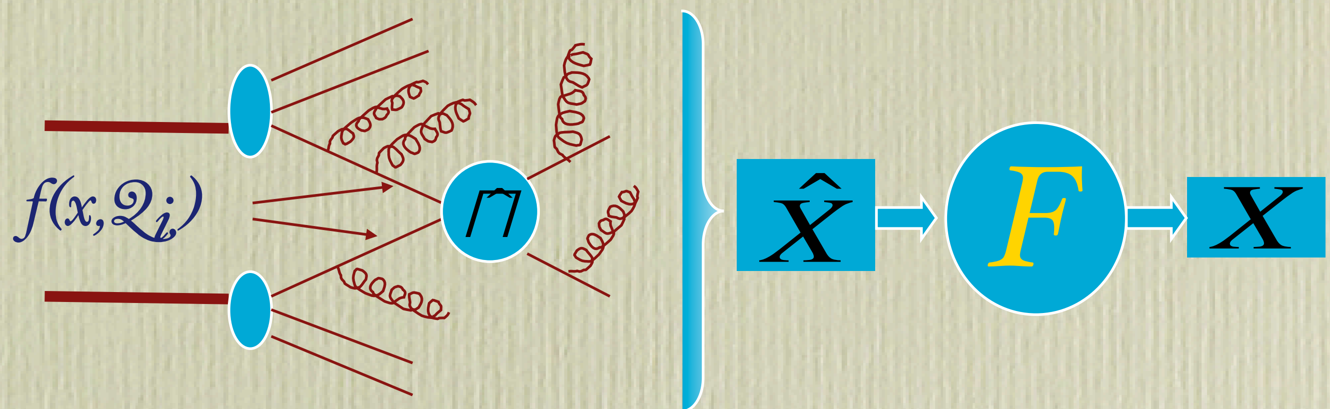
Contents

- 5' review of QCD
- The structure of the proton:
  - parton densities
  - their evolution
- The structure of a hard proton-proton collision:
  - jet evolution
  - hadronization
- Some benchmark SM processes and their applications:
  - Drell-Yan
  - Jets
  - Heavy quark production

Factorization Theorem

$$\frac{d\sigma}{dX} = \sum_{j,k} \sum_{\hat{X}} f_j(x_1, Q_i)\, f_k(x_2, Q_i)\, \frac{d\hat{\sigma}_{jk}(Q_i, Q_f)}{d\hat{X}}\, F(\hat{X} \to X; Q_i, Q_f)$$



$$f_j(x, Q)$$

- sum over all initial state histories leading, at the scale Q, to:

$$\vec{p}_j = x\,\vec{P}_{proton}$$

$$F(\hat{X} \to X; Q_i, Q_f)$$

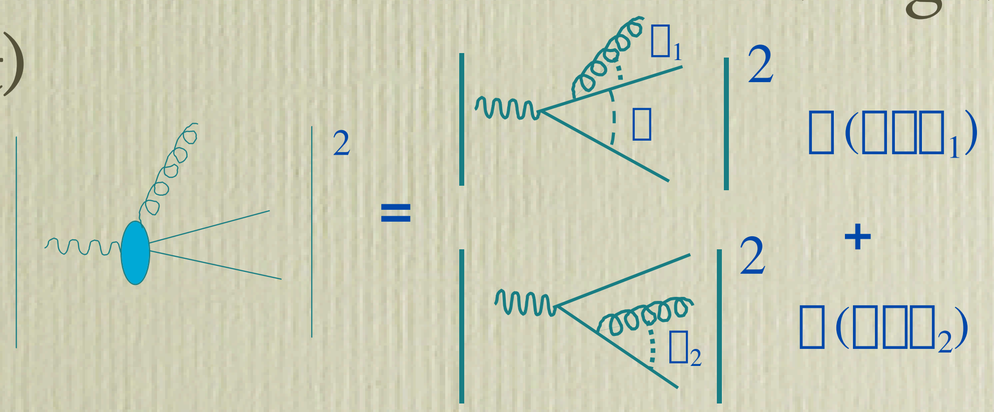
- transition from partonic final state to the hadronic observable (hadronization, fragm. function, jet definition, etc)
- Sum over all histories with *X* in them

Tools to calculate X-sections at the LHC

- Knowledge of partonic structure of the proton
  - extract from available data
  - will improve with LHC data
- Matrix elements for signals and backgrounds:
  - very difficult for high-multiplicity final states

nj	2	3	4	5	6	7	8
diag'	4	25	220	2485	34300	5×10^5	10^7

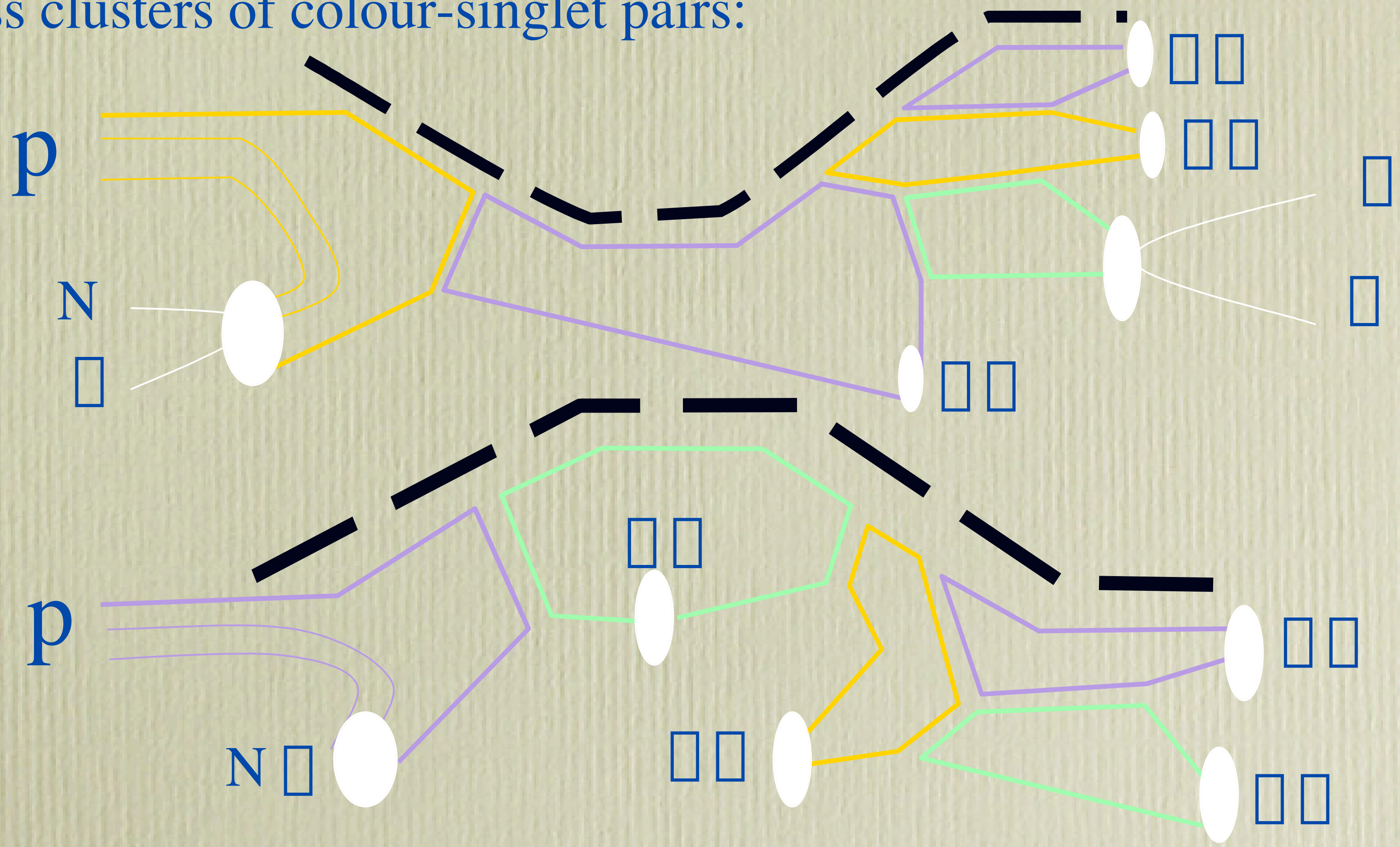
  - very difficult to improve PT beyond LO
- Approximations to the emission of multi-gluons (shower development)



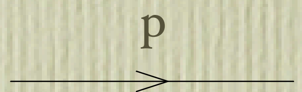
- Models for the final hadronization

Hadronization

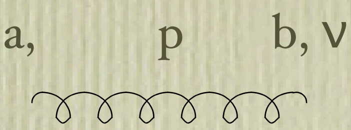
At the end of the perturbative evolution, the final state consists of quarks and gluons, forming, as a result of angular-ordering, low-mass clusters of colour-singlet pairs:



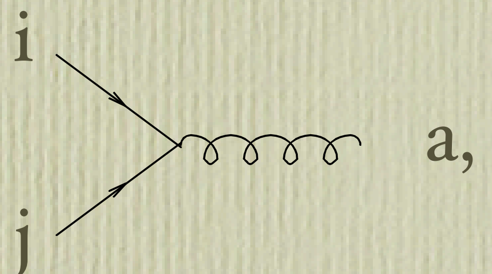
QCD Feynman rules



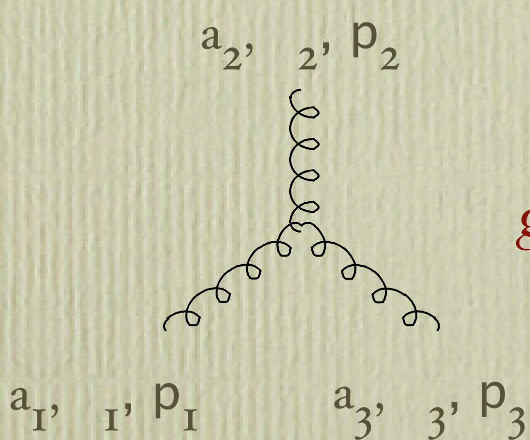
$$\frac{i\Box^{ij}}{\not{p} - m + i\Box}$$



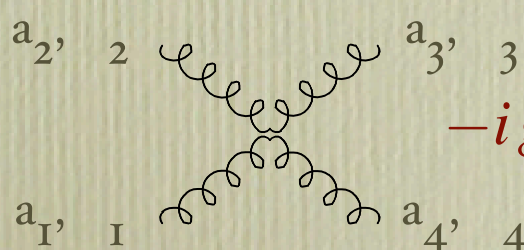
$$\Box^{ab}\frac{-i\,g^{\mu\Box}}{p^2 + i\Box} \quad \text{(Feynman gauge)}$$



$$i\,g\,\Box^{a}_{ij}\,\Box^{\mu}$$



$$g\,f^{a_1a_2a_3}\;[g^{\mu_1\mu_2}\,(p_1 - p_2)^{\mu_3} + g^{\mu_2\mu_3}\,(p_2 - p_3)^{\mu_1} + g^{\mu_3\mu_1}\,(p_3 - p_1)^{\mu_2}]$$



$$-i\,g^2\left[f^{a_1a_2X}\,f^{a_3a_4X}\;(g^{\mu_1\mu_3}g^{\mu_2\mu_4} - g^{\mu_1\mu_4}g^{\mu_2\mu_3}) + (2\leftrightarrow 3) + (2\leftrightarrow 4)\right]$$

Some results for the SU(3) colour algebra

$\square^a_{ij}$: $SU(N_c)$ matrix in the fundamental representation,

$N_c = 3\,, \quad a = 1,\ldots,N_c^2-1\,, \quad i = 1,\ldots,N_c$

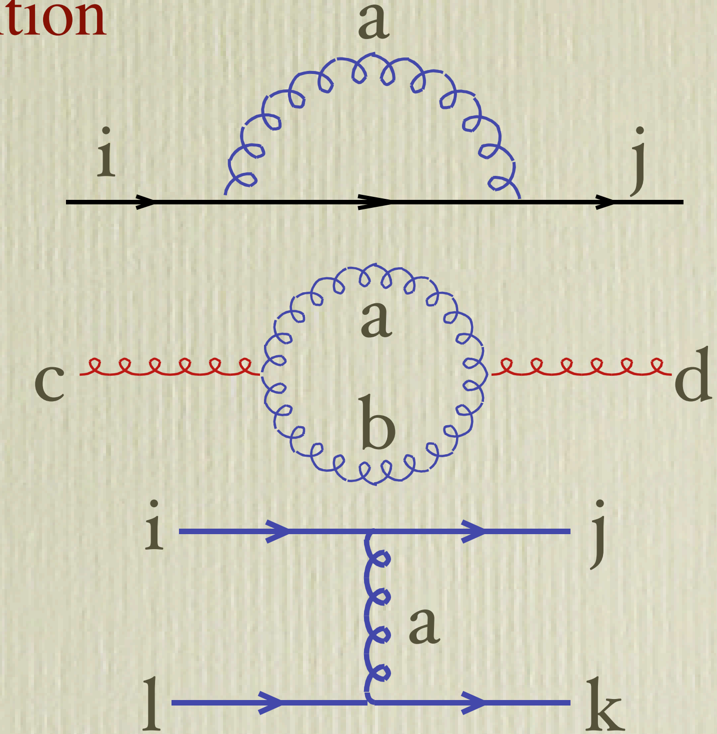
$[\square^a,\square^b] \;=\; i\,f^{abc}\,\square^c \quad (\Rightarrow \mathrm{tr}\,\square^a = 0)$

$\mathrm{tr}\,(\square^a\square^b) \overset{\text{def}}{=} T_F\,\square^{ab}, \quad T_F = 1/2$ by convention

$\underset{a}{\square}\,(\square^a\square^a)_{ij} \overset{\text{def}}{=} C_F\,\square^{ij} \overset{\text{Exercise}}{=} \dfrac{N_c^2-1}{N_c}\square^{ij}$

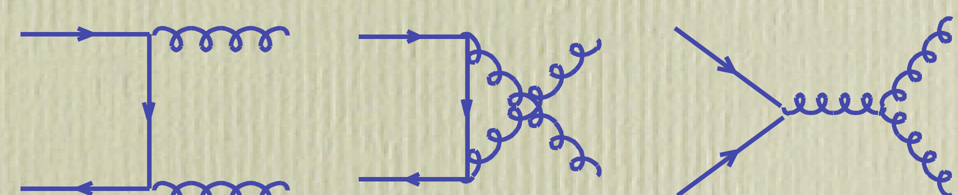
$\underset{a.b}{\square}\;f^{abc}\,f^{abd} \overset{\text{def}}{=} C_A\,\square^{cd} \overset{\text{Exercise}}{=} N_c\,\square^{cd}$

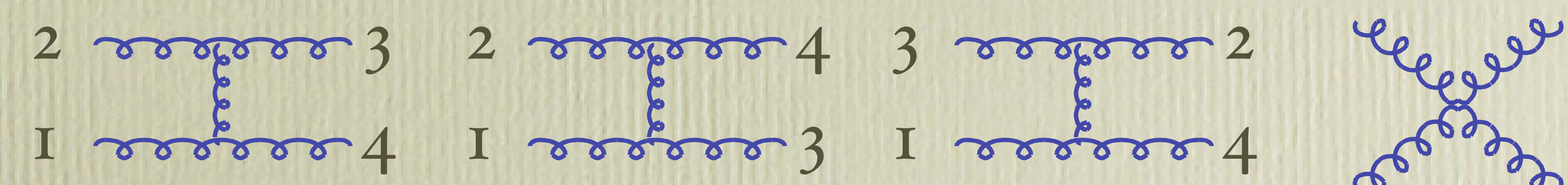
$\underset{a}{\square}\;\square^a_{ij}\,\square^a_{kl} \overset{\text{Exercise}}{=} \dfrac{1}{2}(\square_{ik}\square_{jl} - \dfrac{1}{N}\square_{ij}\square_{kl})$



Exercises

- Verify the properties of the SU(N) algebra given in the previous pages
- Prove that the sums of the following sets of diagrams are gauge invariant, namely the amplitude remains invariant if we replace the polarization vector of any gluon, ϵ , with ϵ + p , p being the gluon momentum:

- 



Running of the coupling constant

$$\alpha_s \stackrel{def}{=} \frac{g_s^2}{4\pi} \qquad \frac{d\alpha_s}{d\log(Q^2)} = \beta(\alpha_s)$$

At 1-loop: $\beta = -b_0\alpha_s^2\,, \quad b_0 = \dfrac{33-2n_f}{12\pi}$

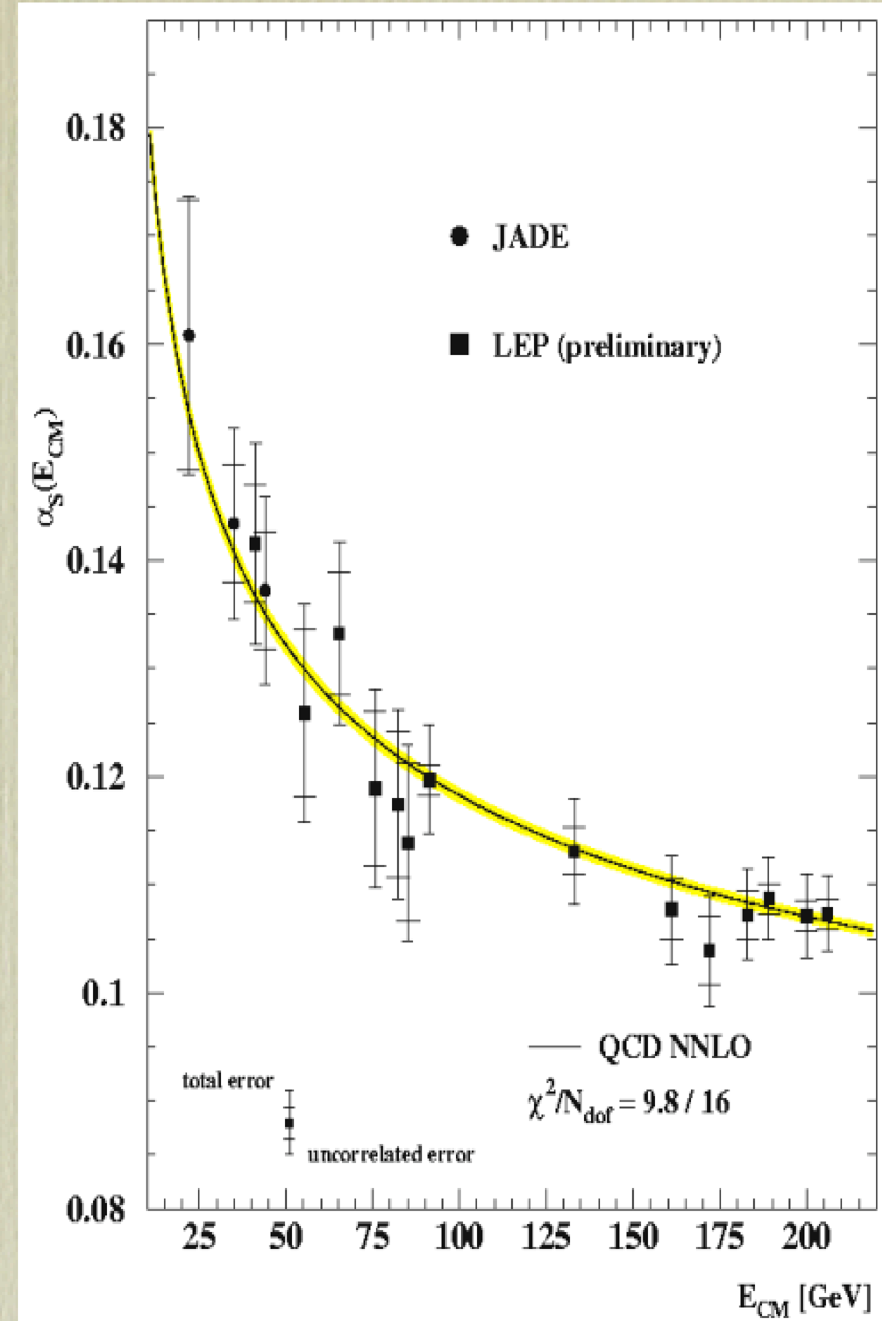
and $\alpha_s(Q) = \dfrac{1}{b_0\log(Q^2/\Lambda^2)}$

At 2-loops:

$$\beta = -b_0\alpha_s^2 - b_1\alpha_s^3\,, \quad b_1 = \frac{153-19n_f}{24\pi^2}$$

and

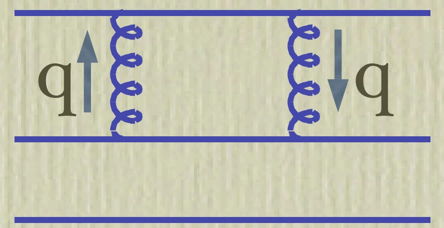
$$\alpha_s(Q) = \frac{1}{b_0\log(Q^2/\Lambda^2)}\left[1-\frac{b_1}{b_0^2}\frac{\log\log(Q^2/\Lambda^2)}{\log Q^2/\Lambda^2}\right]$$



Current World Average (Bethke 2002): $\alpha_s(M_Z) = 0.118 \pm 0.003$

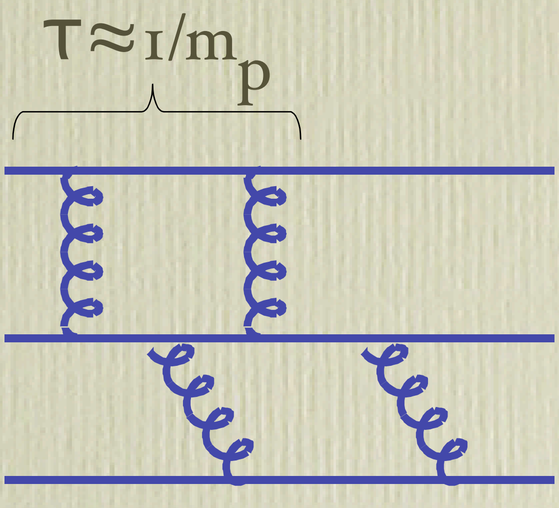
Universality of parton densities and factorization, a naive proof

Exchange of hard gluons among quarks inside the proton is suppressed by powers of $(m_p/Q)^2$



$$\overset{q>Q}{\sim} \int_q^Q \frac{d^4q}{q^6} \sim \frac{1}{Q^2}$$

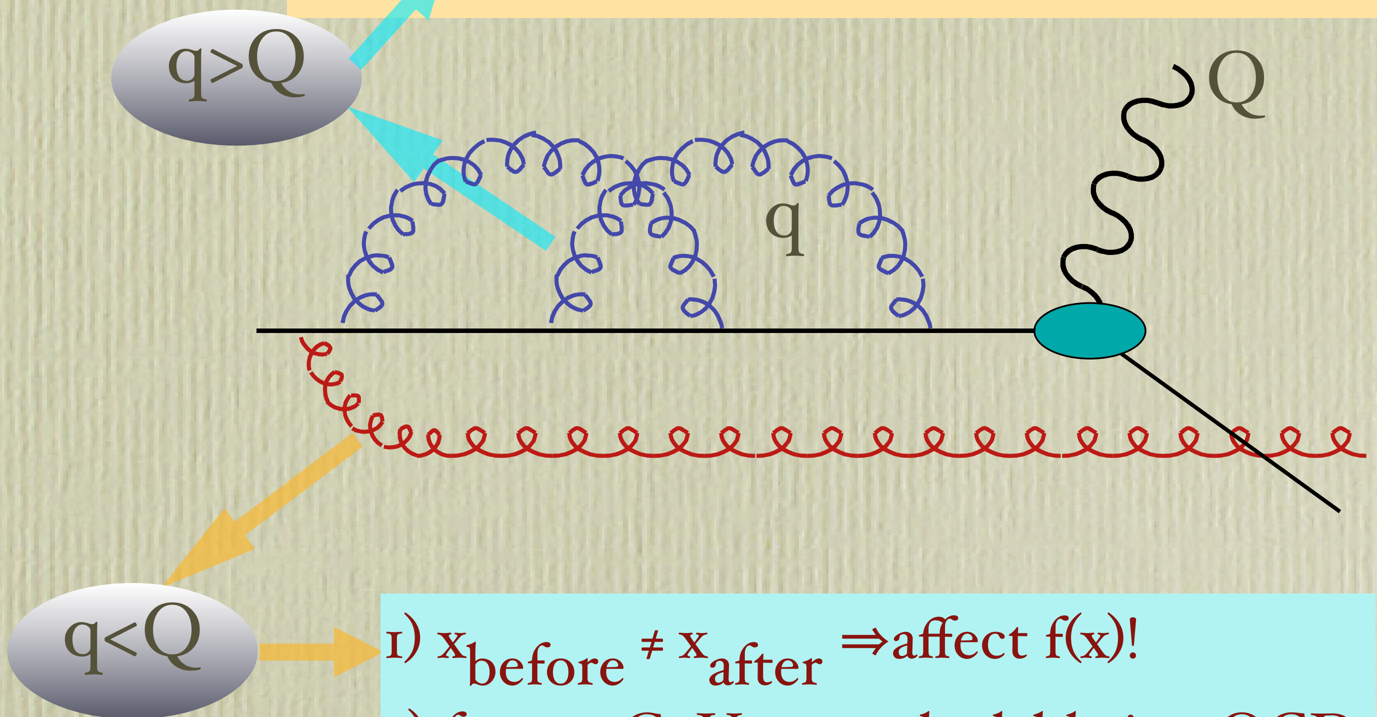
Typical time-scale of interactions binding the proton is therefore of $O(1/m_p)$ (in a frame in which the proton has energy E, $\tau=\gamma/m_p = E/m_p^2$)



If a hard probe ($Q \gg m_p$) hits the proton, on a time scale $=1/Q$, there is no time for quarks to negotiate a coherent response

As a result, to study inclusive processes at large Q it is sufficient to consider the interactions between the external probe and a single parton:

1) calculable in perturbative QCD (pQCD)
2) do not affect f(x): $x_{before} = x_{after}$



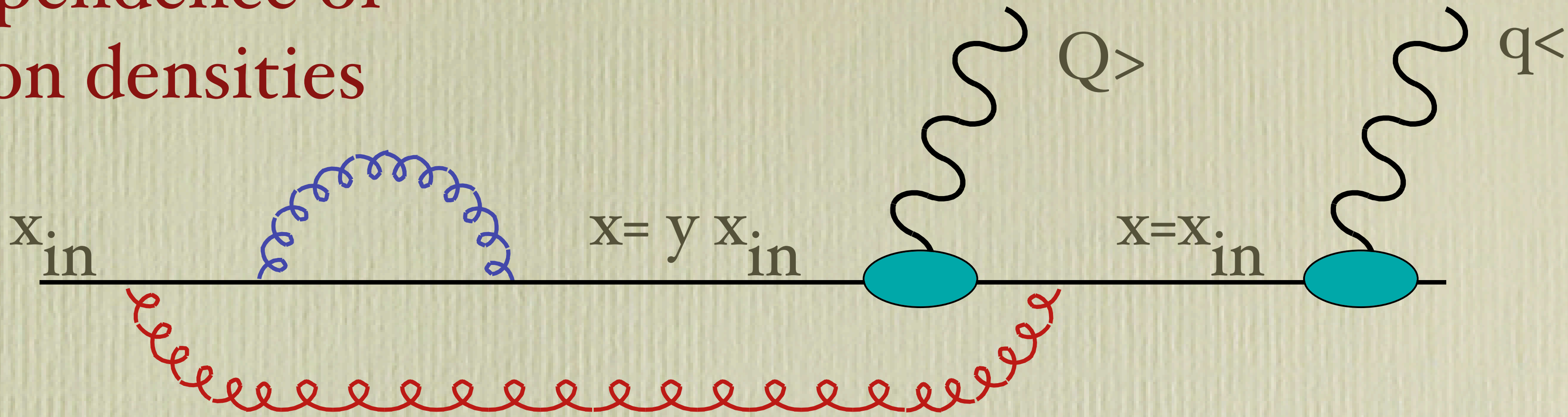
This gluon cannot be reabsorbed because the quark is gone

1) $x_{before} \neq x_{after}$ ⇒affect f(x)!
2) for q≈1 GeV **not** calculable in pQCD

However, since τ(q≈1GeV)>>1/Q, the emission of low-virtuality gluons will take place long before the hard collision, and therefore cannot depend on the detailed nature of the hard probe. While it is not calculable in pQCD, f(q<<Q) can be measured using a reference probe, and used elsewhere ⇒

Universality of f(x)

Q dependence of parton densities



The larger is Q, the more gluons will **not** have time to be reabsorbed

PDF's depend on Q!

$$f(x,Q) = f(x,\mu) + \int_x^1 dx_{in}\, f(x_{in},\mu) \int_\mu^Q dq^2 \int_0^1 dy\, P(y,q^2)\, \delta(x - y x_{in})$$

$$f(x,Q) = f(x,\mu) + \int_x^1 dx_{in} f(x_{in},\mu) \int_\mu^Q dq^2 \int_0^1 dy\, P(y,q^2)\, \delta(x - y x_{in})$$

f(x,Q) should be independent of the intermediate scale considered:

$$\frac{df(x,Q)}{d\mu^2} = 0 \quad \Rightarrow \frac{df(x,\mu)}{d\mu^2} = \int_x^1 \frac{dy}{y} f(y,\mu)\, P(x/y,\mu^2)$$

One can prove that:

calculable in pQCD

$$P(x,Q^2) = \frac{\alpha_s}{2\pi}\frac{1}{Q^2} P(x)$$

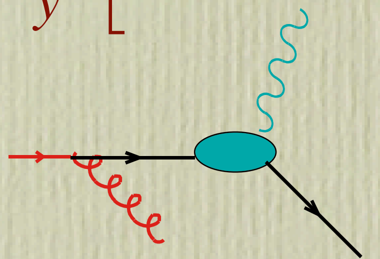
and therefore (Altarelli-Parisi equation):

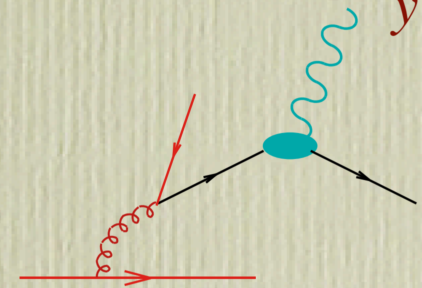
$$\frac{df(x,\mu)}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} f(y,\mu)\, P(x/y)$$

More in general, one should consider additional processes which lead to the evolution of partons at high Q (t=logQ^2):

$$[g(x)]_+ : \quad \int_0^1 dx\, f(x)\, g(x)_+ \equiv \int_0^1 [f(x) - f(1)]\, g(x)\, dx$$

$$\frac{dq(x,Q)}{dt} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left[q(y,Q)\, P_{qq}(\frac{x}{y}) \; + \; g(y,Q)\, P_{qg}(\frac{x}{y}) \right]$$

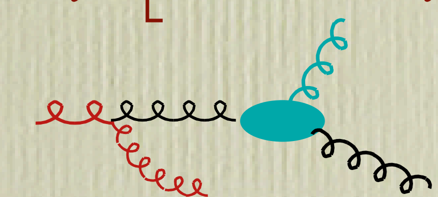


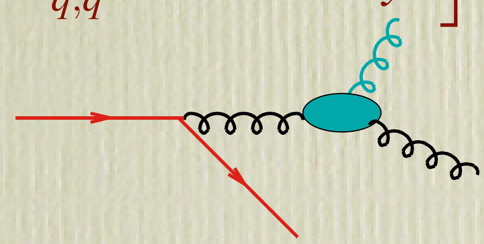


$$P_{qq}(x) = C_F \left(\frac{1+x^2}{1-x} \right)_+$$

$$P_{qg}(x) = \frac{1}{2} \left[x^2 + (1-x)^2 \right]$$

$$\frac{dg(x,Q)}{dt} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left[g(y,Q)\, P_{gg}(\frac{x}{y}) \; + \; \sum_{q,\bar{q}} q(y,Q)\, P_{gq}(\frac{x}{y}) \right]$$





$$P_{gq}(x) = C_F \left(\frac{1+(1-x)^2}{x} \right)$$

$$P_{gg}(x) = 2N_c \left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) \right] + \delta(1-x) \left(\frac{11N_c - 2n_f}{6} \right)$$

Example: charm in the proton

$$\frac{dc(x,Q)}{dt} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y}\, g(y,Q)\, P_{qg}(\frac{x}{y})$$

Assuming a typical behaviour of the gluon density:

$$g(x,Q) \sim A/x$$

we get:

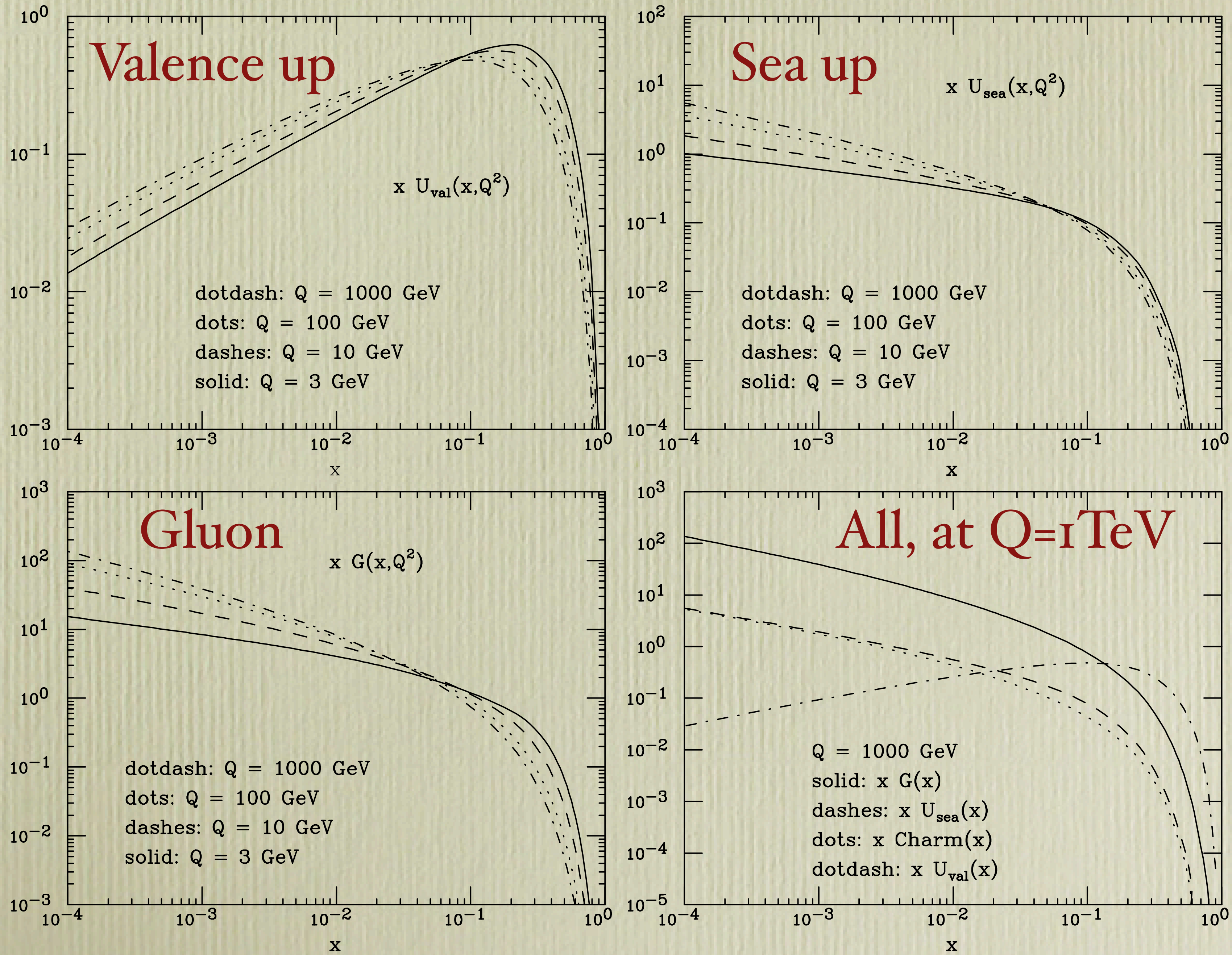
$$\frac{dc(x,Q)}{dt} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y}\, g(x/y,Q)\, P_{qg}(y) = \frac{\alpha_s}{2\pi}\int_x^1 dy \frac{A}{x}\frac{1}{2}[y^2+(1-y)^2] = \frac{\alpha_s}{6\pi}\frac{A}{x}$$

and therefore:

$$c(x,Q) \sim \frac{\alpha_s}{6\pi}\log(\frac{Q^2}{m_c^2})\, g(x,Q)$$

Corrections to this simple formula will arise due to the Q dependence of g(x) and of α_s

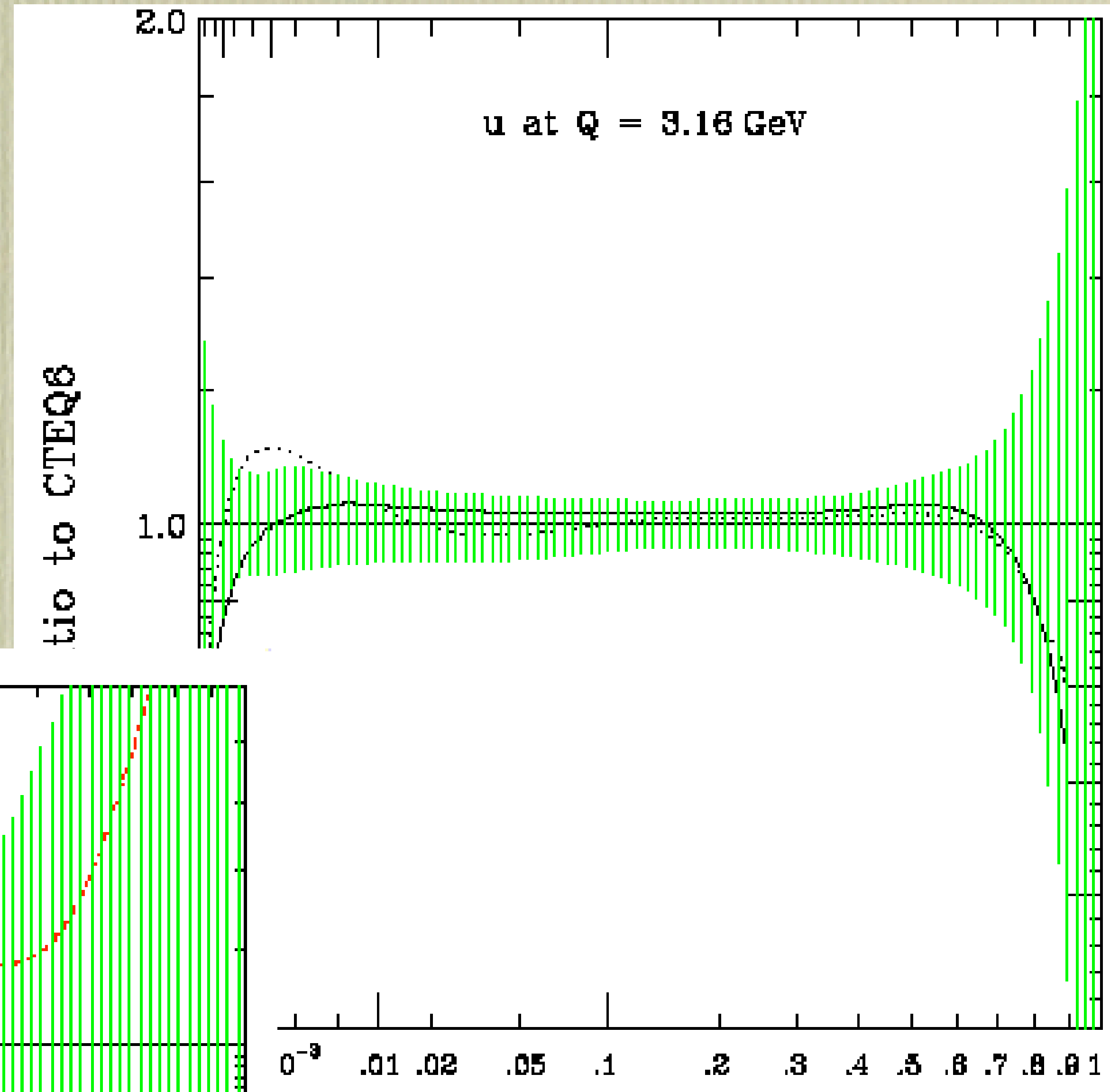
Examples of PDFs and their evolution

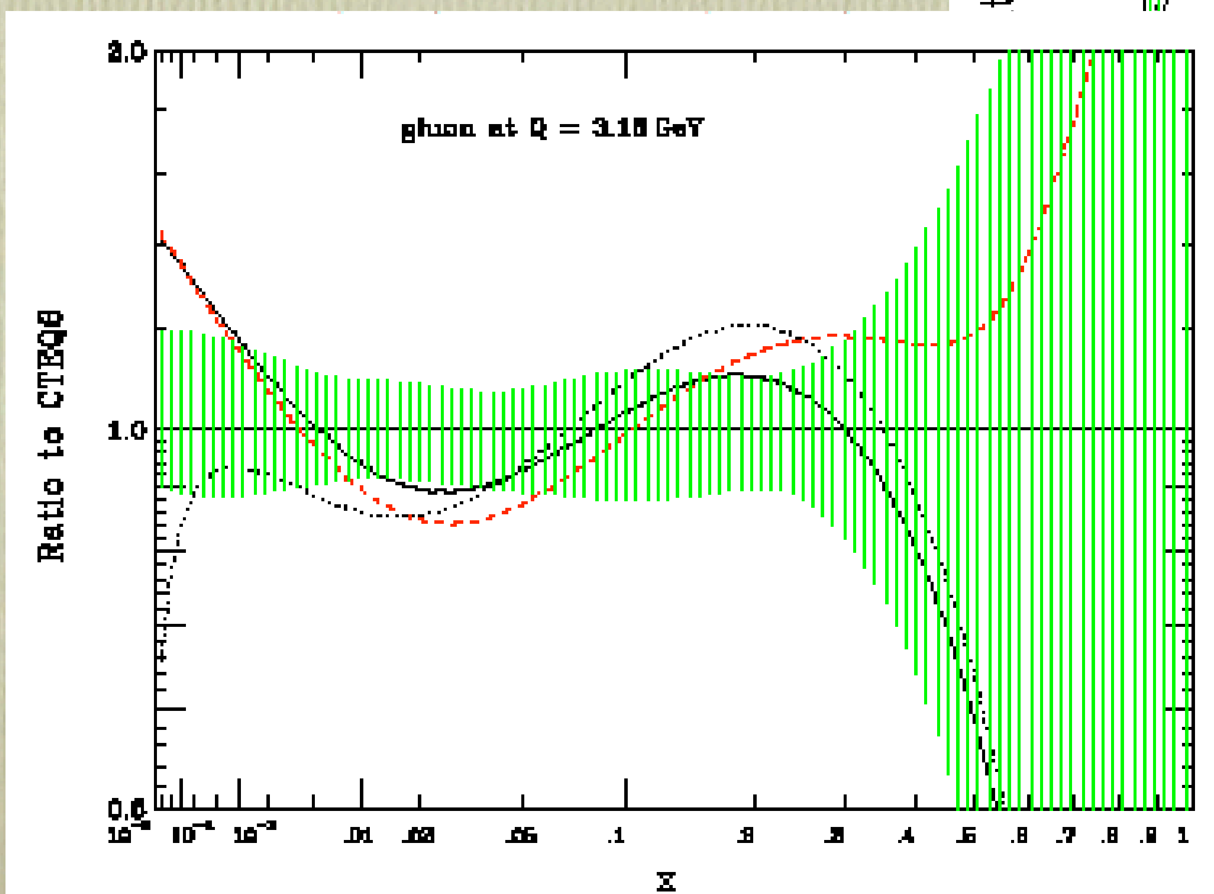


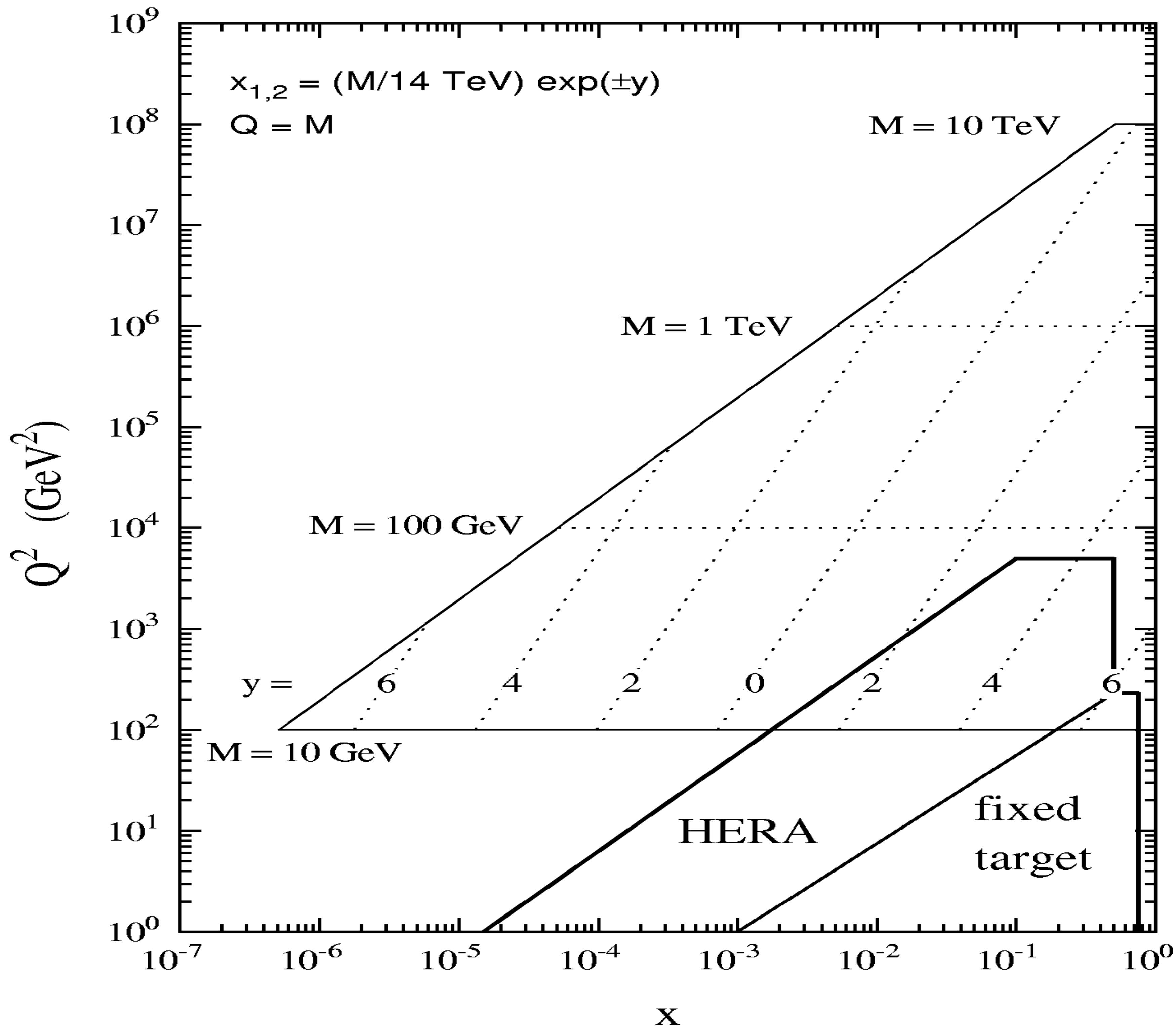
Note:
sea ≈10% glue

Note:
charm≈up at high Q

PDF uncertainties



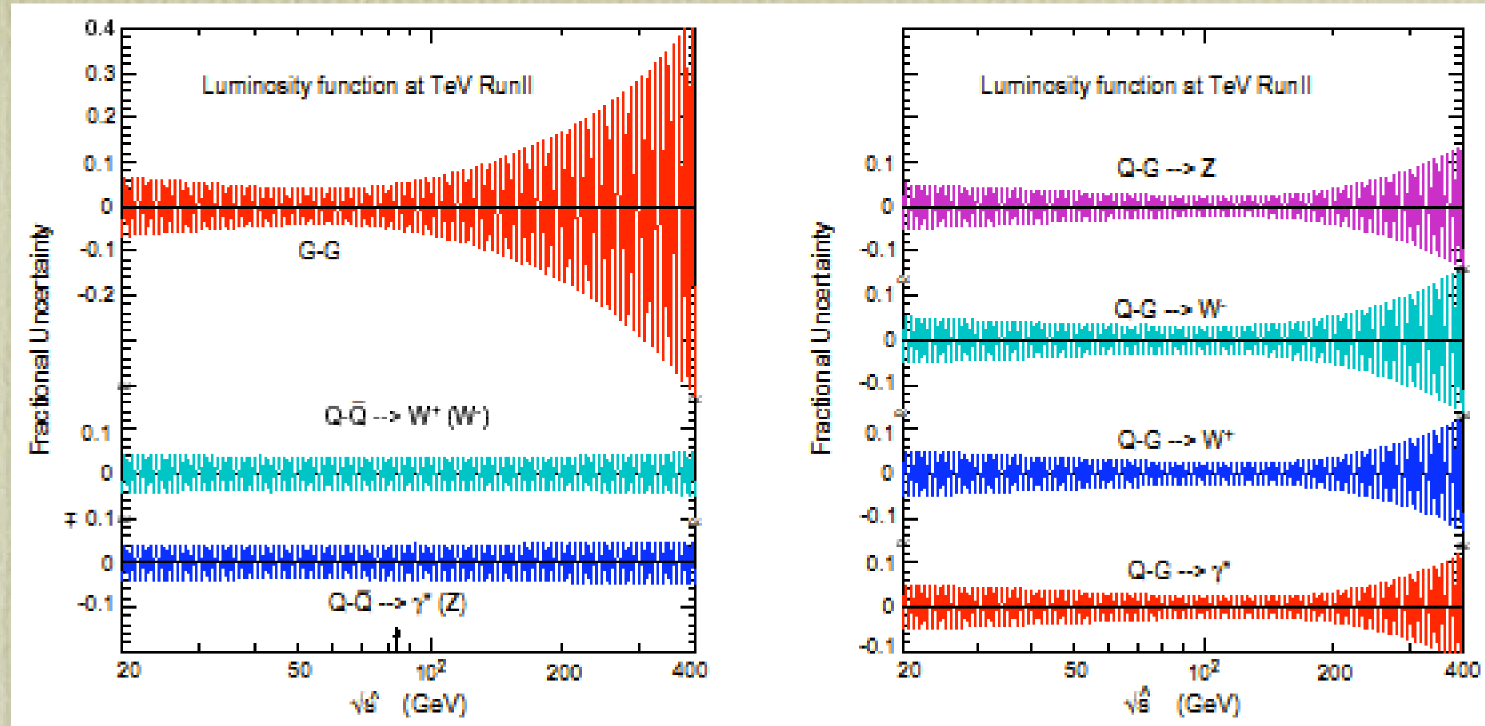


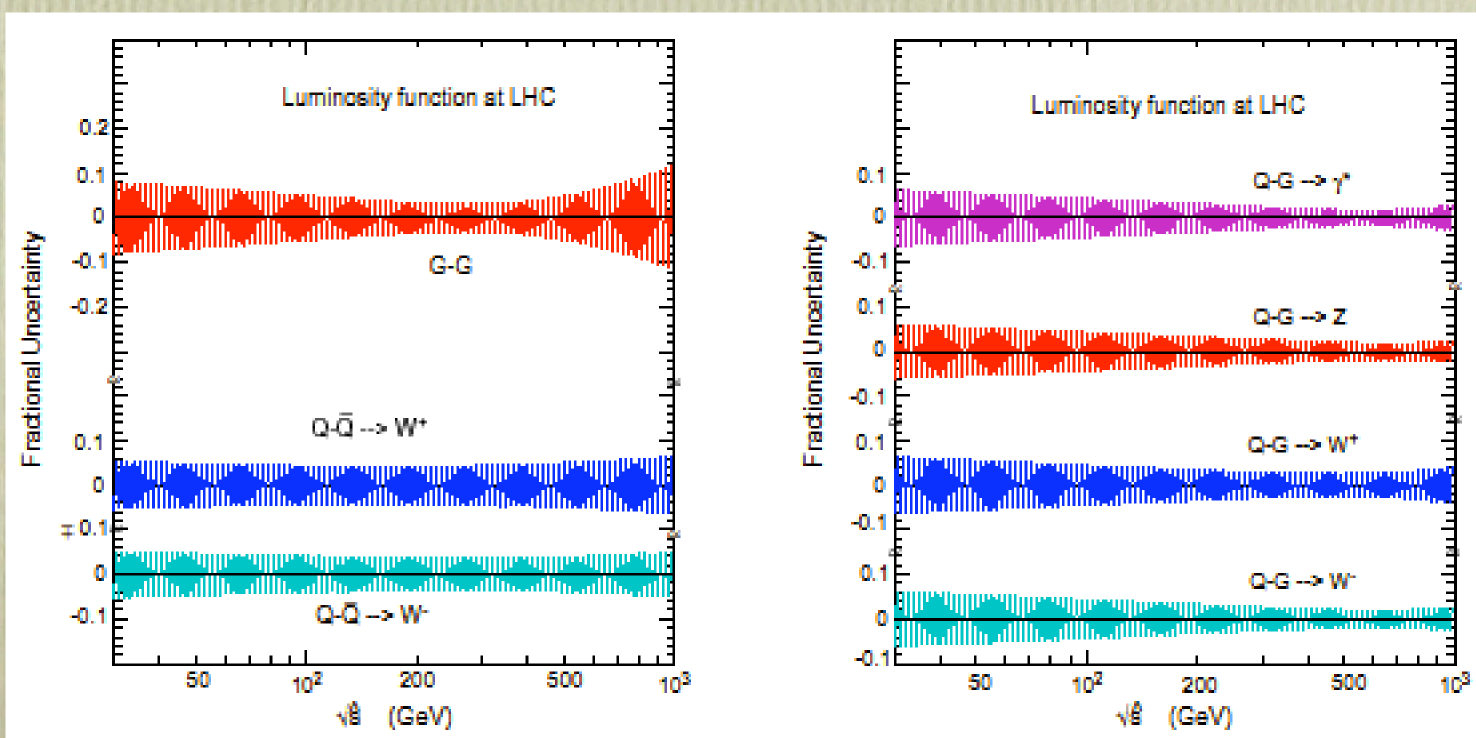


$x_{1,2} = (M/14\ TeV)\ exp(\pm y)$
$Q = M$
M = 10 TeV
M = 1 TeV
M = 100 GeV
M = 10 GeV
y = 6 4 2 0 2 4 6
HERA
fixed target
$Q^2\ (GeV^2)$
x
10^0 10^1 10^2 10^3 10^4 10^5 10^6 10^7 10^8 10^9
10^{-7} 10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^0

PDF luminosity uncertainties

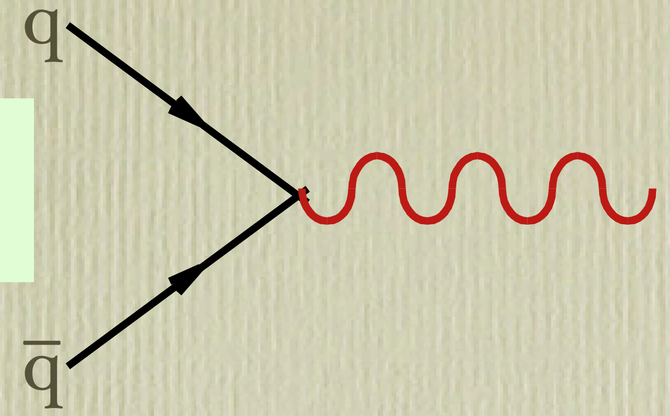
At the Tevatron





At the LHC

Drell-Yan processes:



$W \to \ell \Pi$

$Z \to \ell^+ \ell^-$

Goals:

- Tests of QCD: σ(W,Z) known up to NNLO (2-loops)
- Measure m(W) (➙ constrain m(H))
- constrain PDFs (e.g. $f_{up}(x)/f_{down}(x)$)
- search for new gauge bosons: $q\bar{q} \to W', Z'$
- Probe contact interactions: $q\bar{q}\ell^+\ell^-$

LO Cross-section calculation

$$\sigma(pp\to W) = \sum_{q,q'}\int dx_1 dx_2\, f_q(x_1,Q)\, f_{\bar{q}'}(x_2,Q)\,\frac{1}{2\hat{s}}\int d[PS]\,\overline{\sum_{spin,col}}|M(q\bar{q}'\to W)|^2$$

where:

$$\overline{\sum_{spin,col}}|M(q\bar{q}'\to W)|^2 = \frac{1}{3}\frac{1}{4}8g_W^2|V_{qq'}|^2\hat{s} = \frac{2}{3}\frac{G_F\, m_W^2}{\sqrt{2}}|V_{qq'}|^2\hat{s}$$

$$d[PS] = \frac{d^3 p_W}{(2\pi)^3 p_W^0}(2\pi)^4\delta^4(P_{in}-p_W)$$

$$= 2\pi d^4 p_W\,\delta(p_W^2-m_W^2)\,\delta^4(P_{in}-p_W) = 2\pi\delta(\hat{s}-m_W^2)$$

leading to (exercise!):

$$\sigma(pp\to W) = \sum_{ij}\frac{\pi A_{ij}}{m_W^2}\,\tau\int_\tau^1\frac{dx}{x}f_i(x,Q)\,f_j(\frac{\tau}{x},Q) \equiv \sum_{ij}\frac{\pi A_{ij}}{m_W^2}\,\tau L_{ij}(\tau)$$

where:

$$\frac{\pi A_{u\bar{d}}}{m_W^2} = 6.5\text{nb} \quad \text{and} \quad \tau = \frac{m_W^2}{S}$$

Some useful relations and definitions

Rapidity: $y = \frac{1}{2} \log \frac{E_W + p_W^z}{E_W - p_W^z}$

Pseudorapidity: $\eta = -\log(\tan \frac{\theta}{2})$

where:

$\tan\theta = \frac{p_T}{p^z}$ and $p_T = \sqrt{p_x^2 + p_y^2}$

Exercise: prove that for a massless particle rapidity=pseudorapidity:

Exercise: using $\tau = \frac{\hat{s}}{S} = x_1 x_2$ and

$$\begin{cases} E_W = (x_1 + x_2)\, E_{\text{beam}} \\ p_W^z = (x_1 - x_2)\, E_{\text{beam}} \end{cases} \Rightarrow y = \frac{1}{2} \log \frac{x_1}{x_2}$$

prove the following relations:

$$x_{1.2} = \sqrt{\tau} e^{\pm y} \qquad dx_1\, dx_2 = dy\, d\tau$$

$$dy = \frac{dx_1}{x_1} \qquad d\tau\delta(\hat{s} - m_W^2) = \frac{1}{S}$$

Study the function τL(τ)

Assume, for example, that $f(x) \sim \frac{1}{x^{1+\delta}}, \quad 0 < \delta < 1$

Then: $L(\tau) = \int_\tau^1 \frac{dx}{x} \frac{1}{x^{1+\delta}} (\frac{x}{\tau})^{1+\delta} = \frac{1}{\tau^{1+\delta}} \log(\frac{1}{\tau})$

and: $$\sigma_W = \sigma_W^0 \left(\frac{S}{m_W^2}\right)^\delta \log\left(\frac{S}{m_W}\right)$$

Therefore the W cross-section grows at least logarithmically with the hadronic CM energy. This is a typical behavior of cross-sections for production of fixed-mass objects in hadronic collisions, contrary to the case of e+e- collisions, where cross-sections tend to decrease with CM energy. Note also the following relation, which allows the measurement of the total width of the W boson from the determination of the leptonic rates of W and Z bosons,

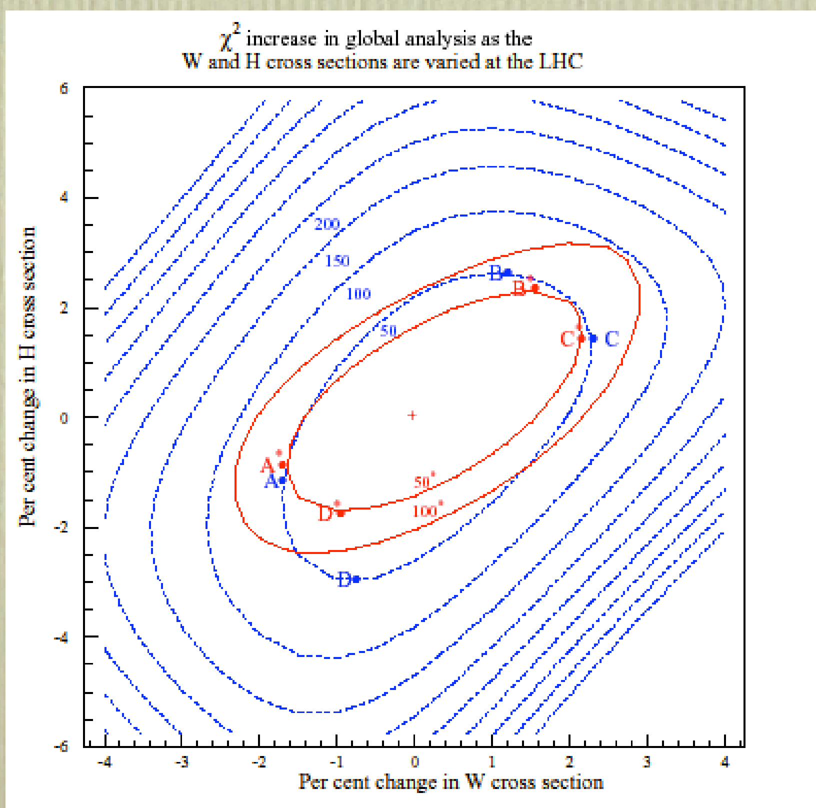
$$\Gamma_W = \frac{N(e^+e^-)}{N(e^\pm\nu)} \left(\frac{\sigma_{W^\pm}}{\sigma_Z}\right) \left(\frac{\Gamma^W_{e\nu}}{\Gamma^Z_{e^+e^-}}\right) \Gamma_Z$$

LHC data — theory — LEP/SLC

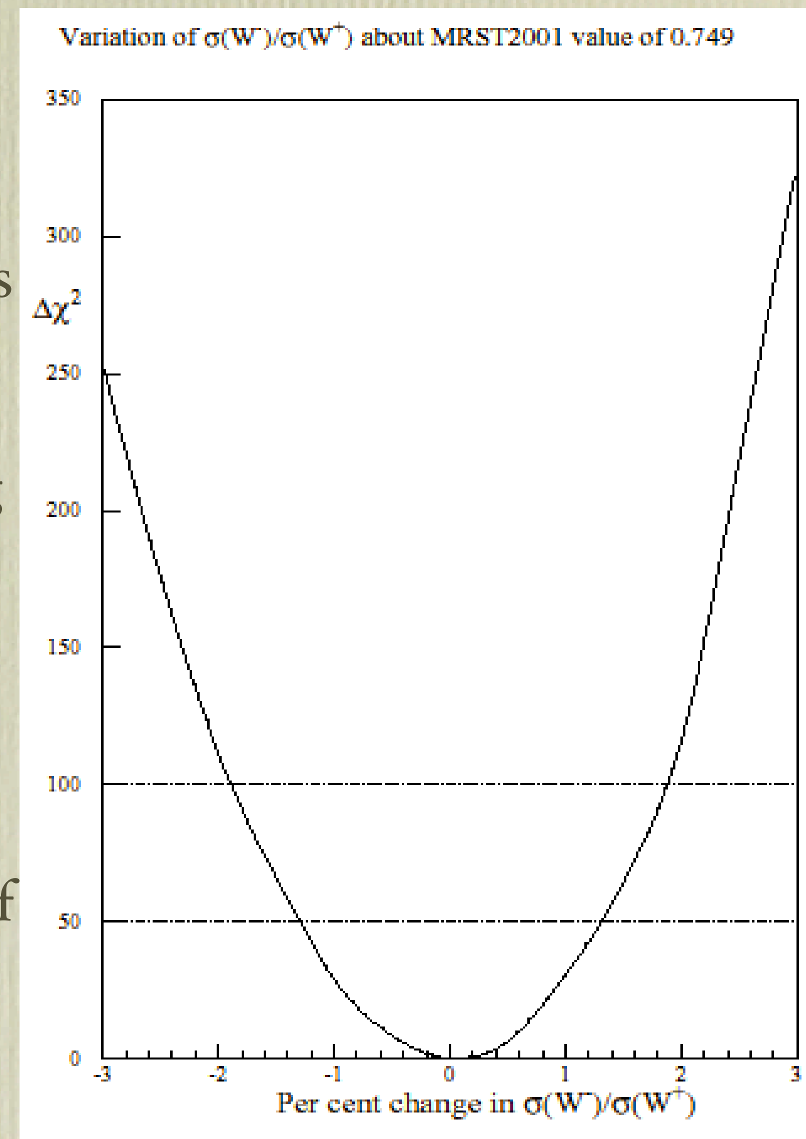
W rates

□(W)	MRST 2000	MRST 2001
NLO		
Fnal	2.39	2.41
LHC	20.5	20.6
NNLO		
Fnal	2.51	2.50
LHC	19.9	20.0

The theoretical uncertainty on the NNLO cross-section prediction is at the level of 2-3%, similar to the uncertainty due to the PDF variations



The prediction for the ratio of W- and W+ rates is much more accurate (below 1%). The leading source of uncertainty here is the ratio u(x)/d(x), which will be measured using the ratio of rapidity distributions for W- and W+



Exercise: W rapidity asymmetries

In proton-antiproton collisions, prove that the charge-rapidity asymmetry:

$$A(y) \; = \; \frac{d\sigma_{W^+}/dy - d\sigma_{W^-}/dy}{d\sigma_{W^+}/dy + d\sigma_{W^-}/dy}$$

is related to the ratio of the **up** and **down** quark PDF ratio $R(x)=f_d(x)/f_u(x)$ via the following relation:

$$A(y) \; = \; \frac{R(x_2) - R(x_1)}{R(x_2) + R(x_1)} \Rightarrow A(y) \neq 0 \leftrightarrow \frac{dR(x)}{dx} \neq 0$$

W rapidity spectra provide therefore useful information on the x-dependence of the up and down density ratio

Exercise: What observables involving W+ and W- are of interest at the LHC?

W mass syst's at LHC (60M W-> l nu /10fb^{-1})

Syst source	Atlas ΔM(W)/MeV
Stat	<2
E-p scale	15
Recoil model	5
Lept ID	5
ptW	5
PDF	10
W width (ΔΓ=30)	7
QED effects	<10
Bg	5
Energy scale	5
Total	<25

Possibility to limit the PDF, recoil, ptW and QED syst using data remains to be fully evaluated

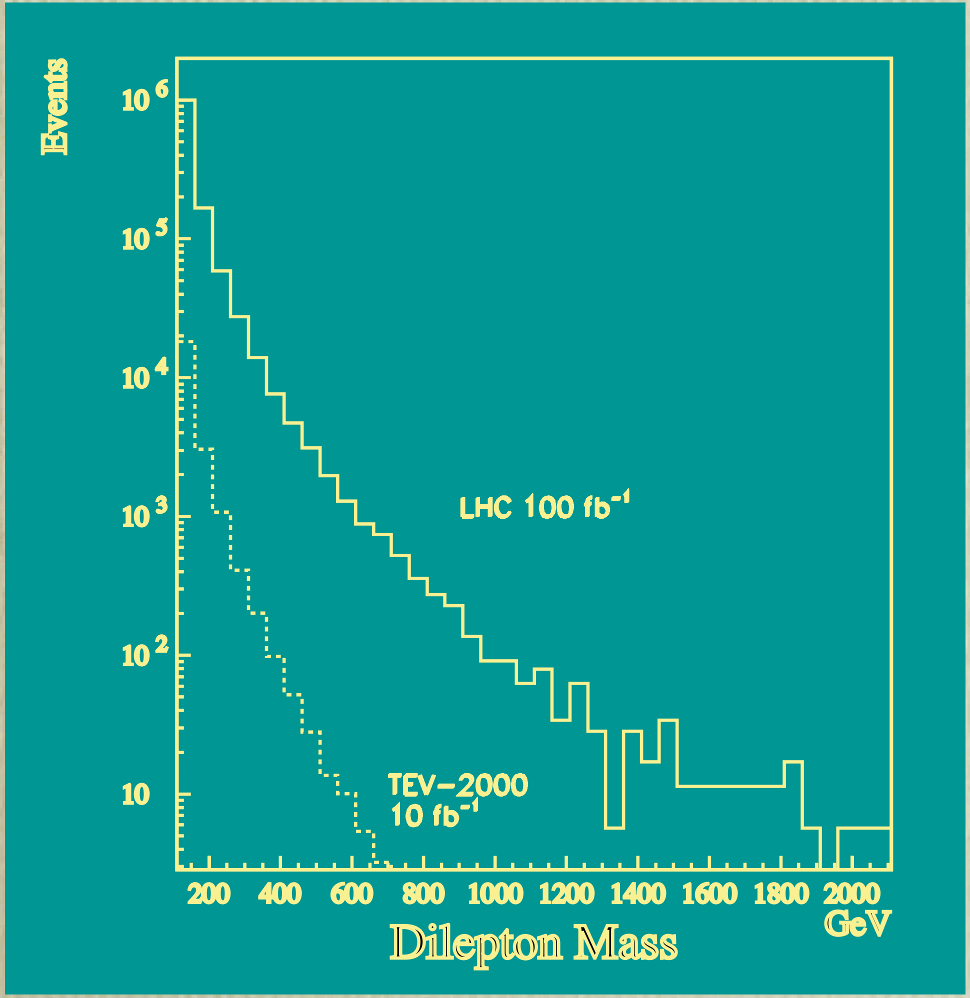
$\sin\theta_W$ from A_{FB}

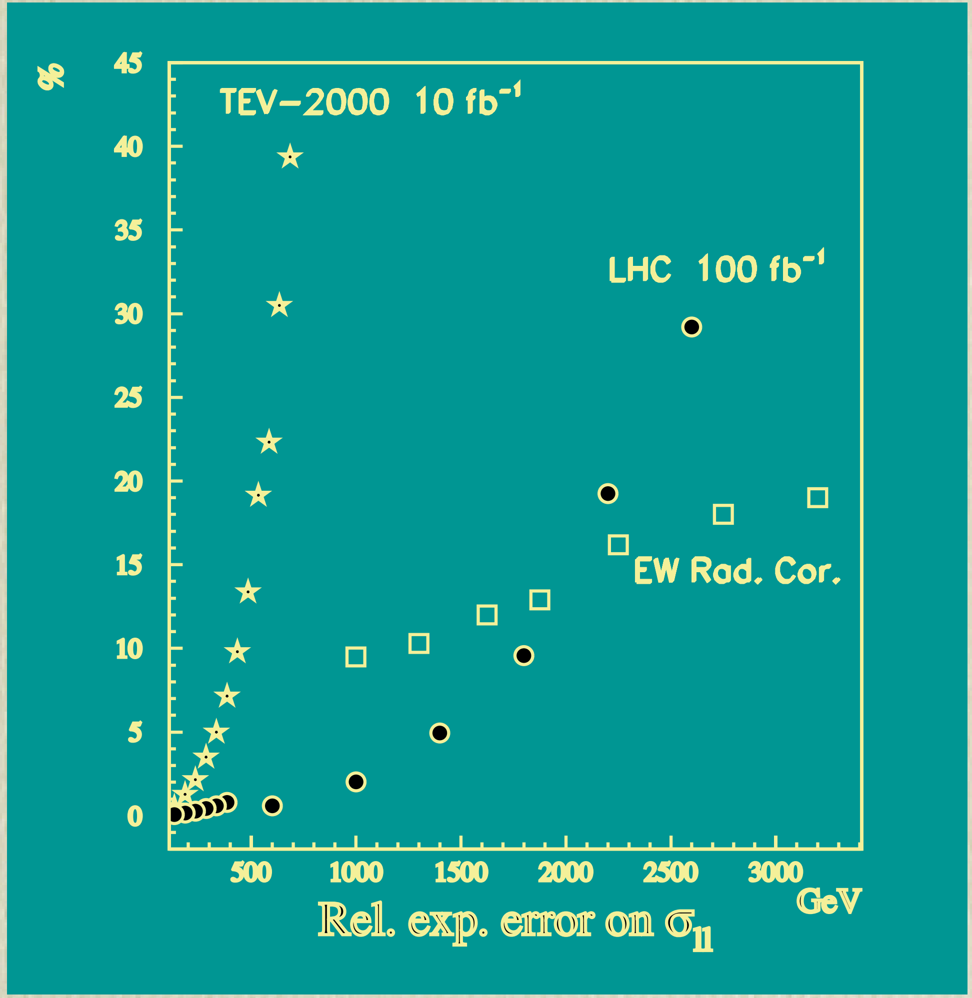
$\sin^{2,\text{eff}}_W = $ 0.2311(2) (lepts)
0.2322(3) (hads)

- y_Z-signed l^+l^- charge asymmetry
- $\Delta_{stat} \sin\theta_W = 2\ 10^{-4}$ for CMS+Atlas e+ with $\eta<2.5$
- $\Delta_{stat} \sin\theta_W = 1.4\ 10^{-4}$ for e, 1 exp only, using fwd e's if jet rejection better than 1/100
- Potential largest syst: PDF effect on acceptance.

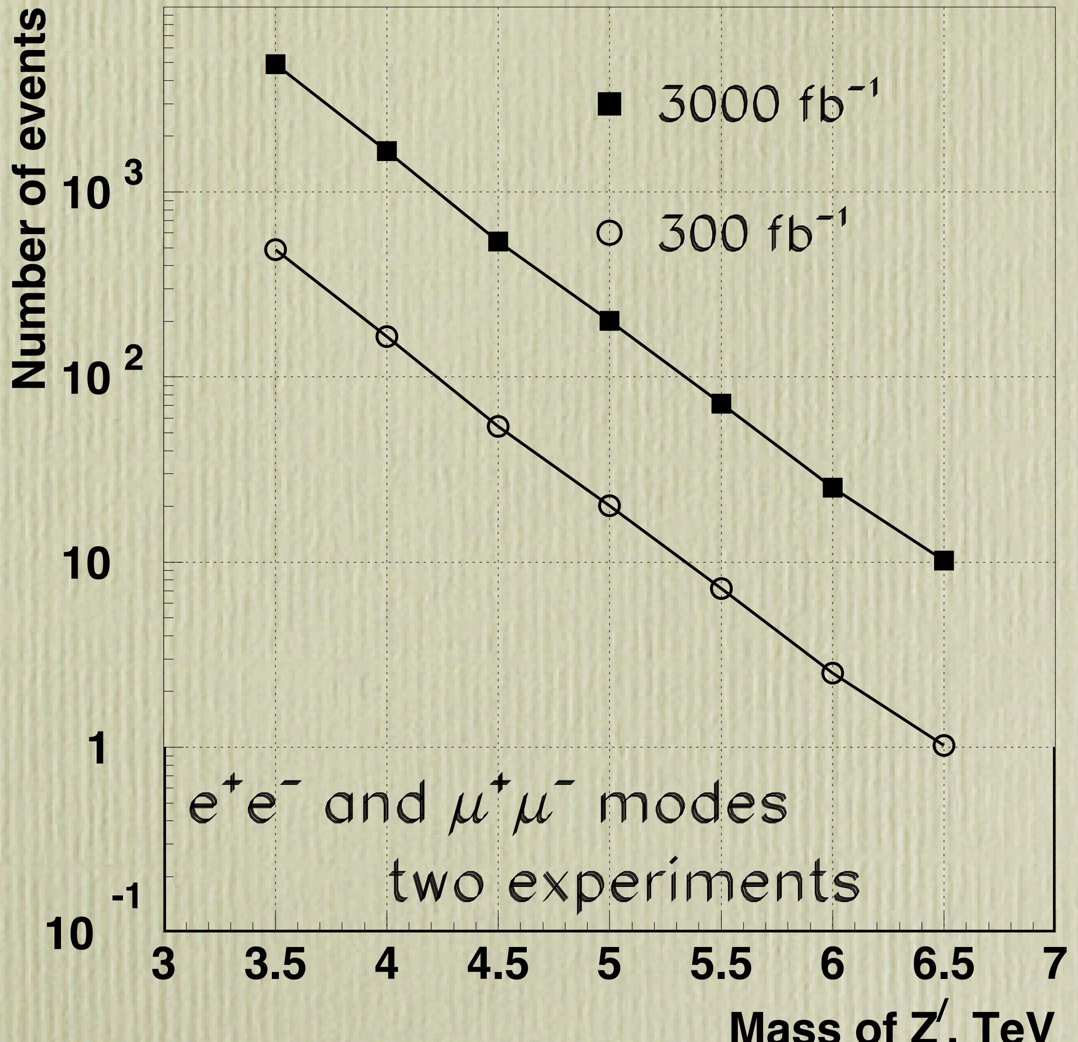
DY final states

m(ll)	Z peak	>110GeV	>400GeV
#(e,)/100fb-1	130M	2.6M	33K





Rates and discovery reach for SM-like new Z bosons



As seen from the plot in the previous page, the SM DY rate falls below 1 event/100 fb^{-1} once above m_{DY}>2TeV. In the high mass region the bg contamination (which includes also dilepton pairs from ttbar events) is totally negligible. A discovery based on observation of 10 events, leads to a reach of

5.3 TeV

for the standard high luminosity option, and of

6.5 TeV

for the super-LHC upgrade

$m_{Z'}$(TeV)	2	3	4	5	6
$\Gamma_{Z'}$(GeV)	62	94	126	158	190

TGC's: WWZ, WWγ, ZZγ

Final states: WZ, WW, Wγ, Zγ

Vertex	Coupling	LHC YR limit	LEP2 limits
WWγ	$\Delta\kappa_\gamma$	0.035	[-0.17, 0.05]
	λ_γ	0.002	[-0.07, 0.03]
WWZ	Δg_Z	0.008	[-0.05, 0.04]
	$\Delta\kappa_Z$	0.07	
	λ_Z	0.006	
ZZγ	h_Z	6.4e-4	

Multiple gauge boson production

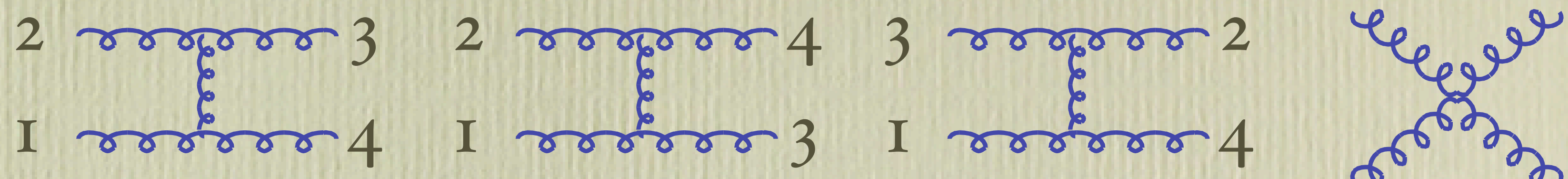
- Triple gauge boson production (#events/100fb^{-1}; rates can increase by 2 or 3 times if H->WW/ZZ is allowed):

WWW	53 (3l 3nu)	64 (2l 2j 2nu)
WWZ	15 (4l 2nu)	26 (3l 2j 1nu)
WZZ	2 (5l 1nu)	2 (4l 2j)
ZZZ	1 (4l 2 nu)	1 (4 2j)

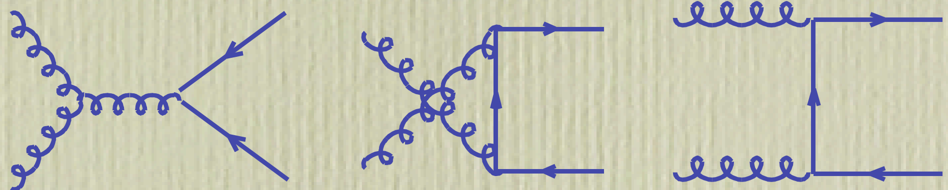
- VV-> VV Gauge boson scattering (only theory parton-level studies available)
- VV-> VVV and VV->VVγ scattering => 5GC's

Jet production

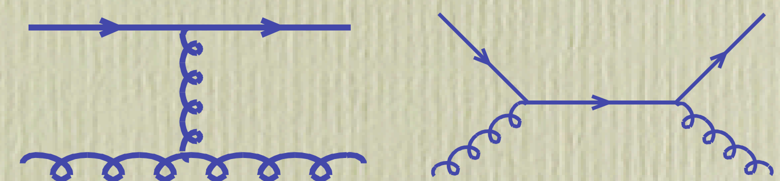
$gg \to gg$



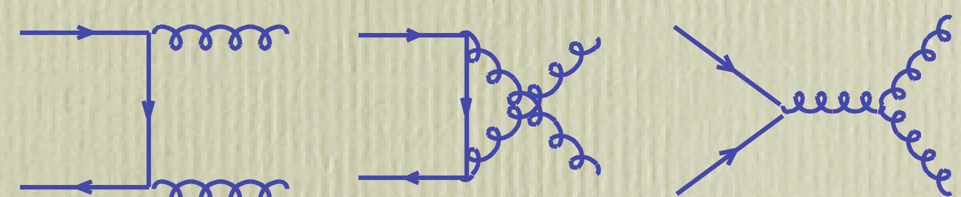
$gg \to q\bar{q}$



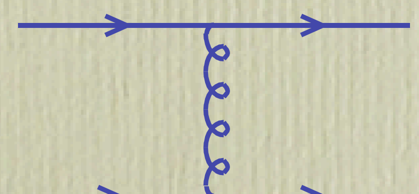
$qg \to qg$



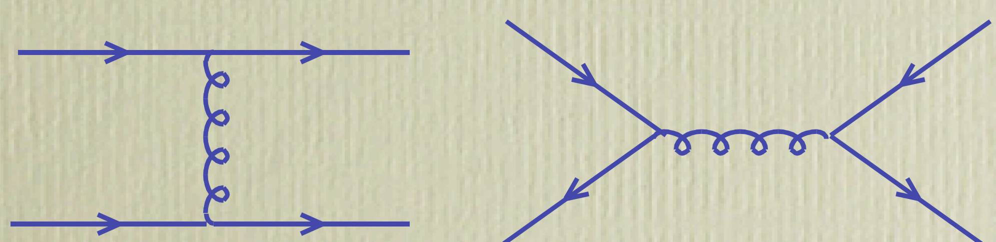
$q\bar{q} \to gg$



$qq' \to qq'$



$q\bar{q} \to q\bar{q}$



- Inclusive production of jets is the largest component of high-Q phenomena in hadronic collisions
- QCD predictions are known up to NLO accuracy
- Intrinsic theoretical uncertainty (at NLO) is approximately 10%
- Uncertainty due to knowledge of parton densities varies from 5-10% (at low transverse momentum, p_T to 100% (at very high p_T, corresponding to high-x gluons)
- Jet are used as probes of the quark structure (possible substructure implies departures from point-like behaviour of cross-section), or as probes of new particles (peaks in the invariant mass distribution of jet pairs)

Phase space and cross-section for LO jet production

$$d[PS] = \frac{d^3 p_1}{(2\pi)^2 2p_1^0}\frac{d^3 p_2}{(2\pi)^2 2p_2^0}(2\pi)^4 \delta^4(P_{in} - P_{out})\, dx_1\, dx_2$$

(a) $\delta(E_{in} - E_{out})\,\delta(P^z_{in} - P^z_{out})\, dx_1\, dx_2 = \frac{1}{2E^2_{beam}}$

(b) $\frac{dp^z}{p^0} = dy \equiv d\eta$

$$d[PS] = \frac{1}{4\pi S}\, p_T\, dp_T\, d\eta_1\, d\eta_2$$

$$\frac{d^3\sigma}{dp_T d\eta_1 d\eta_2} = \frac{p_T}{4\pi S}\sum_{i,j} f_i(x_1)\, f_j(x_2)\, \frac{1}{2\hat{s}}\overline{\sum_{kl}}\, |M(ij \to kl)|^2$$

The measurement of pT and rapidities for a dijet final state uniquely determines the parton momenta x_1 and x_2. Knowledge of the partonic cross-section allows therefore the determination of partonic densities f(x)

Some more kinematics

Prove as an exercise that

$$x_{1,2} = \frac{p_T}{E_{beam}} \cosh y^* \, e^{\pm y_b}$$

where

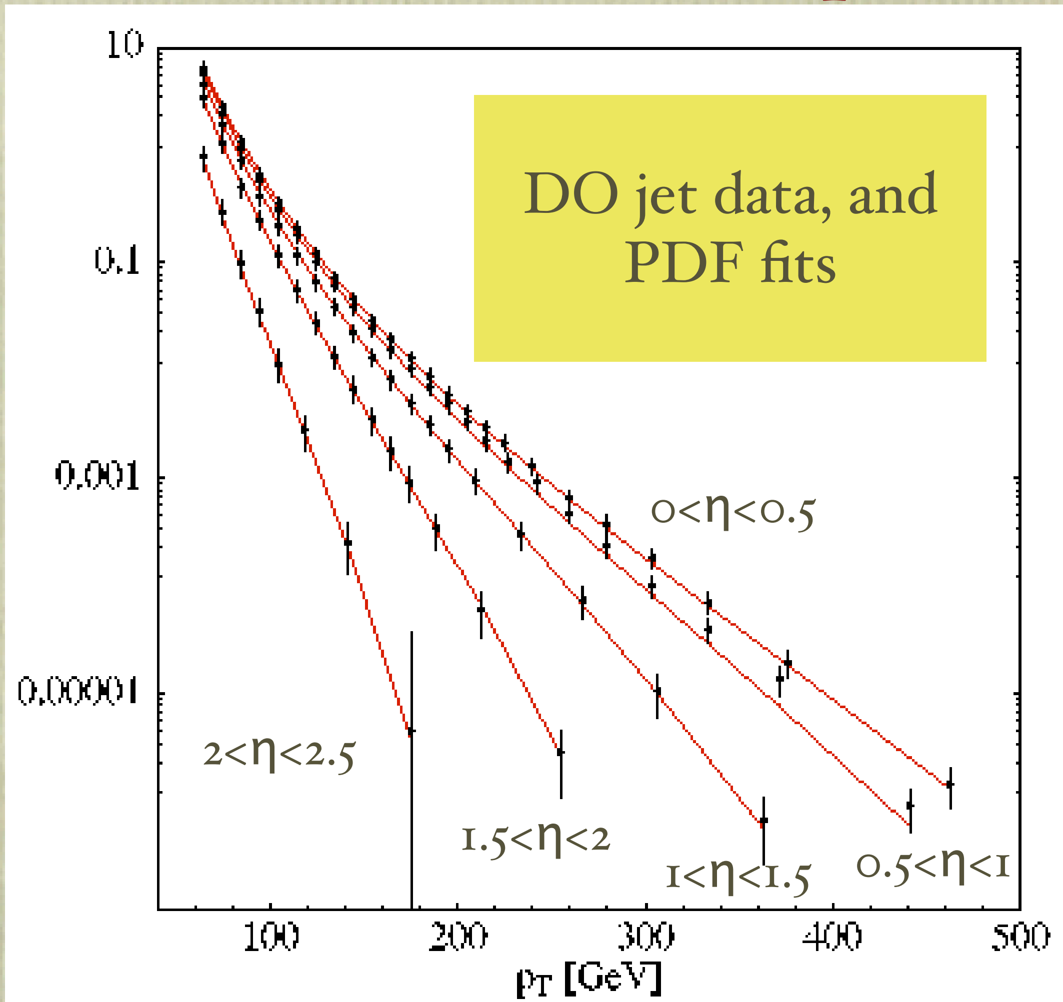
$$y^* = \frac{\sqcup_1 - \sqcup_2}{2}\,, \quad y_b = \frac{\sqcup_1 + \sqcup_2}{2}$$

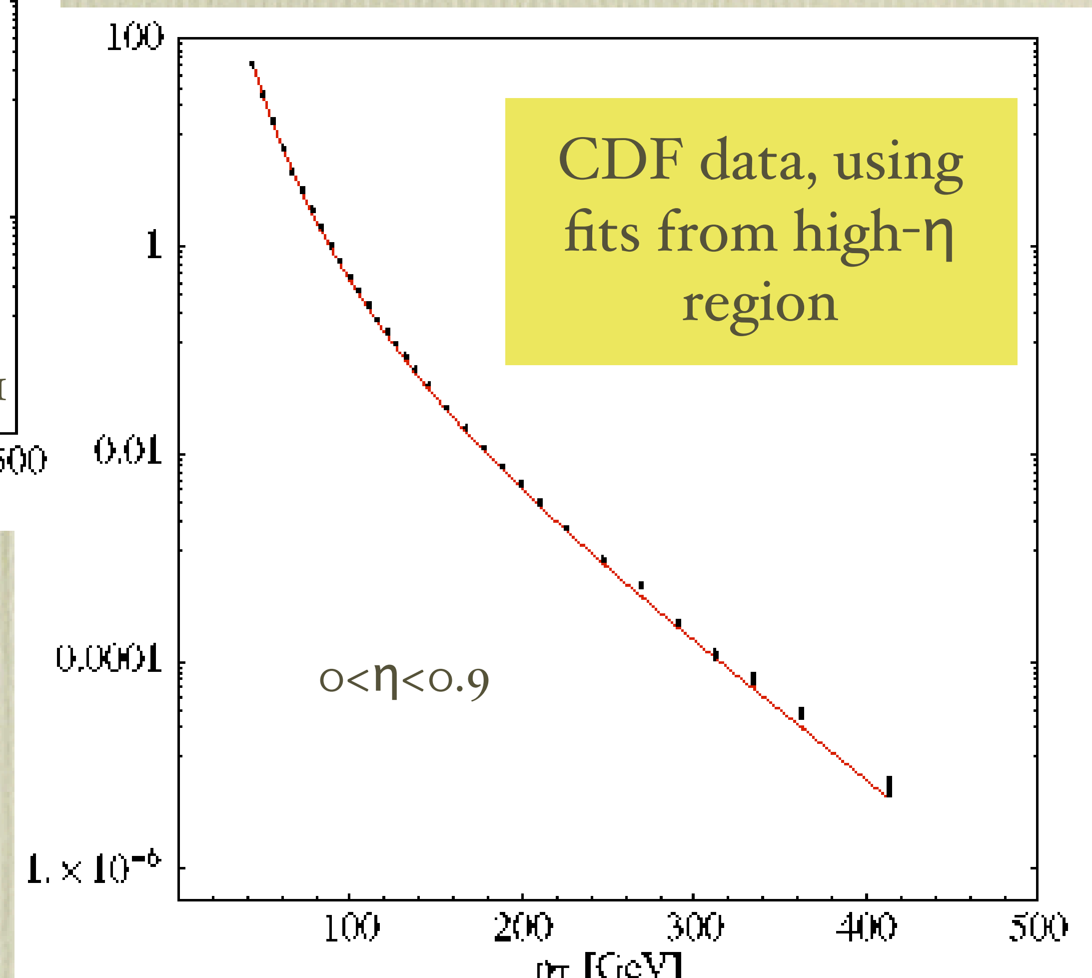
We can therefore reach large values of x either by selecting large invariant mass events:

$$\frac{p_T}{E_{beam}} \cosh y^* \equiv \sqrt{\Box} \rightarrow 1$$

or by selecting low-mass events, but with large boosts (y_b large) in either positive of negative directions. In this case, we probe large-x with events where possible new physics is absent, thus setting consistent constraints on the behaviour of the cross-section in the high-mass region, which could hide new phenomena.

Example, at the Tevatron

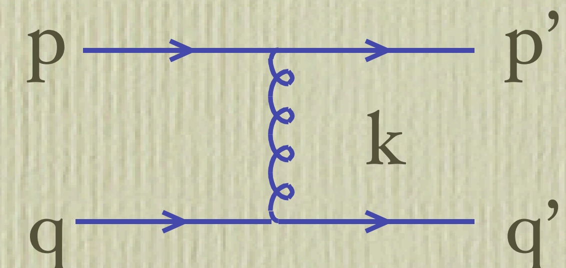




Small-angle jet production, a useful approximation for the determination of the matrix elements and of the cross-section

At small scattering angle, $t = (p_1 - p_3)^2 \sim (1 - \cos\theta) \to 0$

and the 1/t^2 propagators associated with t-channel gluon exchange dominate the matrix elements for all processes. In this limit it is easy to evaluate the matrix elements. For example:



$$\sim (\lambda^a)_{ij}(\lambda^a)_{kl}(2p_\mu)\frac{1}{t}(2q_\mu) = \frac{2s}{t}(\lambda^a)_{ij}(\lambda^a)_{kl}$$

where we used the fact that, for k=p-p'<<p (small angle scattering),

$$\bar{u}(p')\gamma_\mu u(p) \sim \bar{u}(p)\gamma_\mu u(p) = 2p_\mu$$

Using our colour algebra results, we then get: $\overline{\sum_{col,spin}} |M|^2 = \frac{1}{N_c^2}\frac{N_c^2-1}{4}\frac{4s^2}{t^2}$

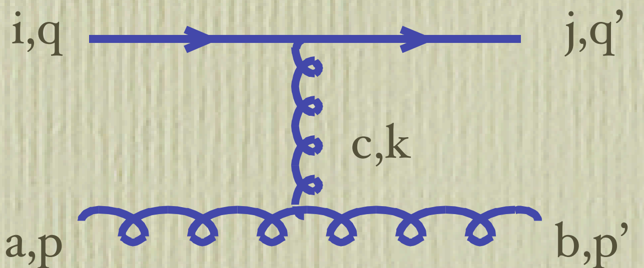
Noting that the result must be symmetric under s↔u exchange, and setting Nc=3, we finally obtain:

$$\overline{\sum_{col,spin}} |M|^2 = \frac{4}{9}\frac{s^2+u^2}{t^2}$$

which turns out to be the exact result!

Quark-gluon and gluon-gluon scattering

We repeat the exercise in the more complex case of qg scattering, assuming the dominance of the t-channel gluon-exchange diagram:



$$\sim f^{abc}\,\square^{c}_{ij}\,2p_\mu \frac{1}{t} 2q_\mu = 2\frac{s}{t} f^{abc}\,\square^{c}_{ij}$$

Using the colour algebra results, and enforcing the s↔u symmetry, we get:

$$\overline{\sum_{col,spin}} |M|^2 = \frac{s^2+u^2}{t^2}$$

which differs by only 20% from the exact result even in the large-angle region, at 90°

$$\overline{\sum_{col,spin}} |M|^2 = \frac{s^2+u^2}{t^2} - \frac{4}{9}\frac{s^2+u^2}{us}$$

In a similar way we obtain for gg scattering (using the t↔u symmetry):

$$\overline{\sum_{col,spin}} |M(gg \to gg)|^2 = \frac{9}{2}\left(\frac{s^2}{t^2} + \frac{s^2}{u^2}\right)$$

compared to the exact result

$$\overline{\sum_{col,spin}} |M(gg \to gg)|^2 = \frac{9}{2}\left(3 - \frac{ut}{s^2} - \frac{us}{t^2} - \frac{st}{u^2}\right)$$

with a 20% difference at 90°

Note that in the leading 1/t approximation we get the following result:

$$\hat{\sigma}_{gg} : \hat{\sigma}_{qg} : \hat{\sigma}_{qq} = \frac{9}{4} : 1 : \frac{4}{9}$$

and therefore

$$d\sigma_{jet} = \int dx_1\, dx_2 \sum_{ij} f_i(x_1)\, f_j(x_2)\, d\hat{\sigma}_{ij} = \int dx_1\, dx_2 \sum_{ij} F(x_1)\, F(x_2)\, d\hat{\sigma}_{gg}$$

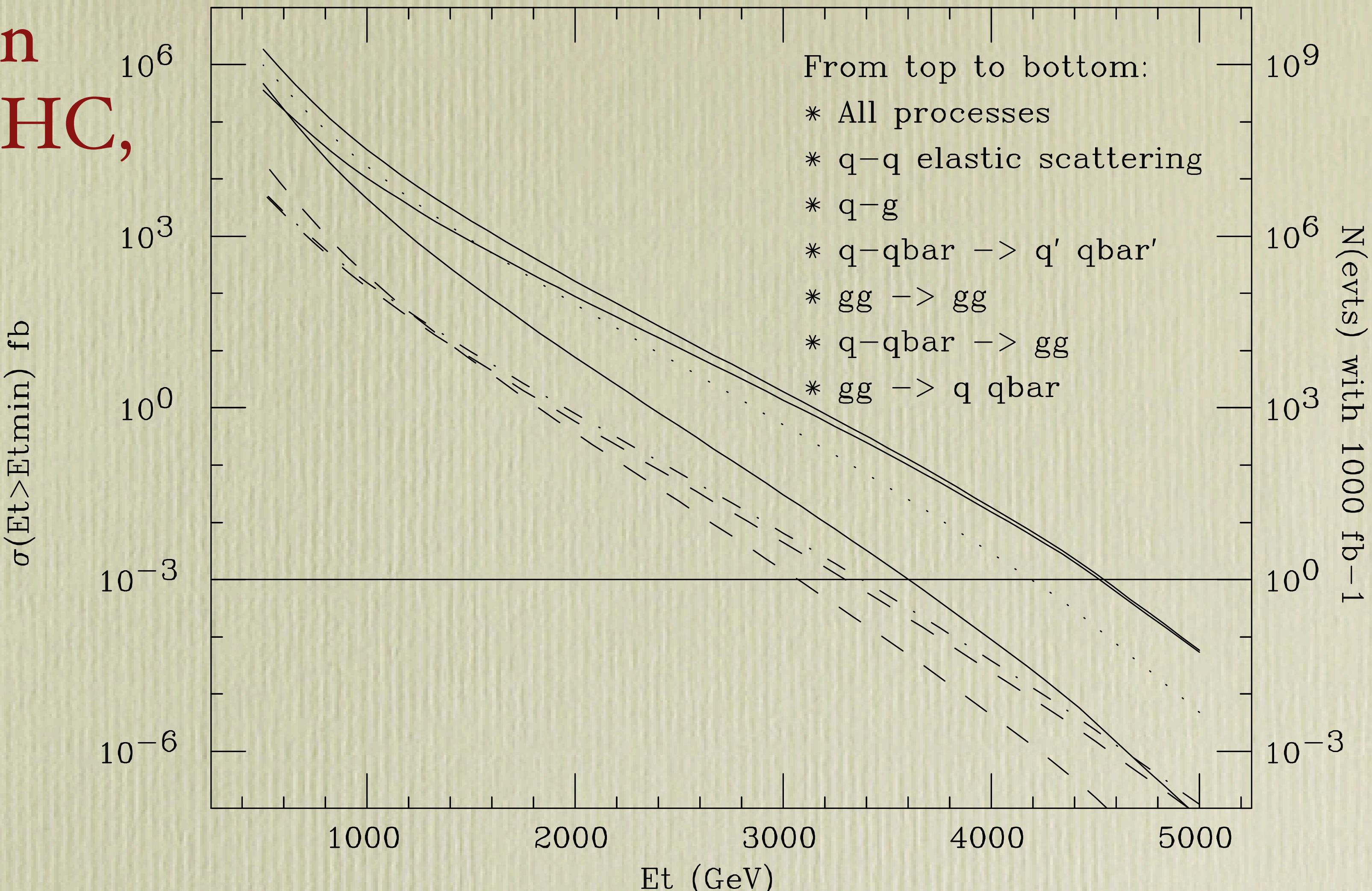
where we defined the `effective parton density' F(x):

$$F(x) = g(x) + \frac{4}{9} \sum_i [q_i(x) + \bar{q}_i(x)]$$

As a result jet data cannot be used to extract separately gluon and quark densities. On the other hand, assuming an accurate knowledge of the quark densities (say from HERA), jet data can help in the determination of the gluon density

Exercise: prove that the $1/t^2$ behaviour of the cross-section implies Rutherford's scattering law.

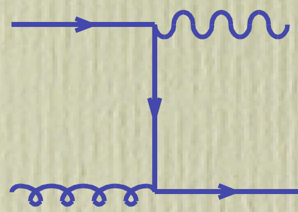
Jet production rates at the LHC, subprocess composition



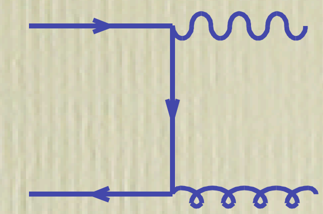
The presence of a quark substructure would manifest itself via contact interactions (as in Fermi's theory of weak interactions). On one side these new interactions would lead to an increase in cross-section, on the other they would affect the jets' angular distributions. In the dijet CMF, QCD implies Rutherford law, and extra point-like interactions can then be isolated using a fit. With the anticipated statistics of 300 fb-1, limits on the scale of the new interactions in excess of 40 TeV should be reached (to increase to 60 TeV with 3000 fb-1)

Photon plus jet production

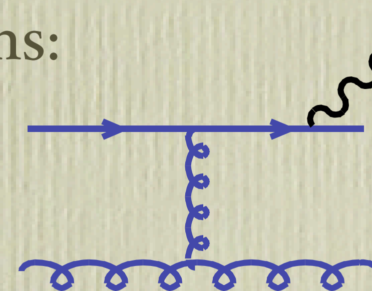
qg initial state:



$q\bar{q}$ initial state:



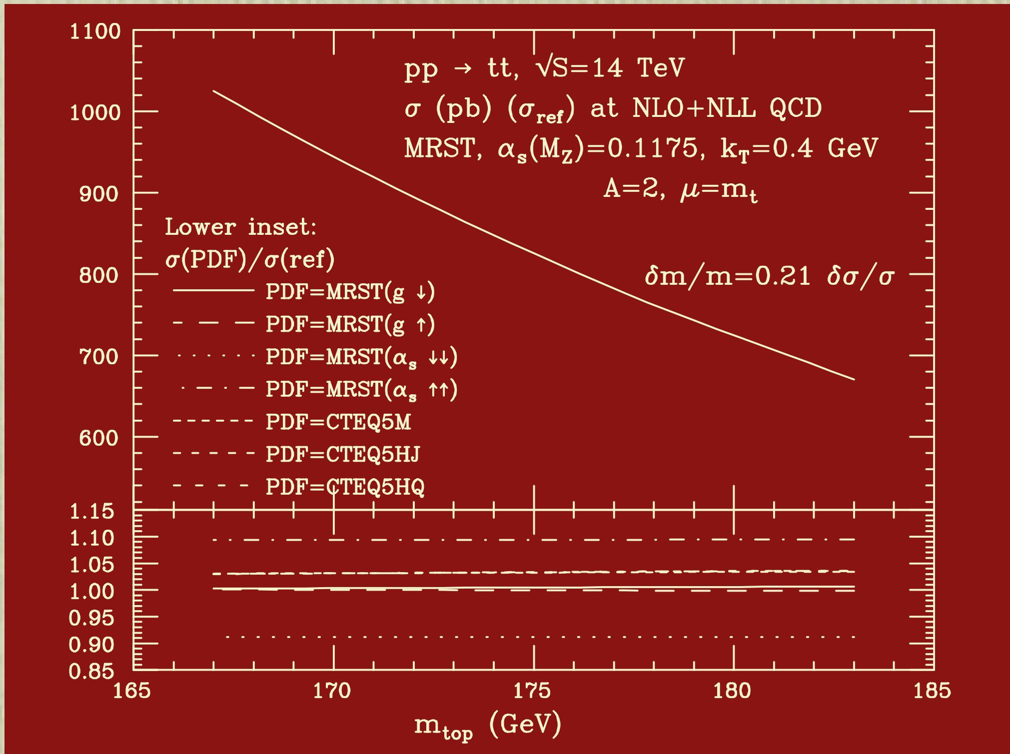
- $g(x) >> \bar{q}(x)$, therefore the first process dominates by at least a factor 10 throughout the phase-space. Potentially a good observable to constrain g(x)! Affected however by large higher-order, bremstrahlung-like corrections:

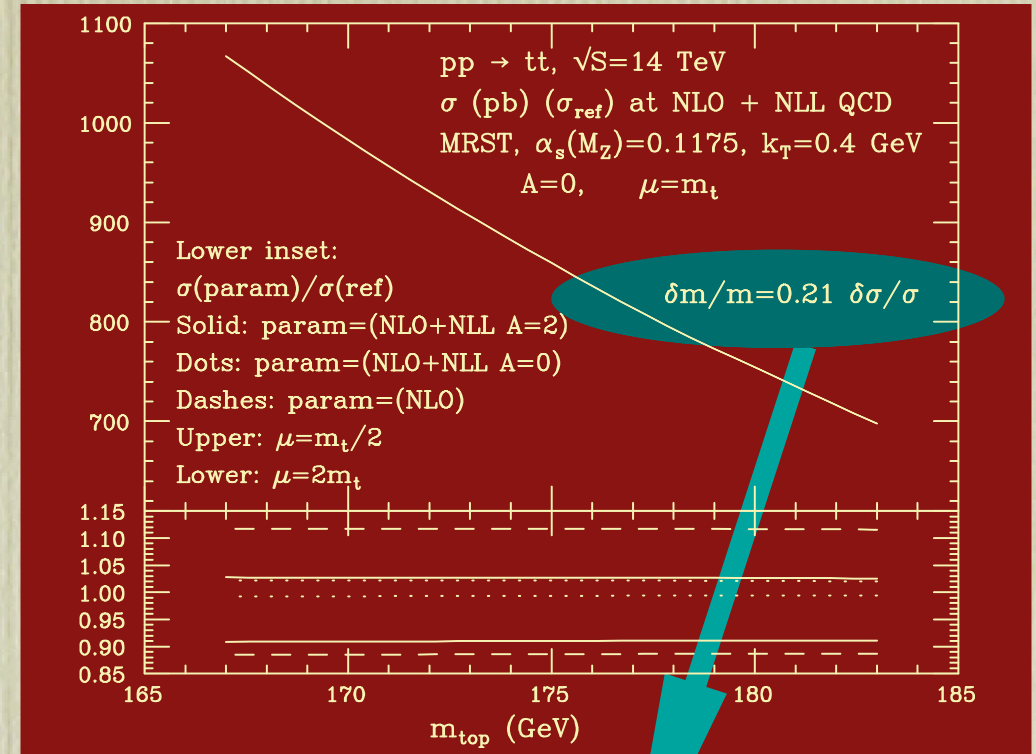


- □ e_q^2 , therefore up-type quarks are enhanced. In particular, the fraction of charm contribution is large, and photon-charm final states are therefore a good probe of the charm parton density.
- Provides a good calibration for the absolute experimental determination of the energy of a jet (jet test beam!)

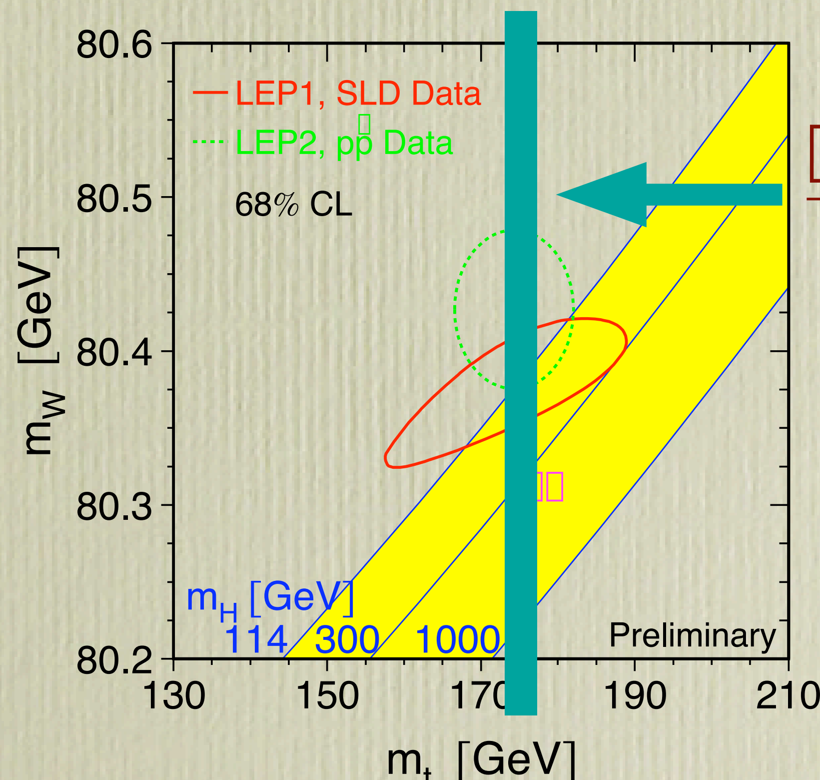
Top quark production

- Heaviest elementary particle known today
- m_{top} 175 GeV ⇒ top Yukawa coupling=1! The most natural value for a fermion mass: a special role in Nature for the top quark?
- LHC will be a “top Factory”: σ~800 pb ⇒10^7 events/yr, 1Hz!
- Large statistics ⇒ statistically accurate determinations of the top properties:
  - mass (crucial to better constrain/predict Higgs mass)
  - production cross-section (accurate QCD tests)
- New physics BSM
  - rare decays (indirect searches for new physics, e.g. FCNC)
  - signal, parent, partner and background for new particle production:
    - gluino → top stop, stop → top neutralino, H^+→t bbar
    - top→H^+b
    - pp→ ttH^0



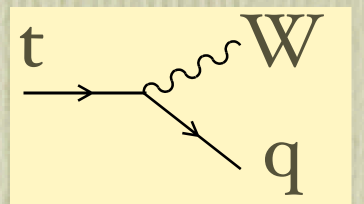


Theoretical systematics dominated today by PDF uncertainties! With the most recent analyses this is now at the level of *5%* (see luminosity plots in previous lecture)

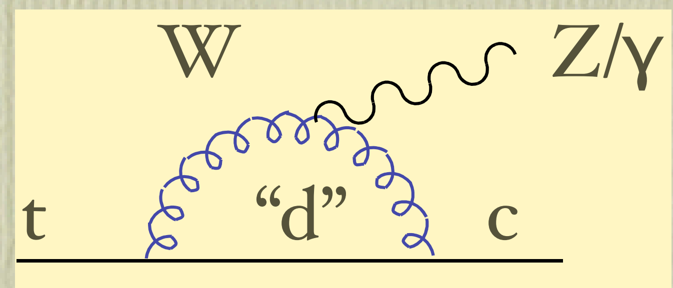


$$\frac{\delta\sigma}{\sigma} \sim 5\% \Leftrightarrow \delta m \sim 2 \text{ GeV}$$

Some rare top decays

BR() $|V_{tq}|^2 = (10^{-4},\ 1.6\,10^{-3},\ 1) \sim (1,\ \Box^4,\ \Box^6)$ for $q = d, s, b$

Probability of not identifying b quark large, BR(t→W+d or s) very hard to measure

BR()

GIM suppression/CKM unitarity

$$\left[\left(\frac{m_b}{m_t} \right)^2 V_{cb}\, \Box_W \right]^2 \sim 10^{-13}$$

Beyond any possible reach, unless new sources of FCNC. E.g., the SUSY partner of the above graph, with charginos and CKM-not-aligned down-type squarks.

t→WZb: m(b)+m(W)+m(Z)=176 GeV implies that the decay is just barely allowed by phase-space, once finite-width effects for the W and Z bosons are included. Very sensitive to m(top), could be an excellent probe of m(top). Unfortunately BR in the range of 10^{-6}, below experimental sensitivity (need to include BR(Z→ee) and BR(W→eν) as well)

Mode	SM BR	Allowed BSM	Wshop est reach
sW	1.6 E-3	0.25 (4th family)	missing
dW	~1 E-4	0.01 (4th family)	missing
bWZ	2 E-6	same	1 E-4
cWW	~1 E-13	1 E-6 (FCNC)	missing
cg	~5 E-11	1 E-3 (MSSM)	2 E-5 (cg->t)
cγ	~5 E-13	1 E-5 (MSSM)	3 E-5
cZ	~1 E-13	1 E-4	1 E-4
cH	< E-13	1 E-4	missing