Introduction to the physics of hard probes in hadron collisions: lecture I

Michelangelo Mangano
TH Division, CERN
michelangelo.mangano@cern.ch

Contents

- The structure of the proton (the “initial-state”):
  - parton densities
  - their evolution
  - nuclear modifications
- Some benchmark SM processes (the “hard probes”)
  - Drell-Yan
  - Jets
  - Heavy quark production
- The structure of a hard proton-proton collision (the “final-state”):
  - jet evolution
  - hadronization

References: Report from the Workshop on Hard Probes in Heavy Ion Collisions at the LHC

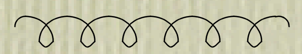
- PDF's, shadowing and pA collisions: hep-ph/0308248
- Heavy quarks: hep-ph/0311048
- Photon physics: hep-ph/0311131
- Jet physics: hep-ph/0310274

Previous lecture notes on QCD

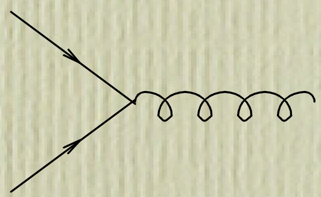
- Introduction to QCD: http://mlm.home.cern.ch/mlm/talks/brazil.ps.gz
- Introduction to heavy quark production: hep-ph/9711337

QCD Feynman rules

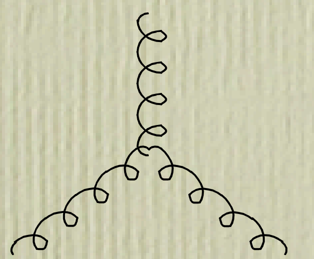
$$\frac{i\delta^{ij}}{\not{p} - m + i\epsilon}$$



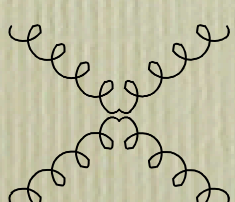
$$\delta^{ab}\frac{-i\,g^{\mu\nu}}{p^2 + i\epsilon}$$ (Feynman gauge)



$$i\,g\,t^a_{ij}\,\gamma^\mu$$



$$g\,f^{a_1a_2a_3}\;[g^{\mu_1\mu_2}\,(p_1-p_2)^{\mu_3}+g^{\mu_2\mu_3}\,(p_2-p_3)^{\mu_1}+g^{\mu_3\mu_1}\,(p_3-p_1)^{\mu_2}]$$



$$-i\,g^2\left[f^{a_1a_2X}\,f^{a_3a_4X}\;(g^{\mu_1\mu_3}g^{\mu_2\mu_4}-g^{\mu_1\mu_4}g^{\mu_2\mu_3})+(2\leftrightarrow 3)+(2\leftrightarrow 4)\right]$$

Some results for the SU(3) colour algebra

$\square^{a}_{ij}$: $SU(N_c)$ matrix in the fundamental representation,

$N_c = 3\,, \quad a = 1,\ldots,N_c^2-1\,, \quad i = 1,\ldots,N_c$

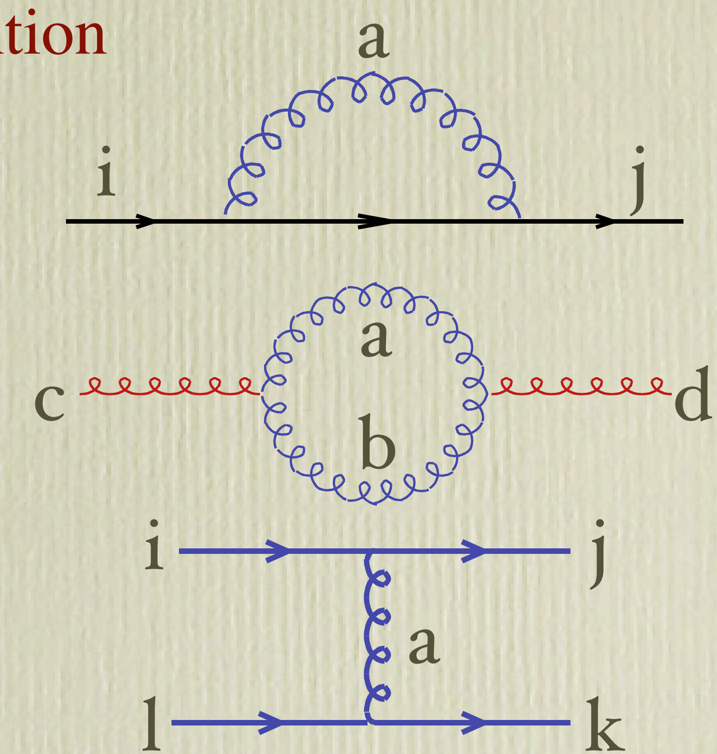
$[\square^a,\square^b] \;=\; i\,f^{abc}\,\square^c \quad (\Rightarrow \mathrm{tr}\,\square^a = 0)$

$\mathrm{tr}\,(\square^a\square^b) \overset{\text{def}}{=} T_F\,\square^{ab}, \quad T_F = 1/2$ by convention

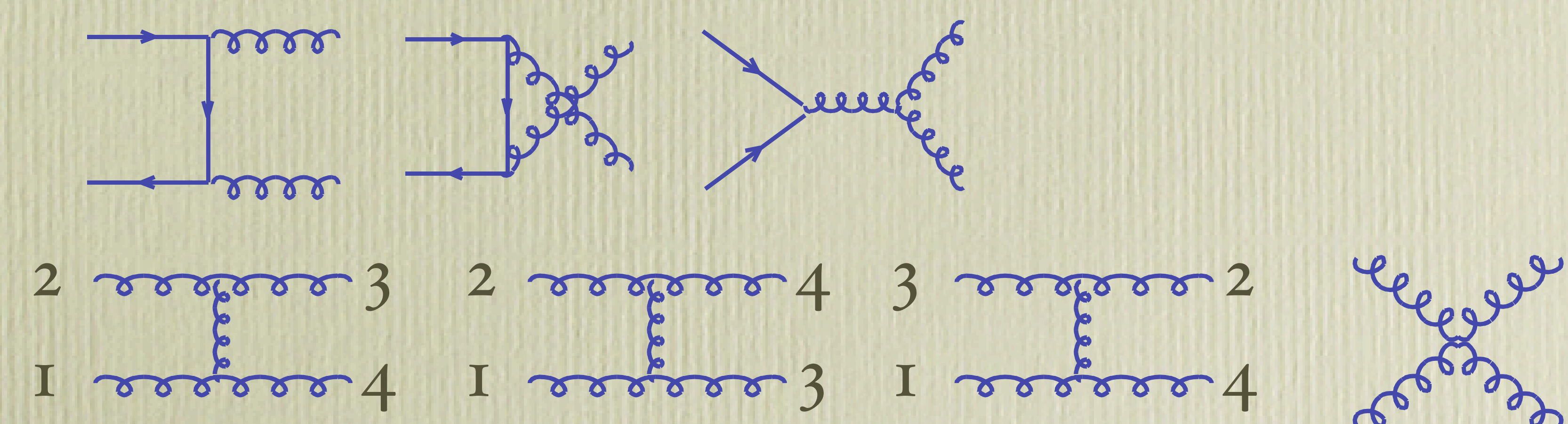
$$\underset{a}{\square}\,(\square^a\square^a)_{ij} \overset{\text{def}}{=} C_F\,\square^{ij} \overset{\text{Exercise}}{=} \frac{N_c^2-1}{N_c}\square^{ij}$$

$$\underset{a.b}{\square}\, f^{abc} f^{abd} \overset{\text{def}}{=} C_A\,\square^{cd} \overset{\text{Exercise}}{=} N_c\,\square^{cd}$$

$$\underset{a}{\square}\,\square^a_{ij}\square^a_{kl} \overset{\text{Exercise}}{=} \frac{1}{2}(\square_{ik}\square_{jl} - \frac{1}{N}\square_{ij}\square_{kl})$$



Exercises

- Verify the properties of the SU(N) algebra given in the previous pages
- Prove that the sums of the following sets of diagrams are gauge invariant, namely the amplitude remains invariant if we replace the polarization vector of any gluon, ϵ , with ϵ + p , p being the gluon momentum:
- 

Running of the coupling constant

$$\alpha_s \stackrel{def}{=} \frac{g_s^2}{4\pi} \qquad \frac{d\alpha_s}{d\log(Q^2)} = \beta(\alpha_s)$$

At 1-loop: $\beta = -b_0\,\alpha_s^2\,, \quad b_0 = \dfrac{33-2n_f}{12\pi}$

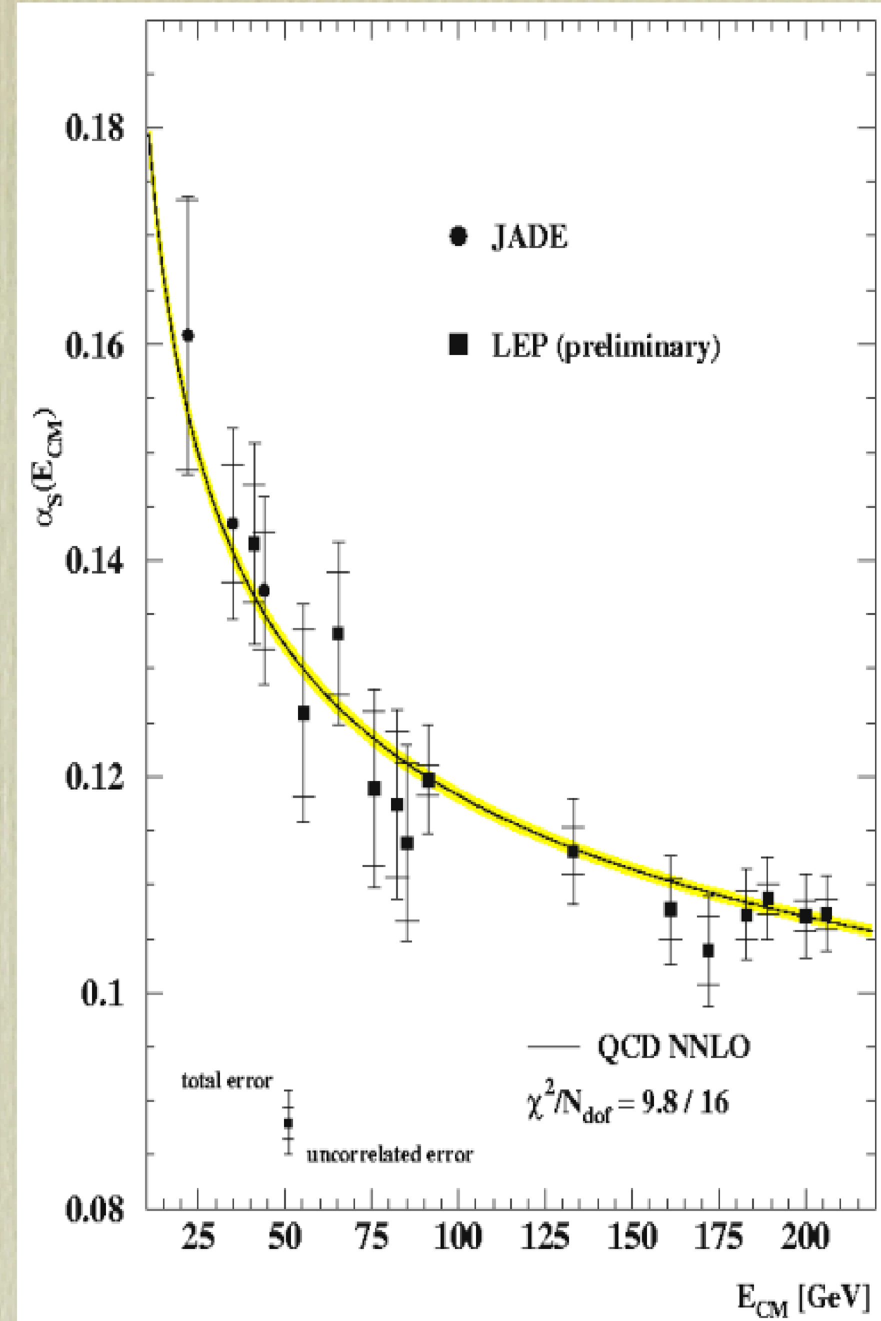
and $\alpha_s(Q) = \dfrac{1}{b_0 \log(Q^2/\Lambda^2)}$

At 2-loops:

$$\beta = -b_0\,\alpha_s^2 - b_1\,\alpha_s^3\,, \quad b_1 = \frac{153-19n_f}{24\pi^2}$$

and

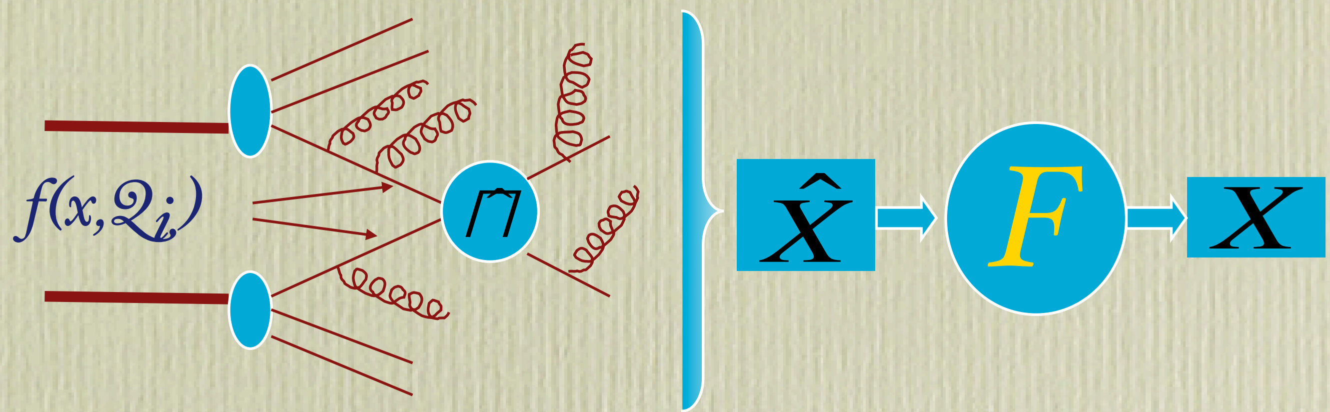
$$\alpha_s(Q) = \frac{1}{b_0\log(Q^2/\Lambda^2)}\left[1 - \frac{b_1}{b_0^2}\frac{\log\log(Q^2/\Lambda^2)}{\log Q^2/\Lambda^2}\right]$$



Current World Average (Bethke 2002): $\alpha_s(M_Z) = 0.118 \pm 0.003$

Factorization Theorem

$$\frac{d\sigma}{dX} = \sum_{j,k} \sum_{\hat{X}} f_j(x_1,Q_i)\, f_k(x_2,Q_i)\, \frac{d\hat{\sigma}_{jk}(Q_i,Q_f)}{d\hat{X}}\, F(\hat{X} \to X;Q_i,Q_f)$$



$$f_j(x,Q)$$

- sum over all initial state histories leading, at the scale Q, to:

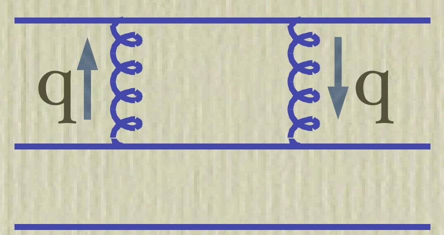
$$\vec{p}_j = x\,\vec{P}_{proton}$$

$$F(\hat{X} \to X;Q_i,Q_f)$$

- transition from partonic final state to the hadronic observable (hadronization, fragm. function, jet definition, etc)
- Sum over all histories with X in them

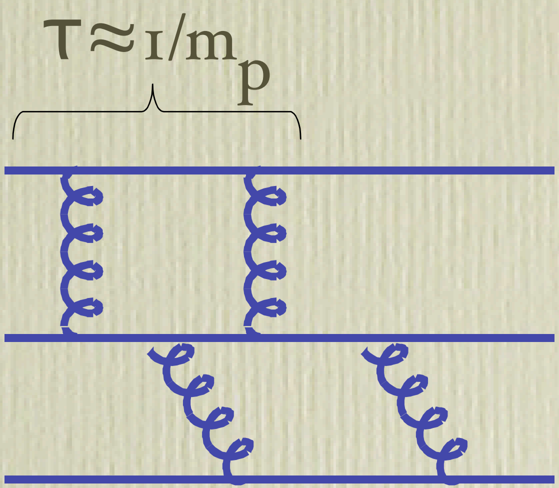
Universality of parton densities and factorization, a naive proof

Exchange of hard gluons among quarks inside the proton is suppressed by powers of $(m_p/Q)^2$



$$\overset{q>Q}{\sim} \int_q^Q \frac{d^4q}{q^6} \sim \frac{1}{Q^2}$$

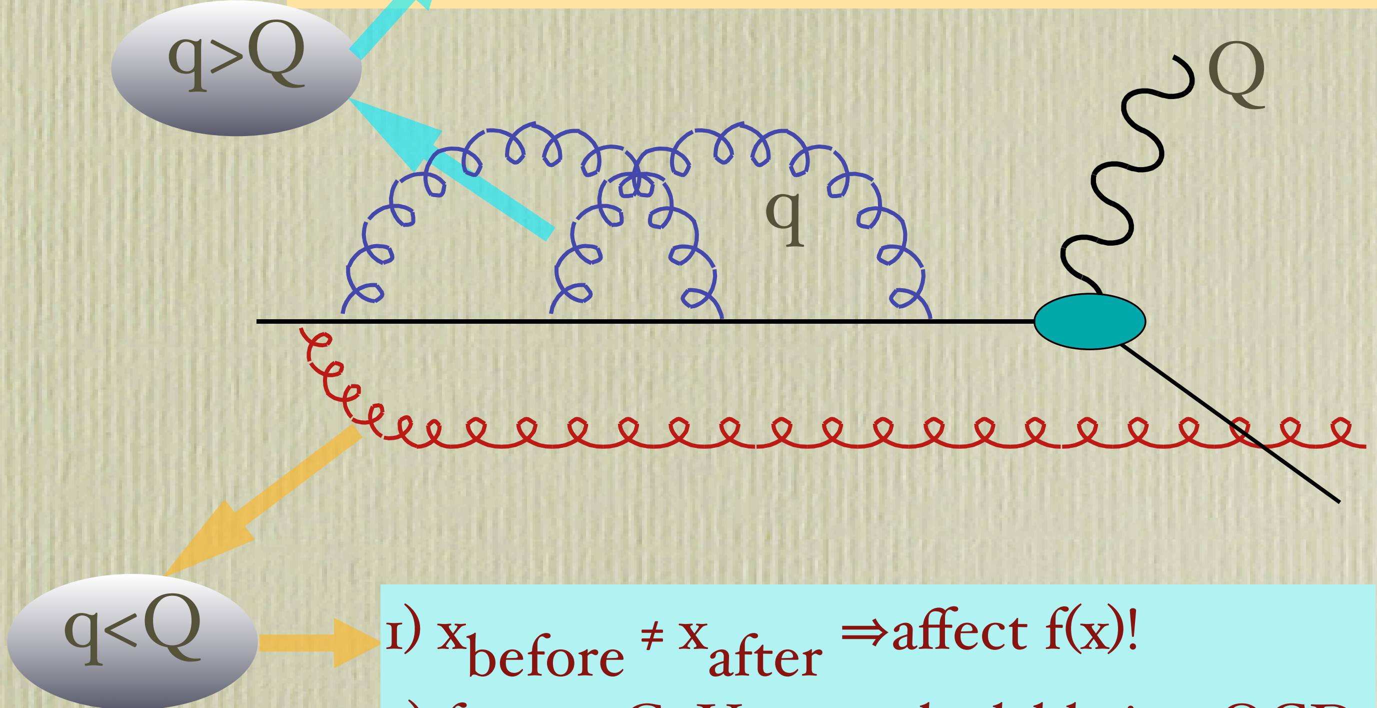
Typical time-scale of interactions binding the proton is therefore of $O(1/m_p)$ (in a frame in which the proton has energy E, $\tau=\gamma/m_p = E/m_p^2$)



If a hard probe ($Q>>m_p$) hits the proton, on a time scale $=1/Q$, there is no time for quarks to negotiate a coherent response

As a result, to study inclusive processes at large Q it is sufficient to consider the interactions between the external probe and a single parton:

1) calculable in perturbative QCD (pQCD)
2) do not affect f(x): $x_{before} = x_{after}$



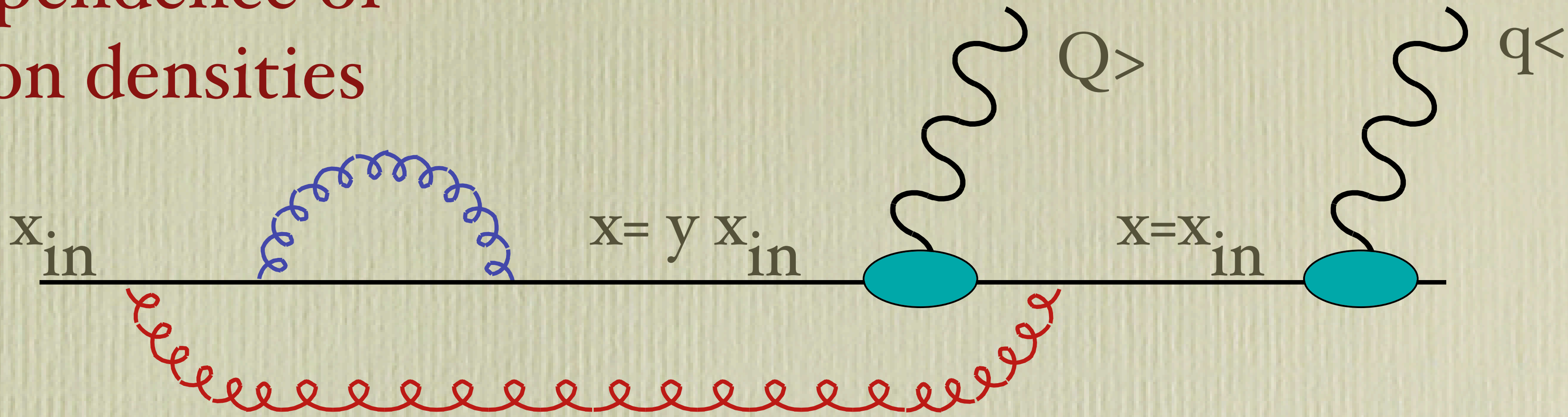
This gluon cannot be reabsorbed because the quark is gone

1) $x_{before} \neq x_{after}$ ⇒affect f(x)!
2) for q≈1 GeV **not** calculable in pQCD

However, since τ(q≈1GeV)>>1/Q, the emission of low-virtuality gluons will take place long before the hard collision, and therefore cannot depend on the detailed nature of the hard probe. While it is not calculable in pQCD, f(q<<Q) can be measured using a reference probe, and used elsewhere ⇒

Universality of f(x)

Q dependence of parton densities



The larger is Q, the more gluons will **not** have time to be reabsorbed

PDF's depend on Q!

$$f(x,Q) \;=\; f(x,\mu) + \int_x^1 dx_{in}\, f(x_{in},\mu) \int_\mu^Q dq^2 \int_0^1 dy\, P(y,q^2)\, \delta(x - y x_{in})$$

$$f(x,Q) = f(x,\mu) + \int_x^1 dx_{in}\, f(x_{in},\mu) \int_\mu^Q dq^2 \int_0^1 dy\, P(y,q^2)\, \delta(x - y x_{in})$$

f(x,Q) should be independent of the intermediate scale considered:

$$\frac{df(x,Q)}{d\mu^2} = 0 \quad \Rightarrow \frac{df(x,\mu)}{d\mu^2} = \int_x^1 \frac{dy}{y} f(y,\mu)\, P(x/y,\mu^2)$$

One can prove that:

calculable in pQCD

$$P(x,Q^2) = \frac{\alpha_s}{2\pi} \frac{1}{Q^2} P(x)$$

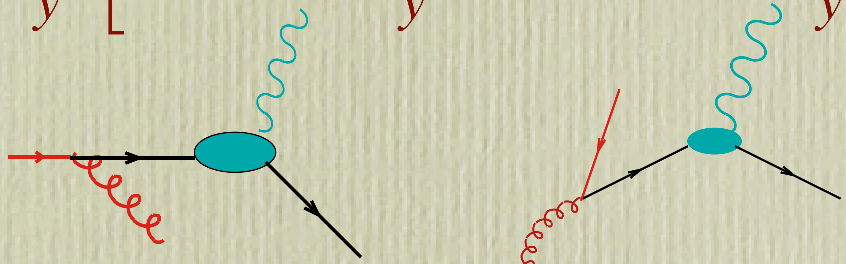
and therefore (Altarelli-Parisi equation):

$$\frac{df(x,\mu)}{d\log\mu^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} f(y,\mu)\, P(x/y)$$

More in general, one should consider additional processes which lead to the evolution of partons at high Q ($t=\log Q^2$):

$$[g(x)]_+ : \quad \int_0^1 dx\, f(x)\, g(x)_+ \equiv \int_0^1 [f(x) - f(1)]\, g(x)\, dx$$

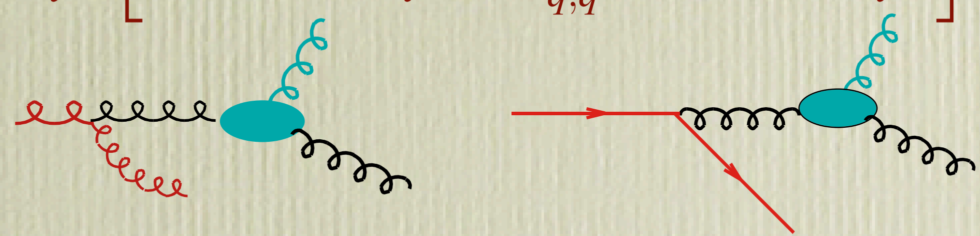
$$\frac{dq(x,Q)}{dt} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left[q(y,Q)\, P_{qq}\left(\frac{x}{y}\right) + g(y,Q)\, P_{qg}\left(\frac{x}{y}\right) \right]$$



$$P_{qq}(x) = C_F \left(\frac{1+x^2}{1-x} \right)_+$$

$$P_{qg}(x) = \frac{1}{2} \left[x^2 + (1-x)^2 \right]$$

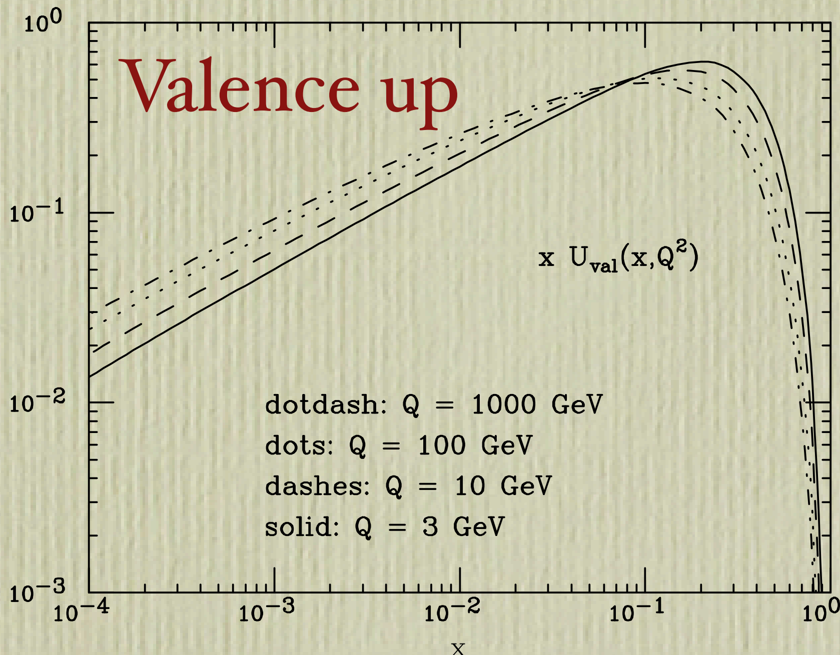
$$\frac{dg(x,Q)}{dt} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left[g(y,Q)\, P_{gg}\left(\frac{x}{y}\right) + \sum_{q,\bar{q}} q(y,Q)\, P_{gq}\left(\frac{x}{y}\right) \right]$$

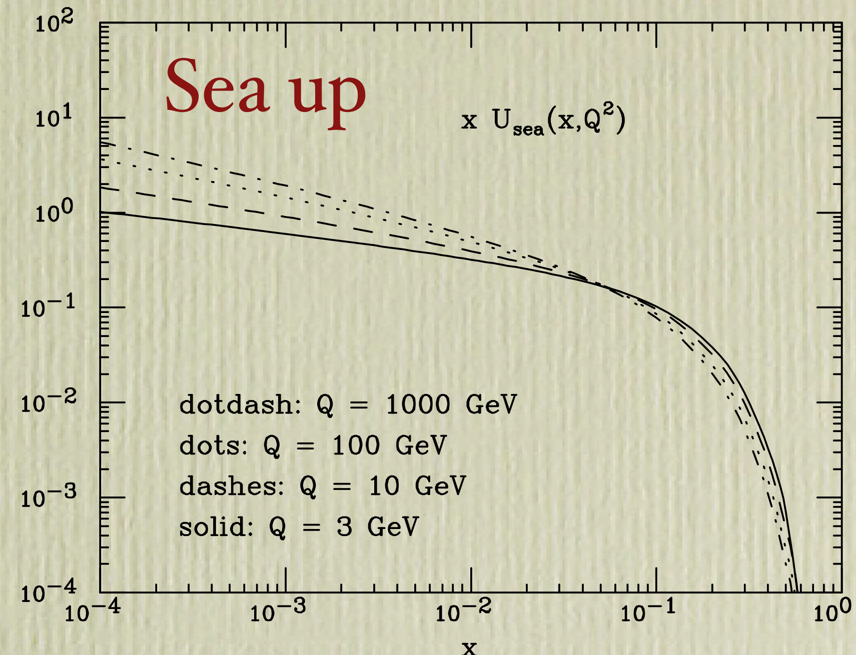


$$P_{gq}(x) = C_F \left(\frac{1+(1-x)^2}{x} \right)$$

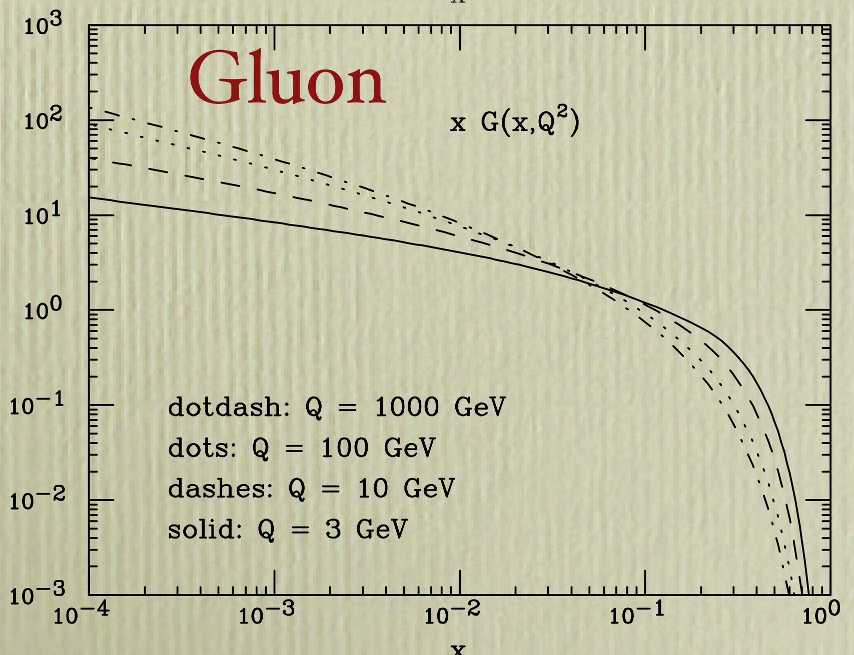
$$P_{gg}(x) = 2N_c \left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) \right] + \delta(1-x) \left(\frac{11N_c - 2n_f}{6} \right)$$

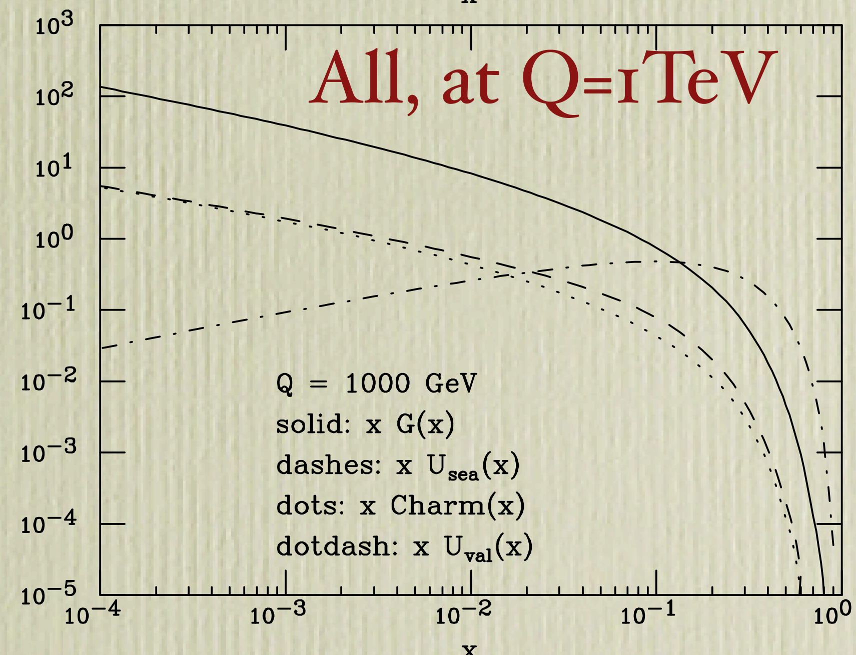
Examples of PDFs and their evolution





Note:
sea ≈10% glue





Note:
charm≈up at high Q

Example: charm in the proton

$$\frac{dc(x,Q)}{dt} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y}\, g(y,Q)\, P_{qg}(\frac{x}{y})$$

Assuming a typical behaviour of the gluon density:

$$g(x,Q) \sim A/x$$

we get:

$$\frac{dc(x,Q)}{dt} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y}\, g(x/y,Q)\, P_{qg}(y) = \frac{\alpha_s}{2\pi}\int_x^1 dy \frac{A}{x}\frac{1}{2}[y^2+(1-y)^2] = \frac{\alpha_s}{6\pi}\frac{A}{x}$$

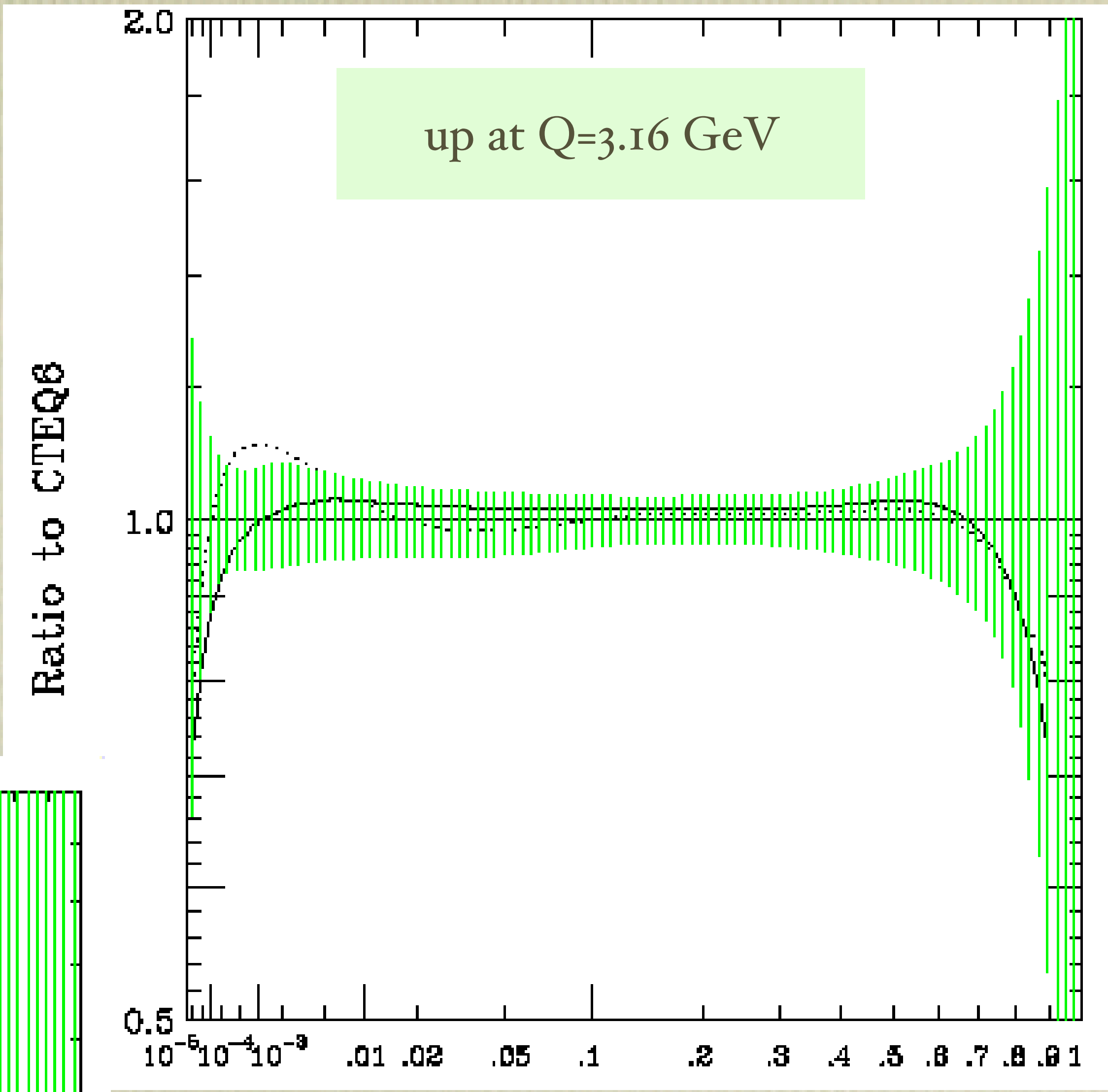
and therefore:

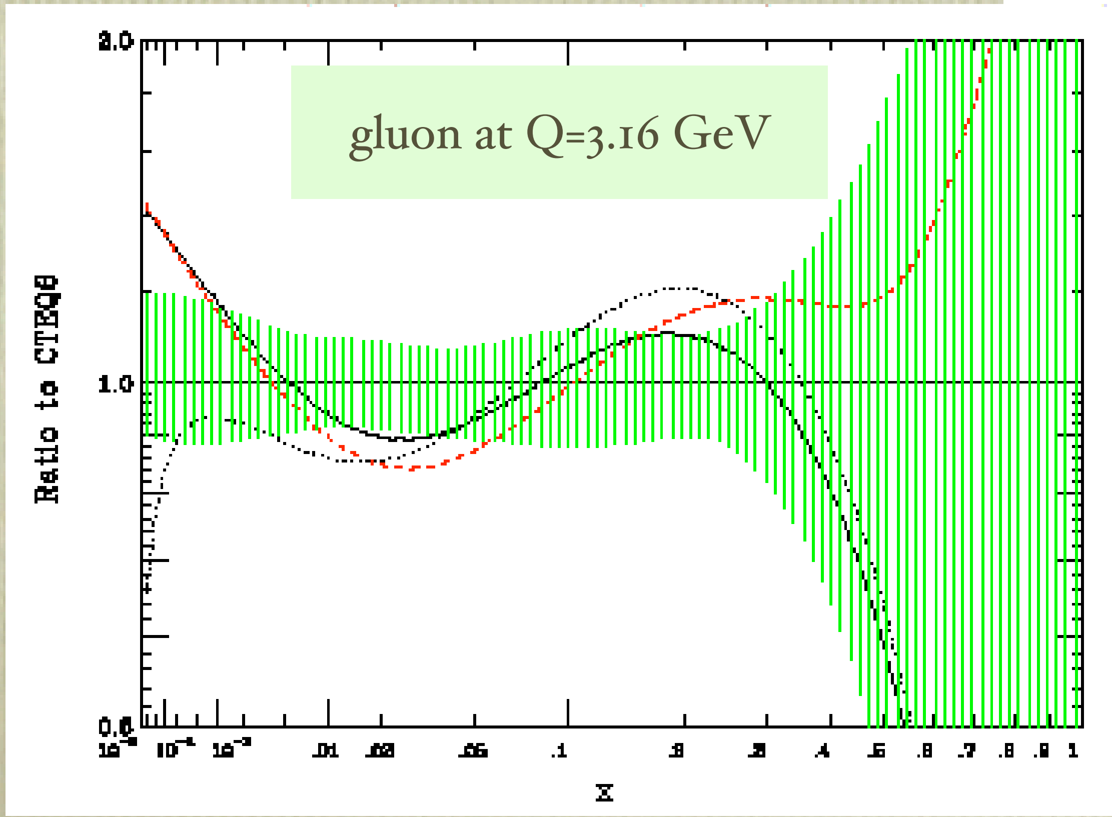
$$c(x,Q) \sim \frac{\alpha_s}{6\pi}\log(\frac{Q^2}{m_c^2})\, g(x,Q)$$

Corrections to this simple formula will arise due to the Q dependence of g(x) and of α_s

PDF uncertainties

Green bands represent the convolution of theoretical and experimental systematics in the determination of PDFs



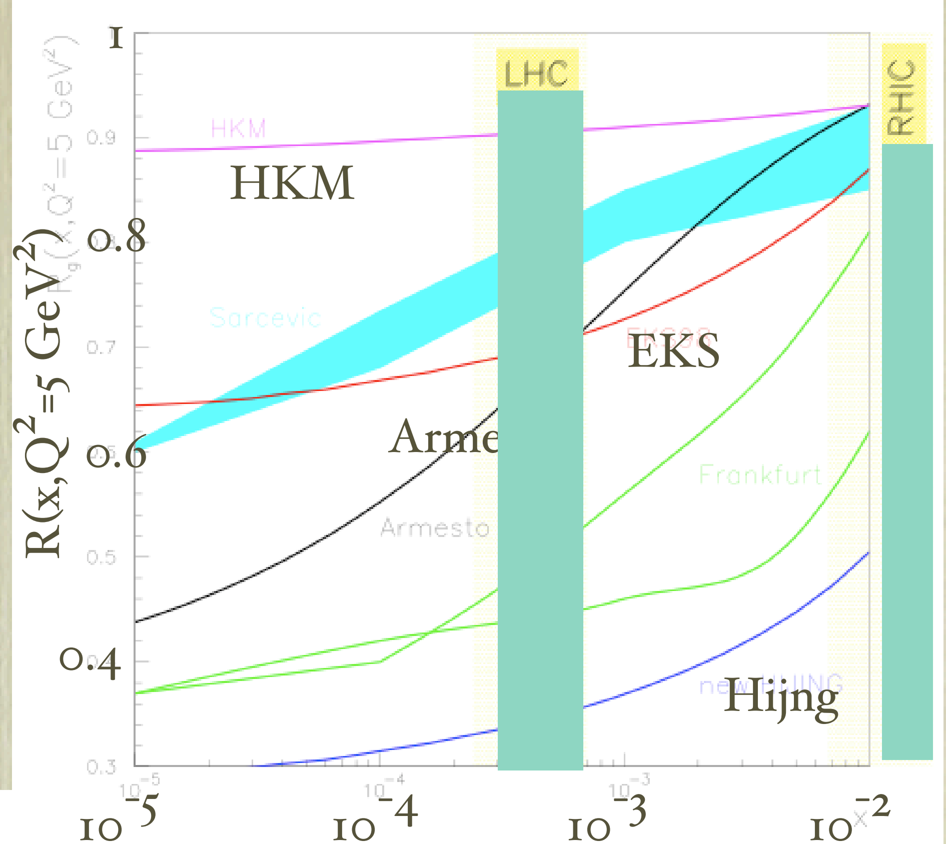


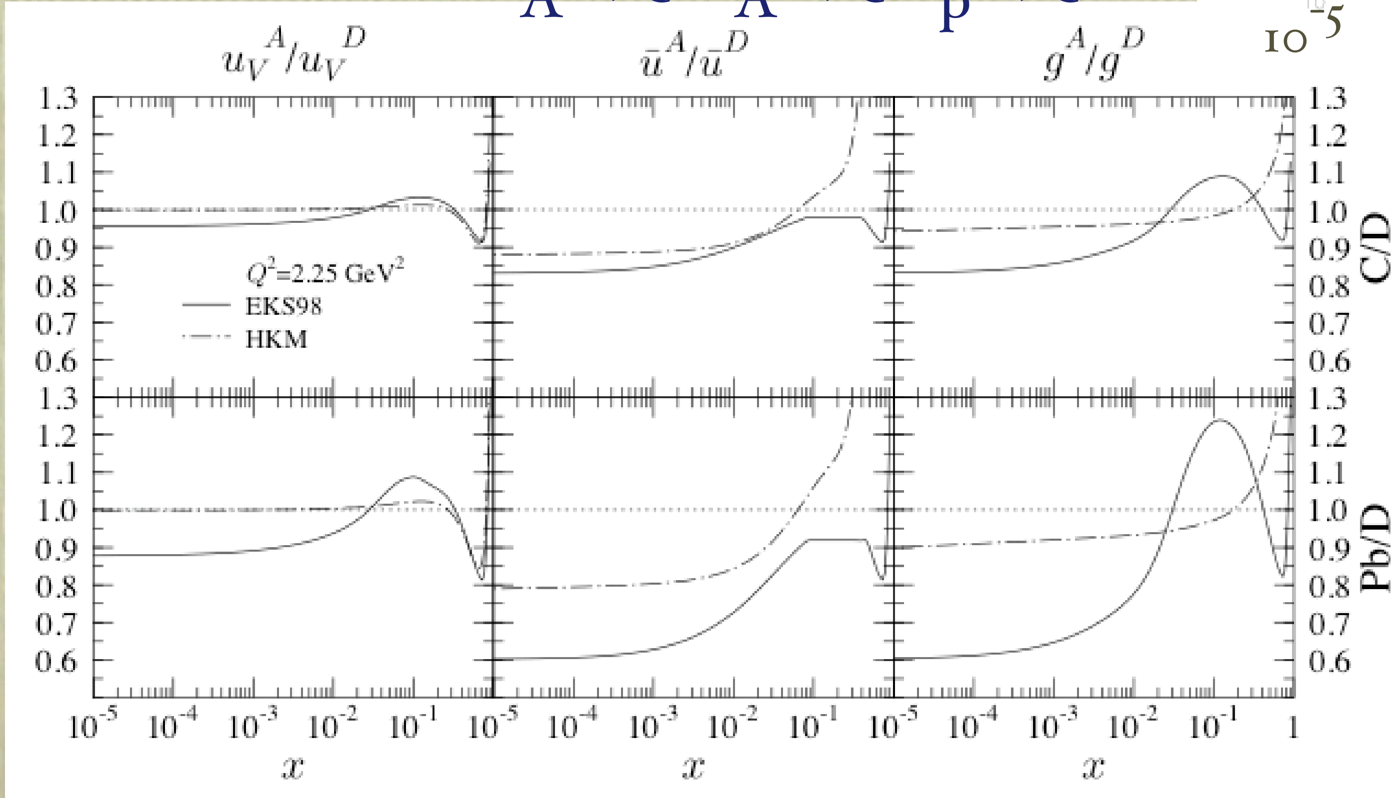
Proton PDFs known to 10-20% for $10^{-3}<x<0.3$, with uncertainties getting smaller at larger Q

Nuclear modifications

- Interactions among various nucleons may change the parton densities (shadowing, Cronin effect, etc)
- The evolution at high-Q, however, should be decoupled from the nuclear environment. Nuclear PDFs for hard processes can therefore be obtained from AP evolution of PDFs fit at low Q.
- Usually parameterized by

$$R_A(x,Q)=f_A(x,Q)/f_p(x,Q)$$

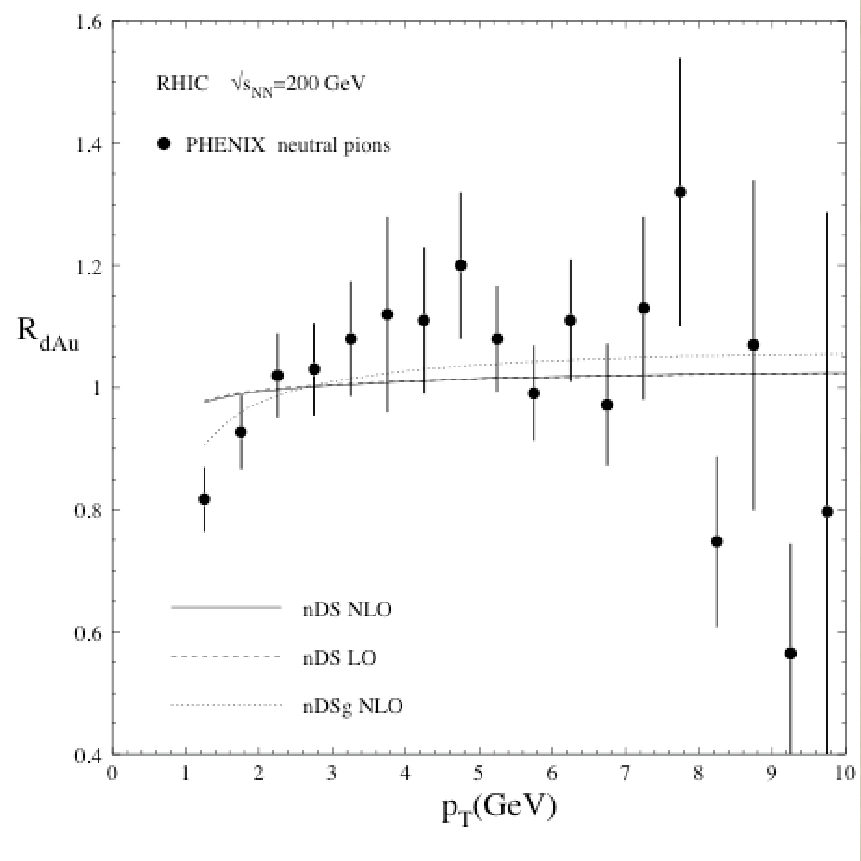




Large differences between different parameterizations, approaches. Only data will provide a solid basis for extrapolation from pp to PbPb at the LHC

Recent NLO analysis of nuclear PDF's:

DeFlorian, Sassot, hep-ph/0311227



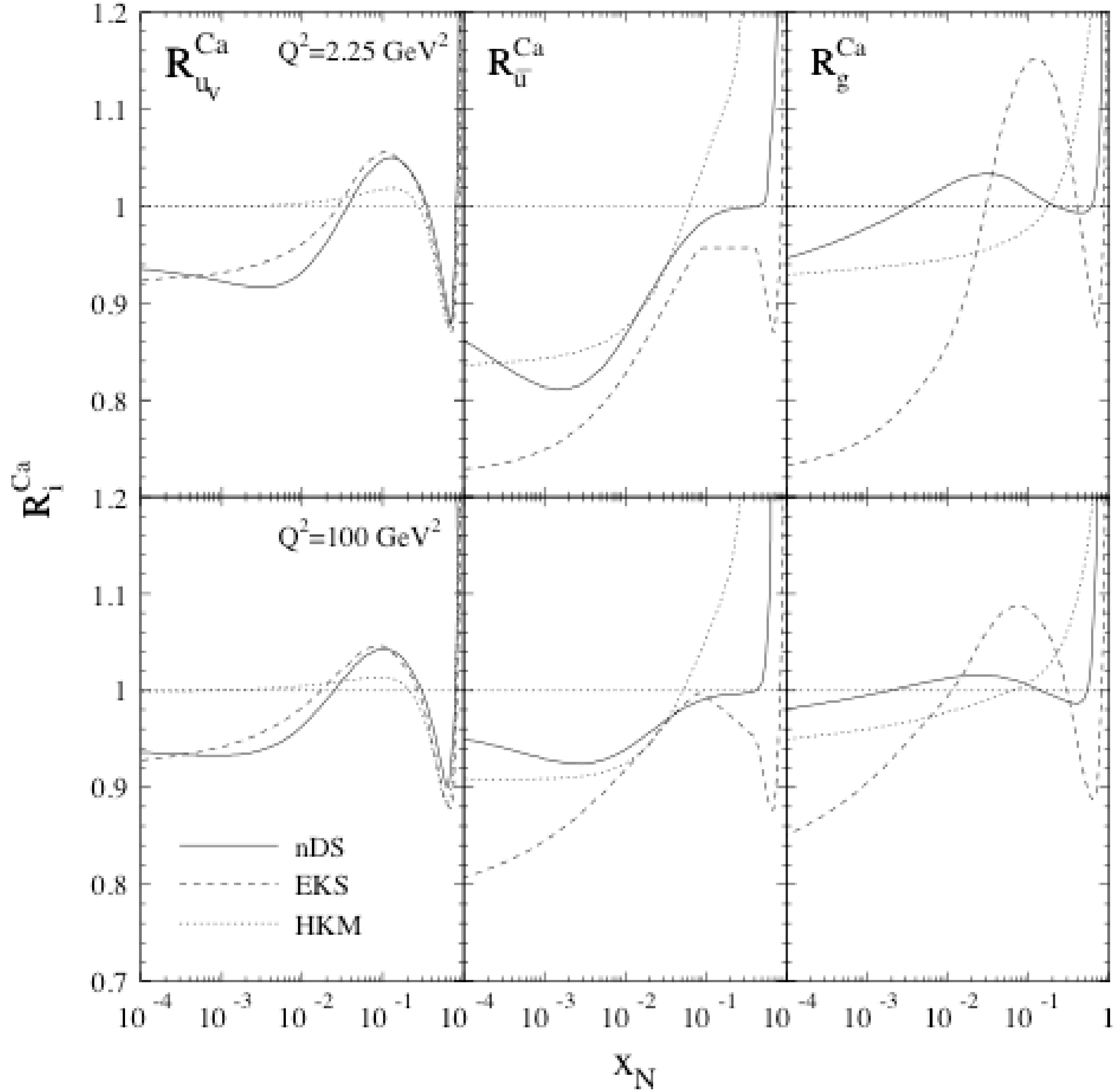
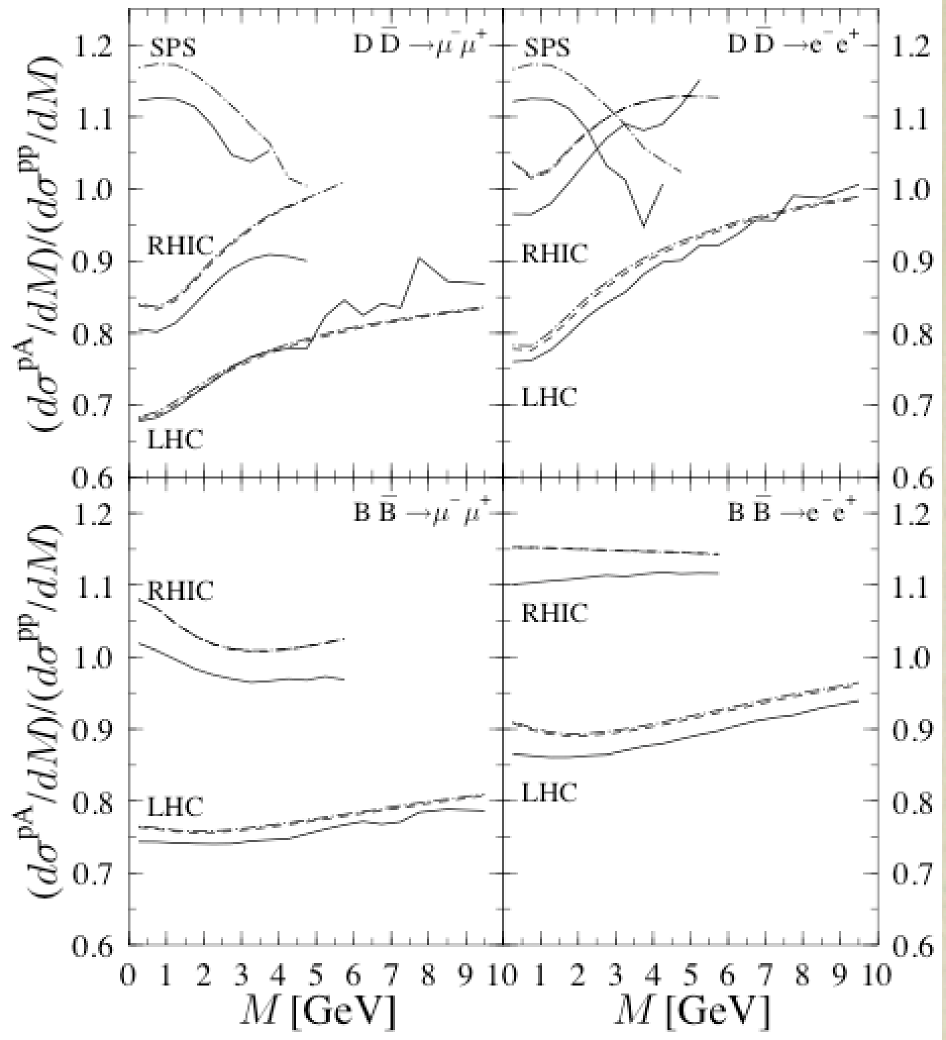


FIG. 7: *Ratios coming from different nPDF sets*

Examples of impact of shadowing on some hard-probe observables at RHIC and LHC



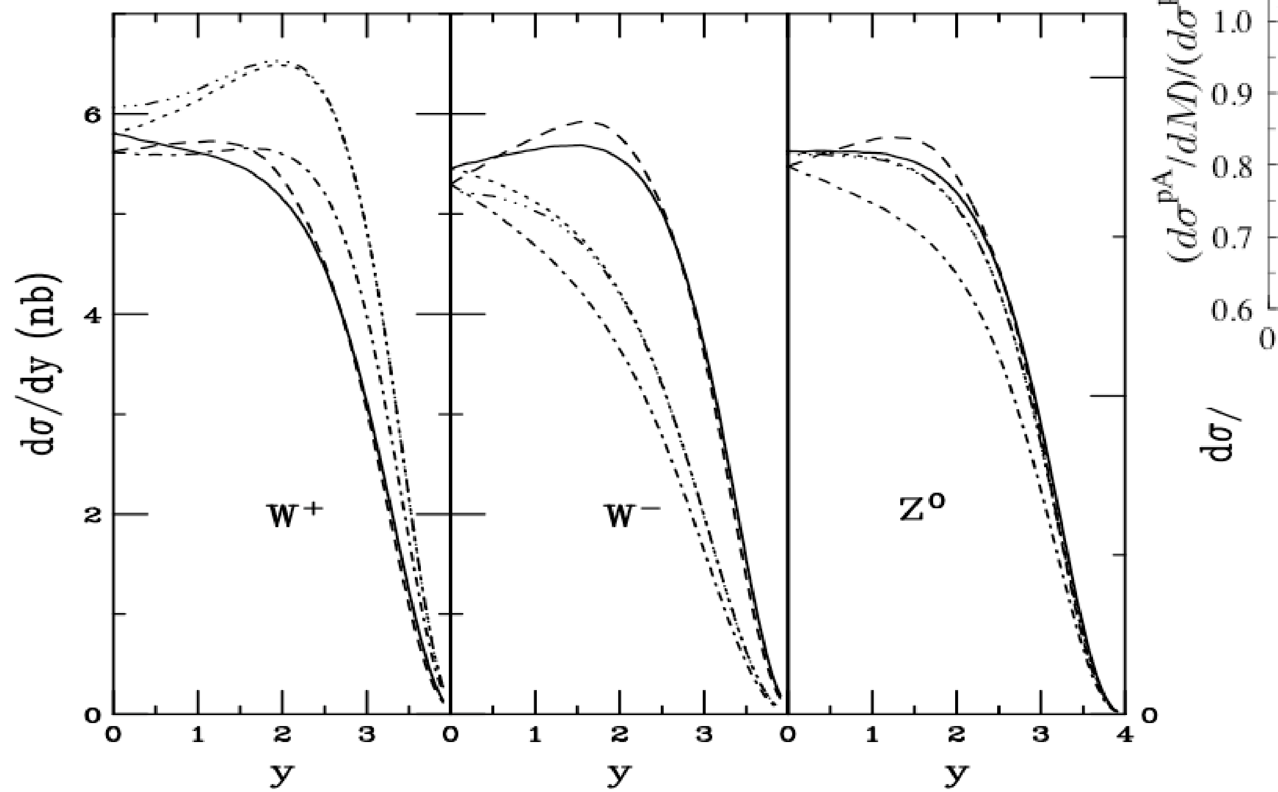


Fig. 22: The W^+, W^- and Z^0 rapidity distributions in pp, pPb and Pbp collisions at 5.5 TeV/nucleon evaluated at $Q = M_V$. The solid and dashed curves show the results without and with shadowing respectively in Pbp collisions while the dotted and dot-dashed curves give the results without and with shadowing for pPb collisions. The dot-dot-dot-dashed curve is the pp result.

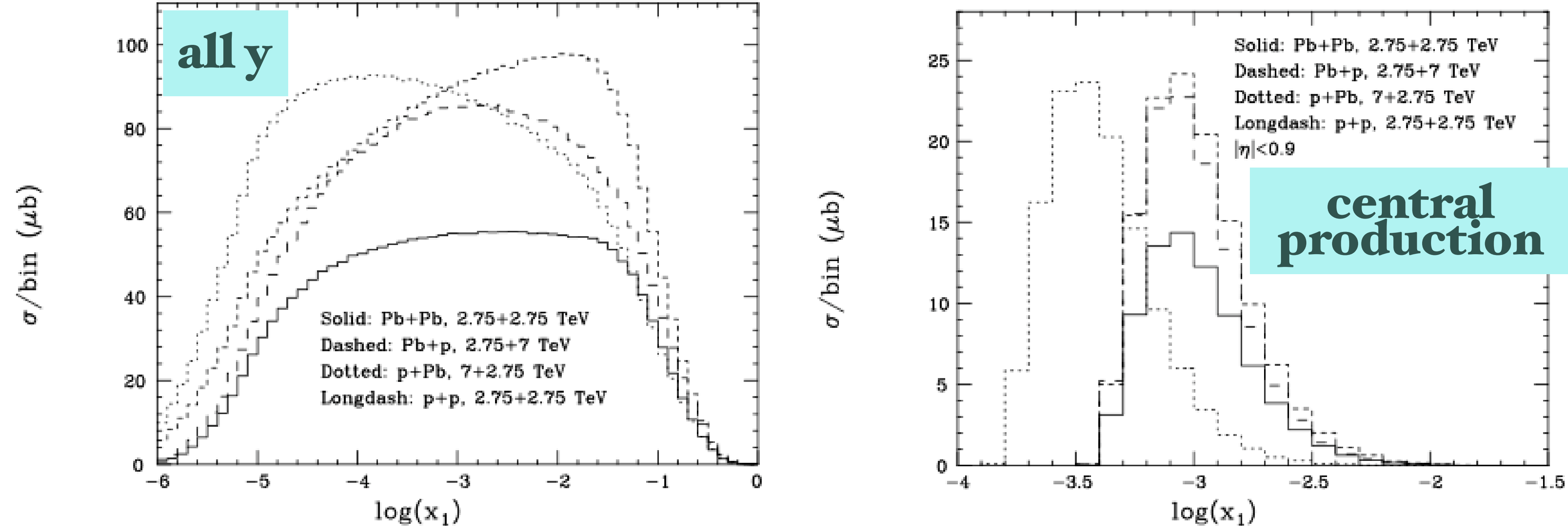


Fig. 1: Parton-x_1 distributions for charm production in the full pseudorapidity range (left) and in the central region (right). The cross section per nucleon is given.

Examples of x ranges probed by charm production at the LHC

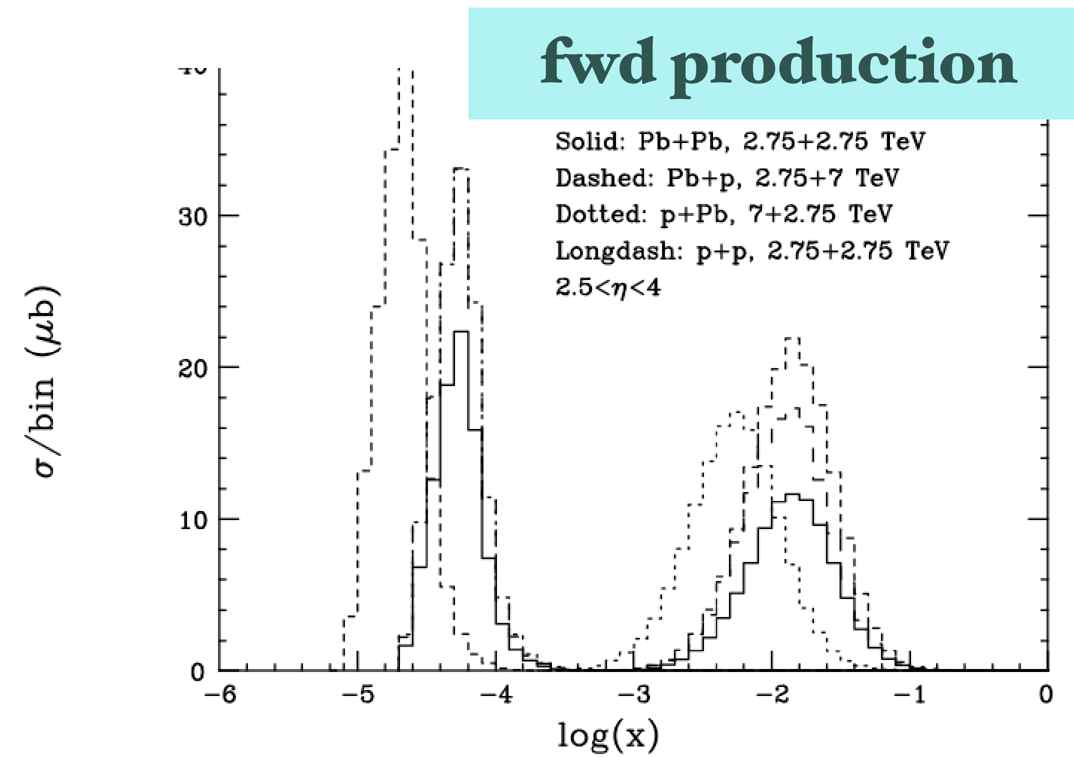
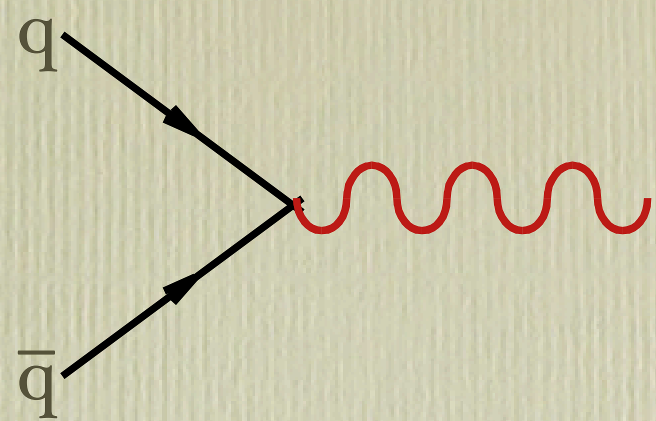


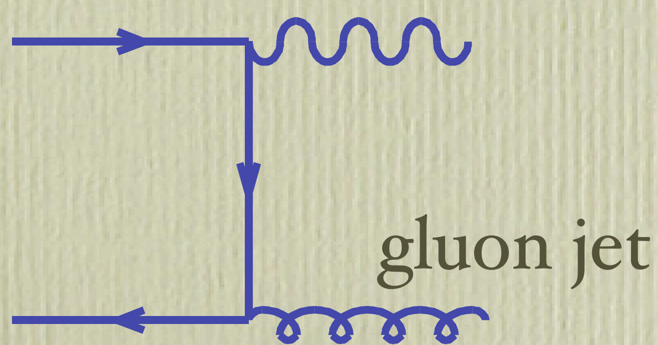
Fig. 2: Parton-x distributions for charm production in the forward pseudorapidity range $2.5 < \eta < 4$. The cross section per nucleon is given.

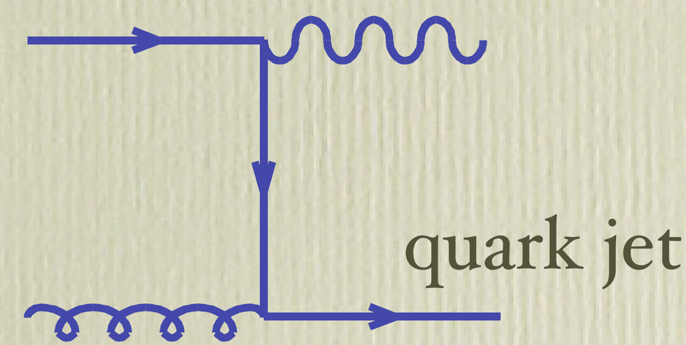
Drell-Yan processes:



$$W \to \ell\Pi$$
$$Z \to \ell^+\ell^-$$

- Very clean probe of the initial state: no interaction with the plasma!
- Very well understood theoretically: σ(W,Z) known up to NNLO (2-loops)
- Excellent experimental monitor of energy-scale for jets, when produced at large Et:





#(events)/month detected by CMS:

	barrel	barrel+ endcap
$Z(\to \mu^+\mu^-)$+jet, $E_T^{\rm jet}, p_T^Z > 100$ GeV	30	45
$Z(\to \mu^+\mu^-)$+jet, $E_T^{\rm jet}, p_T^Z > 50$ GeV	180	300

LO Cross-section calculation

$$\square(pp \to W) = \sum_{q,q'} \int dx_1 dx_2\, f_q(x_1,Q)\, f_{\bar{q}'}(x_2,Q)\, \frac{1}{2\hat{s}} \int d[PS]\, \overline{\sum_{spin,col}} |M(q\bar{q}' \to W)|^2$$

where:

$$\overline{\sum_{spin,col}} |M(q\bar{q}' \to W)|^2 = \frac{1}{3}\frac{1}{4} 8 g_W^2 |V_{qq'}|^2 \hat{s} = \frac{2}{3}\frac{G_F\, m_W^2}{\sqrt{2}} |V_{qq'}|^2 \hat{s}$$

$$d[PS] = \frac{d^3 p_W}{(2\square)^3 p_W^0}(2\square)^4 \square^4(P_{in} - p_W)$$

$$= 2\square d^4 p_W\, \square(p_W^2 - m_W^2)\, \square^4(P_{in} - p_W) = 2\square\square(\hat{s} - m_W^2)$$

leading to (exercise!):

$$\square(pp \to W) = \sum_{ij} \frac{\square A_{ij}}{m_W^2}\, \square \int_{\square}^{1} \frac{dx}{x} f_i(x,Q)\, f_j(\frac{\square}{x},Q) \equiv \sum_{ij} \frac{\square A_{ij}}{m_W^2}\, \square L_{ij}(\square)$$

where:

$$\frac{\square A_{u\bar{d}}}{m_W^2} = 6.5\text{nb} \quad \text{and} \quad \square = \frac{m_W^2}{S}$$

Some useful relations and definitions

Rapidity: $y = \frac{1}{2}\log\frac{E_W + p_W^z}{E_W - p_W^z}$

Pseudorapidity: $\square = -\log(\tan\frac{\square}{2})$

where:

$\tan\square = \frac{p_T}{p^z}$ and $p_T = \sqrt{p_x^2 + p_y^2}$

Exercise: prove that for a massless particle rapidity=pseudorapidity:

Exercise: using $\square = \frac{\hat{s}}{S} = x_1 x_2$ and

$$\begin{cases} E_W = (x_1 + x_2)\,E_{\text{beam}} \\ p_W^z = (x_1 - x_2)\,E_{\text{beam}} \end{cases} \Rightarrow y = \frac{1}{2}\log\frac{x_1}{x_2}$$

prove the following relations:

$$x_{1.2} = \sqrt{\square}\,e^{\pm y} \qquad dx_1\,dx_2 = dy\,d\square$$

$$dy = \frac{dx_1}{x_1} \qquad d\square\square(\hat{s} - m_W^2) = \frac{1}{S}$$

Study the function τL(τ)

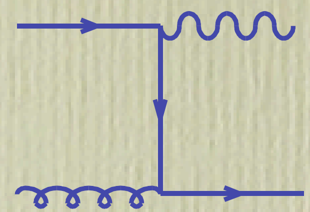
Assume, for example, that $f(x) \sim \dfrac{1}{x^{1+\delta}}, \quad 0 < \delta < 1$

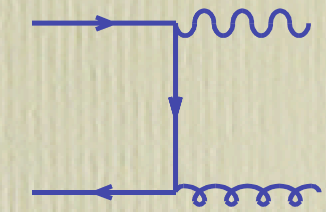
Then: $L(\tau) = \displaystyle\int_\tau^1 \frac{dx}{x}\frac{1}{x^{1+\delta}}\left(\frac{x}{\tau}\right)^{1+\delta} = \frac{1}{\tau^{1+\delta}}\log\left(\frac{1}{\tau}\right)$

and: $$\sigma_W = \sigma_W^0 \left(\frac{S}{m_W^2}\right)^{\delta} \log\left(\frac{S}{m_W}\right)$$

Therefore the W cross-section grows at least logarithmically with the hadronic CM energy. This is a typical behavior of cross-sections for production of fixed-mass objects in hadronic collisions, contrary to the case of e+e- collisions, where cross-sections tend to decrease with CM energy.

Photon plus jet production

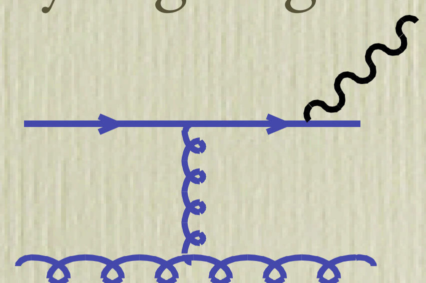
qg initial state: 

$q\bar{q}$ initial state: 

- As in the case of Z+jet, provides a good calibration for the absolute experimental determination of the energy of the recoil jet. Rates are larger than for Z's:

γ+jet, $E_T^{\mathrm{jet},\gamma} > 100$ GeV	1.6×10^3	3.0×10^3

- g(x)>>$\bar{q}$(x), therefore the first process dominates by at least a factor 10 throughout the phase-space. Potentially a good observable to constrain g(x)! Affected however by large higher-order, bremstrahlung-like corrections:



- $\sigma \propto e^2{}_q$, therefore up-type quarks are enhanced. In particular, the fraction of charm contribution is large, and a good fraction of recoling jets is charm-like.