Introduction to the physics of hard probes in hadron collisions:
lecture I

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The structure of the proton !the “initial-state”#
- parton densities
- their evolution
- nuclear modifications

Some benchmark SM processes !the “hard probes”#
- Drell-Yan
- Jets
- Heavy quark production

The structure of a hard proton-proton collision !the “final-state”#
- jet evolution
- hadronization
References: Report from the Workshop on Hard Probes in Heavy Ion Collisions at the LHC

- PDF’s, shadowing and pA collisions: hep-ph/0308248
- Heavy quarks: hep-ph/0311048
- Photon physics: hep-ph/0311131
- Jet physics: hep-ph/0310274

Previous lecture notes on QCD

- Introduction to QCD: http://mlm.home.cern.ch/mlm/talks/brazil.ps.gz
- Introduction to heavy quark production: hep-ph/9711337
\[ \left( \frac{i \Box^{ij}}{p - m + i \Box} \right)^{ab} \left( \frac{-i g^\mu}{p^2 + i \Box} \right) \]

\[ g f^{a_1 a_2 a_3} \left[ g^{\mu_1 \mu_2} (p_1 - p_2)^{\mu_3} + g^{\mu_2 \mu_3} (p_2 - p_3)^{\mu_1} + g^{\mu_3 \mu_1} (p_3 - p_1)^{\mu_2} \right] \]

\[ -i g^2 \left[ f^{a_1 a_2 X} f^{a_3 a_4 X} \left( g^{\mu_1 \mu_3} g^{\mu_2 \mu_4} - g^{\mu_1 \mu_4} g^{\mu_2 \mu_3} \right) + (2 \leftrightarrow 3) + (2 \leftrightarrow 4) \right] \]
Some results for the $SU(3)$\#colour algebra

$\square^a_{ij} : SU(N_c)$ matrix in the fundamental representation,

$$N_c = 3, \quad a = 1, \ldots, N_c^2 - 1, \quad i = 1, \ldots, N_c$$

$$[\square^a, \square^b] = i f^{abc} \square^c \quad (\Rightarrow \text{tr}\square^a = 0)$$

$$\text{tr}(\square^a \square^b) \overset{\text{def}}{=} T_F \square^{ab}, \quad T_F = 1/2 \text{ by convention}$$

$$\square_a \left(\square^a \square^a\right)_{ij} \overset{\text{def}}{=} C_F \square^j \quad \text{Exercise} \quad \frac{N_c^2 - 1}{N_c} \square^j$$

$$f_{abc} f_{abd} \overset{\text{def}}{=} C_A \square^{cd} \quad \text{Exercise} \quad N_c \square^{cd}$$

$$\square_a \square^a_{ij} \square^a_{kl} \quad \text{Exercise} \quad \frac{1}{2} \left(\square_{ik} \square_{jl} - \frac{1}{N} \square_{ij} \square_{kl}\right)$$
Exercises

• Verify the properties of the SU(N) algebra given in the previous pages
• Prove that the sums of the following sets of diagrams are gauge invariant, namely the amplitude remains invariant if we replace the polarization vector of any gluon, \( \varepsilon \), with \( \varepsilon + p \), \( p \) being the gluon momentum:
Running of the coupling constant

\[ \Box_s \overset{\text{def}}{=} \frac{g_s^2}{4\pi} \]

\[ \frac{d\Box_s}{d\log(Q^2)} = \Box(\Box_s) \]

At 1-loop:

\[ \Box = -b_0 \Box_s^2, \quad b_0 = \frac{33 - 2n_f}{12\pi} \]

and

\[ \Box_s(Q) = \frac{1}{b_0 \log(Q^2/\Box^2)} \]

At 2-loops:

\[ \Box = -b_0 \Box_s^2 - b_1 \Box_s^3, \quad b_1 = \frac{153 - 19n_f}{24\pi^2} \]

and

\[ \Box_s(Q) = \frac{1}{b_0 \log(Q^2/\Box^2)} \left[ 1 - \frac{b_1 \log \log(Q^2/\Box^2)}{b_0^2 \log Q^2/\Box^2} \right] \]

Current World Average !Bethke 2002# \[ \Box_s(M_Z) = 0.118 \pm 0.003 \]
Factorization Theorem

\[
\frac{d\sigma}{dX} = \sum_{j,k} f_j(x_1, Q_i) f_k(x_2, Q_i) \frac{d\sigma_{jk}(Q_i, Q_f)}{d\hat{X}} F(\hat{X} \, X; Q_i, Q_f)
\]

- sum over all initial state histories leading, at the scale \( Q \), to:
  \[ p_j = x \, \vec{P}_{proton} \]

- transition from partonic final state to the hadronic observable
  - hadronization, fragm. function, jet definition, etc

- Sum over all histories with \( X \) in them
Universality of parton densities and factorization, a naive proof

Exchange of hard gluons among quarks inside the proton is

\[ q \sim Q \int_{q}^{Q} \frac{d^{4}q}{q^{6}} \sim \frac{1}{Q^{2}} \]

suppressed by powers of \( \frac{m_{p}}{Q} \)

Typical time-scale of interactions binding the proton is therefore of \( O(\frac{1}{m_{p}}) \) in a frame in which the proton has energy \( E, \tau = \frac{\gamma}{m_{p}} = \frac{E}{m_{p}} \)

If a hard probe \( Q \gg m_{p} \) hits the proton, on a time scale \( = \frac{1}{Q} \), there is no time for quarks to negotiate a coherent response
As a result, to study inclusive processes at large $Q$ it is sufficient to consider the interactions between the external probe and a single parton:

1. Calculable in perturbative QCD and not calculable in pQCD
2. Do not affect $f(x)$, where $x_{\text{before}} = x_{\text{after}}$

This gluon cannot be reabsorbed because the quark is gone.

However, since $\tau q \approx 1 \text{ GeV} > 1/Q$, the emission of low-virtuality gluons will take place long before the hard collision, and therefore cannot depend on the detailed nature of the hard probe. While it is not calculable in pQCD, $f(q << Q)$ can be measured using a reference probe, and used elsewhere.

**Universality of $f(x)$**
Q dependence of parton densities

The larger is $Q$, the more gluons will not have time to be reabsorbed.

PDF’s depend on $Q$!

$$f(x, Q) = f(x, \mu) + \int_x^1 dx_{\text{in}} f(x_{\text{in}}, \mu) \int^Q_\mu dq^2 \int_0^1 dy P(y, q^2) \delta(x - yx_{\text{in}})$$
\[ f(x, Q) = f(x, \mu) + \int_x^1 dx_{in} f(x_{in}, \mu) \int_{\mu}^Q dq^2 \int_0^1 dy P(y, q^2) \delta(x - yx_{in}) \]

\( f!x,Q\) should be independent of the intermediate scale considered:

\[ \frac{d f(x, Q)}{d \mu^2} = 0 \quad \Rightarrow \quad \frac{d f(x, \mu)}{d \mu^2} = \int_x^1 \frac{dy}{y} f(y, \mu) P(x/y, \mu^2) \]

One can prove that:

\[ P(x, Q^2) = \frac{\delta_s}{2 \delta} \frac{1}{Q^2} P(x) \]

and therefore Altarelli-Parisi equation:

\[ \frac{d f(x, \mu)}{d \log \mu^2} = \frac{\delta_s}{2 \delta} \int_x^1 \frac{dy}{y} f(y, \mu) P(x/y) \]
More in general, one should consider additional processes which lead to the evolution of partons at high $Q ! t=\log Q^2$#

\[
\frac{dq(x, Q)}{dt} = \frac{\Box_s}{2\Box} \int_1^x \frac{dy}{y} \left[ q(y, Q) P_{qq}(\frac{x}{y}) + g(y, Q) P_{qg}(\frac{x}{y}) \right]
\]

\[
P_{qq}(x) = C_F \left( \frac{1 + x^2}{1 - x} \right) + \frac{1}{2} \left[ x^2 + (1 - x)^2 \right]
\]

\[
P_{qg}(x) = \frac{1}{2 N_c} \left[ \frac{x}{(1 - x)_+} + \frac{1 - x}{x} + x(1 - x) \right] + \Box(1 - x) \left( \frac{11N_c - 2n_f}{6} \right)
\]

\[
[g(x)]_+ : \int_0^1 dx f(x) g(x) \equiv \int_0^1 [f(x) - f(1)] g(x) dx
\]
Examples of PDFs and their evolution

Valence up

x $U_{\text{val}}(x,Q^2)$

dotdash: $Q = 1000$ GeV
dots: $Q = 100$ GeV
dashes: $Q = 10$ GeV
solid: $Q = 3$ GeV

Sea up

x $U_{\text{sea}}(x,Q^2)$

dotdash: $Q = 1000$ GeV
dots: $Q = 100$ GeV
dashes: $Q = 10$ GeV
solid: $Q = 3$ GeV

Gluon

x $G(x,Q^2)$

dotdash: $Q = 1000$ GeV
dots: $Q = 100$ GeV
dashes: $Q = 10$ GeV
solid: $Q = 3$ GeV

All, at $Q = 1$ TeV

Q = 1000 GeV
solid: x $G(x)$
dashes: x $U_{\text{sea}}(x)$
dots: x Charm$(x)$
dotdash: x $U_{\text{val}}(x)$

Note:
sea $\approx 10\%$ glue

Note:
charm $\approx$ up at high $Q$
**Example: charm in the proton**

\[
\frac{dc(x, Q)}{dt} = \frac{\Box_s}{2\Box} \int_x^1 \frac{dy}{y} g(y, Q) P_{qg}(\frac{x}{y})
\]

Assuming a typical behaviour of the gluon density:

\[
g(x, Q) \sim \frac{A}{x}
\]

we get:

\[
\frac{dc(x, Q)}{dt} = \frac{\Box_s}{2\Box} \int_x^1 \frac{dy}{y} g(x/y, Q) P_{qg}(y) = \frac{\Box_s}{2\Box} \int_x^1 dy \frac{A}{2} \frac{1}{x} \left[ y^2 + (1 - y)^2 \right] = \frac{\Box_s A}{6\Box x}
\]

and therefore:

\[
c(x, Q) \sim \frac{\Box_s}{6\Box} \log(\frac{Q^2}{m_c^2}) g(x, Q)
\]

Corrections to this simple formula will arise due to the Q dependence of g!x#and of \(\alpha_s\)
PDF uncertainties

Green bands represent the convolution of theoretical and experimental systematics in the determination of PDFs

Proton PDFs known to 10-20% for $10^{-3} < x < 0.3$, with uncertainties getting smaller at larger Q
Nuclear modifications

- Interactions among various nucleons may change the parton densities!shadowing, Cronin effect, etc#
- The evolution at high-$Q$, however, should be decoupled from the nuclear environment. Nuclear PDFs for hard processes can therefore be obtained from AP evolution of PDFs fit at low $Q$.
- Usually parameterized by

$$ R_A^{x,Q} \frac{f_A^{x,Q} f_p^{x,Q}}{f_p^{x,Q}} $$

Large differences between different parameterizations, approaches. Only data will provide a solid basis for extrapolation from pp to PbPb at the LHC.
Recent NLO analysis of nuclear PDF’s:
DeFlorian, Sassot, hep-ph/0311227
Examples of impact of shadowing on some hard-probe observables at RHIC and LHC

Fig. 22: The $W^+$, $W^-$ and $Z^0$ rapidity distributions in $pp$, $pPb$ and $PbPb$ collisions at 5.5 TeV/nucleon evaluated at $Q = M_N$.

The solid and dashed curves show the results without and with shadowing respectively in $PbPb$ collisions while the dotted and dot-dashed curves give the results without and with shadowing for $pPb$ collisions. The dot-dot-dot-dashed curve is the $pp$ result.
Examples of x ranges probed by charm production at the LHC

Fig. 1: Parton-$x_1$ distributions for charm production in the full pseudorapidity range (left) and in the central region (right). The cross section per nucleon is given.

Fig. 2: Parton-$x$ distributions for charm production in the forward pseudorapidity range $2.5 < \eta < 4$. The cross section per nucleon is given.
Drell-Yan processes:

- Very clean probe of the initial state: no interaction with the plasma!
- Very well understood theoretically: $\sigma(W, Z)$ known up to NNLO $\ell^2$-loops#
- Excellent experimental monitor of energy-scale for jets, when produced at large $E_T$: #events/month detected by CMS:

<table>
<thead>
<tr>
<th>$Z(\rightarrow \mu^+\mu^-)+$jet, $E_T^{jet}, p_T^Z &gt; 100$ GeV</th>
<th>barrel</th>
<th>barrel+ endcap</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z(\rightarrow \mu^+\mu^-)+$jet, $E_T^{jet}, p_T^Z &gt; 50$ GeV</td>
<td>30</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td>180</td>
<td>300</td>
</tr>
</tbody>
</table>
LO Cross-section calculation

\[ \Box(pp \rightarrow W) = \Box \int d_{x_{1}}dx_{2}f_{q}(x_{1},Q)f_{q'}(x_{2},Q) \frac{1}{2\hat{s}} \int d[PS] \Box |M(q\bar{q}' \rightarrow W)|^{2} \]

where:

\[ \Box |M(q\bar{q}' \rightarrow W)|^{2} = \frac{1}{3} \frac{1}{4} 8g_{W}^{2} |V_{qq'}|^{2} \hat{s} = \frac{2G_{F}m_{W}^{2}}{3} \sqrt{2} |V_{qq'}|^{2} \hat{s} \]

\[ d[PS] = \frac{d^{3}p_{W}}{(2\Box)^{3}p_{W}^{0}} (2\Box)^{4} (P_{in} - p_{W}) \]

\[ = 2\Box d^{4}p_{W} \Box (p_{W}^{2} - m_{W}^{2}) \Box^{4}(P_{in} - p_{W}) = 2\Box\Box(\hat{s} - m_{W}^{2}) \]

leading to !exercise!#

\[ \Box(pp \rightarrow W) = \Box \frac{\Box A_{ij}}{m_{W}^{2}} \Box \int_{\Box}^{1} dx f_{i}(x,Q)f_{j}(\frac{\Box}{x},Q) \equiv \Box \frac{\Box A_{ij}}{m_{W}^{2}} \Box L_{i,j}(\Box) \]

where:

\[ \frac{\Box A_{ud}}{m_{W}^{2}} = 6.5\text{nb} \quad \text{and} \quad \Box = \frac{m_{W}^{2}}{S} \]
Some useful relations and definitions

Rapidity: \( y = \frac{1}{2} \log \frac{E_W + p_W^z}{E_W - p_W^z} \)

Pseudorapidity: \( \Box = -\log(\tan \frac{\Box}{2}) \)

where:
\[
\tan \Box = \frac{p_T}{p_W^z} \quad \text{and} \quad p_T = \sqrt{p_x^2 + p_y^2}
\]

Exercise: prove that for a massless particle rapidity=pseudorapidity:

Exercise: using \( \Box = \frac{\hat{s}}{S} = x_1 x_2 \) and

\[
\begin{cases}
E_W = (x_1 + x_2)E_{\text{beam}} \\
p_W^z = (x_1 - x_2)E_{\text{beam}}
\end{cases} \quad \Rightarrow \quad y = \frac{1}{2} \log \frac{x_1}{x_2}
\]

prove the following relations:

\[
\begin{align*}
x_{1,2} &= \sqrt{\Box} e^{\pm y} \\
dx_1 \, dx_2 &= dy \, d\Box \\
dy &= \frac{dx_1}{x_1} \\
d\Box \, d(\hat{s} - m_W^2) &= \frac{1}{S}
\end{align*}
\]
Study the function $\tau L(\tau)$

Assume, for example, that $f(x) \sim \frac{1}{x^{1+\tau}}$, $0 < \tau < 1$.

Then: $L(\tau) = \int_{\tau}^{1} \frac{dx}{x} \frac{1}{x^{1+\tau}} \left( \frac{x}{\tau} \right)^{1+\tau} = \frac{1}{\tau^{1+\tau}} \log \left( \frac{1}{\tau} \right)$

and: $\Box_w = \Box_0 \left( \frac{S}{m_W^2} \right) \log \left( \frac{S}{m_W} \right)$

Therefore the $W$ cross-section grows at least logarithmically with the hadronic CM energy. This is a typical behavior of cross-sections for production of fixed-mass objects in hadronic collisions, contrary to the case of $e^+e^-$ collisions, where cross-sections tend to decrease with CM energy.
Photon plus jet production

qg initial state:  

q̅q initial state:  

• As in the case of Z+jet, provides a good calibration for the absolute experimental determination of the energy of the recoil jet. Rates are larger than for Z’s:

| γ+jet, $E_T^{jet,γ}$ > 100 GeV | 1.6 × 10³ | 3.0 × 10³ |

• $g!x# > q!x#$ therefore the first process dominates by at least a factor 10 throughout the phase-space. Potentially a good observable to constrain $g!x#$ affected however by large higher-order, bremsstrahlung-like corrections:

• $σ \propto e^2 Q$, therefore up-type quarks are enhanced. In particular, the fraction of charm contribution is large, and a good fraction of recoling jets is charm-like.