Hadron Collider Physics
- Experimental Overview – Part I -

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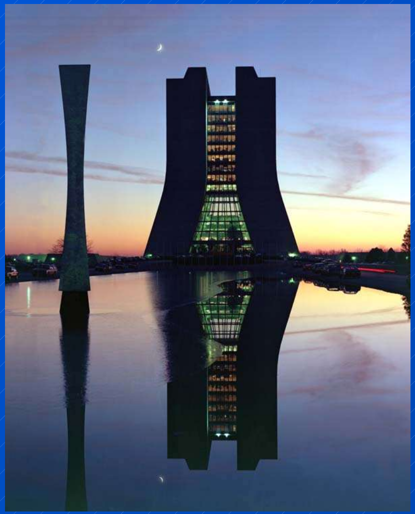


Outline

- Part I — Introduction to Hadron Colliders
  Studies of the Strong Interaction (QCD)

- Part II — Electroweak Physics
  Top Physics

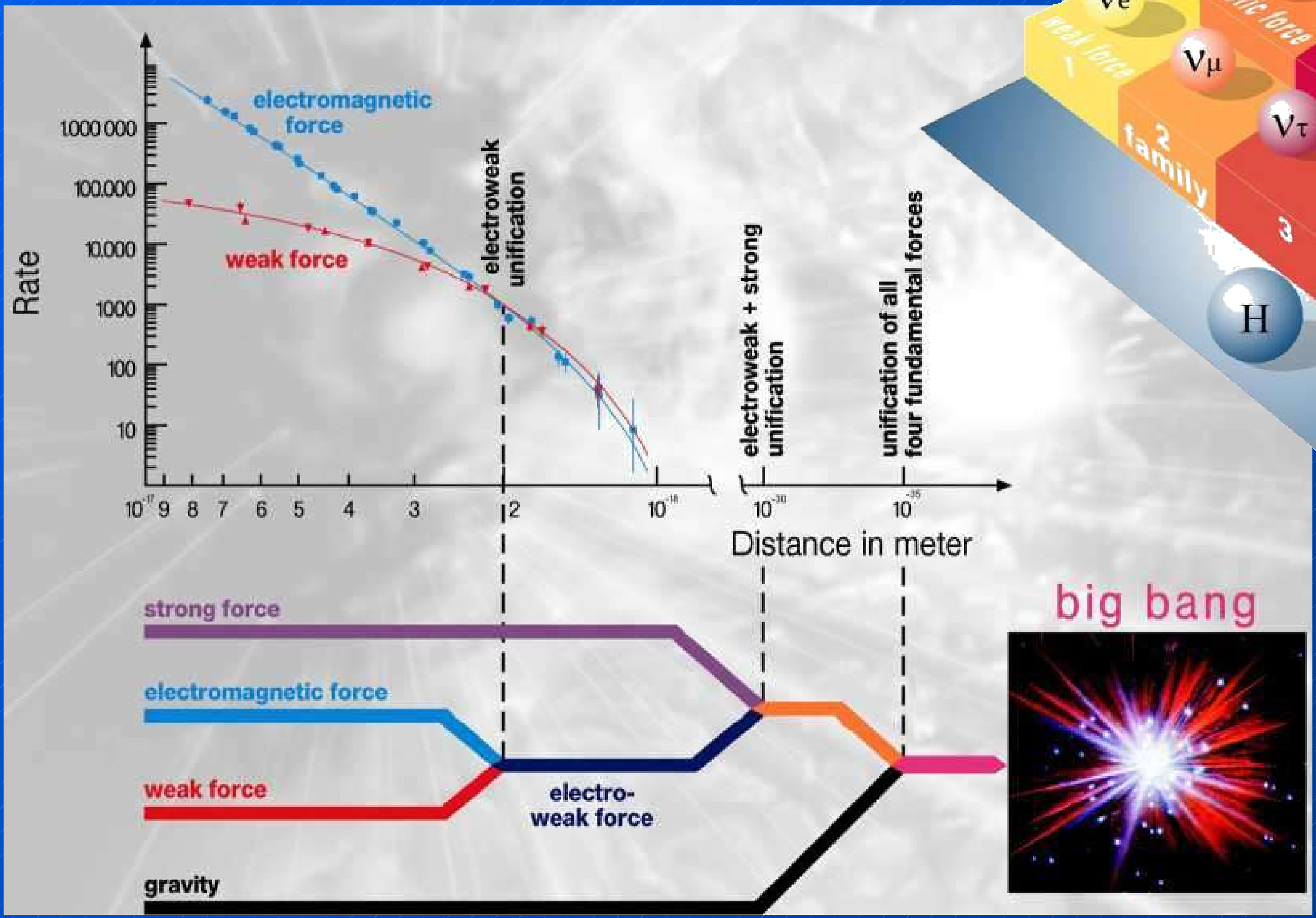
- Part III — Search for the Standard Model Higgs Boson
  Search for the Higgs beyond the Standard Model
  Search for New Phenomena

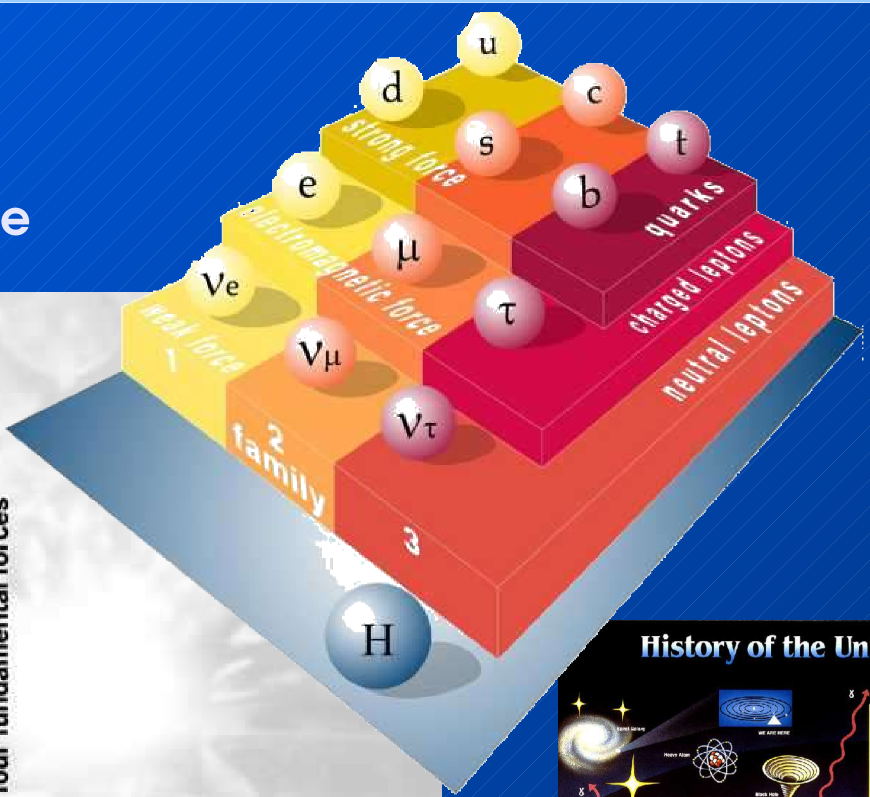
Part I

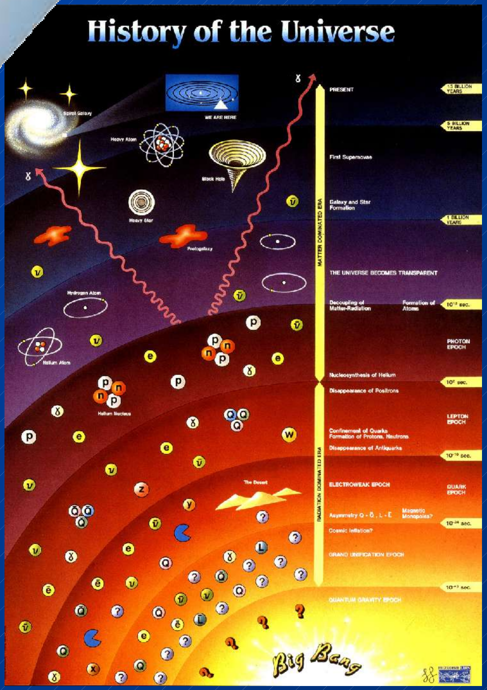
- Brief Summary of the Standard Model
- Structure of the Proton
- Hadron Colliders :
  PS – SPPS – Tevatron – LHC
- The Experiments :
  CDF – DØ – ATLAS – CMS
- Studies of the Strong Interaction (QCD)

The Standardmodel

- 2 * 3 * 2 fundamental particles (fermions)
- 4 fundamental forces + Higgs boson
- gauge theories with local gauge invariance



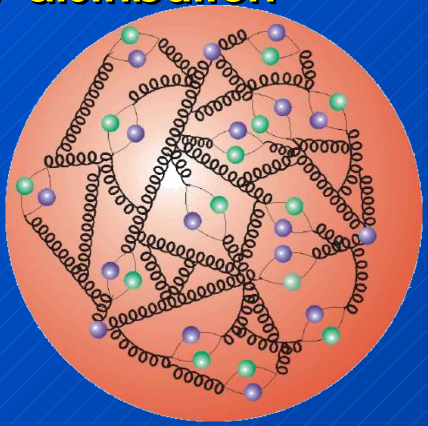




The Structure of the Proton (1)

Protons (hadrons) are not elementary particles

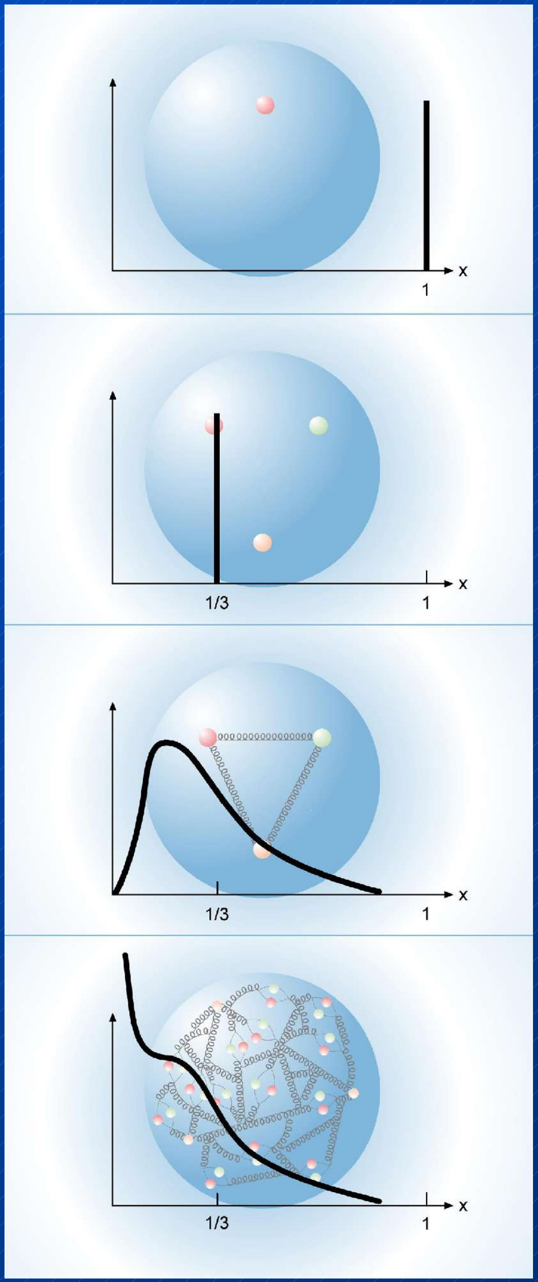
- they have substructure (partons = quarks and gluons)
- not well defined parton energy, but energy distribution
- various parton flavours can collide
- data analysis more complex than e^+e^-, QCD always involved



Why do we collide them ?

- much less synchroton radiation losses
- not limited by cavity power
- but by ring size / magnet strength
- essentially no strict kinematic limit, only luminosity limited
- protons are easy to produce in vast quantities
- antiproton production ? (cooling techniques)

$$\Delta E = \frac{4\pi\alpha\hbar c}{3R} \cdot \left(\frac{E}{m}\right)^4$$

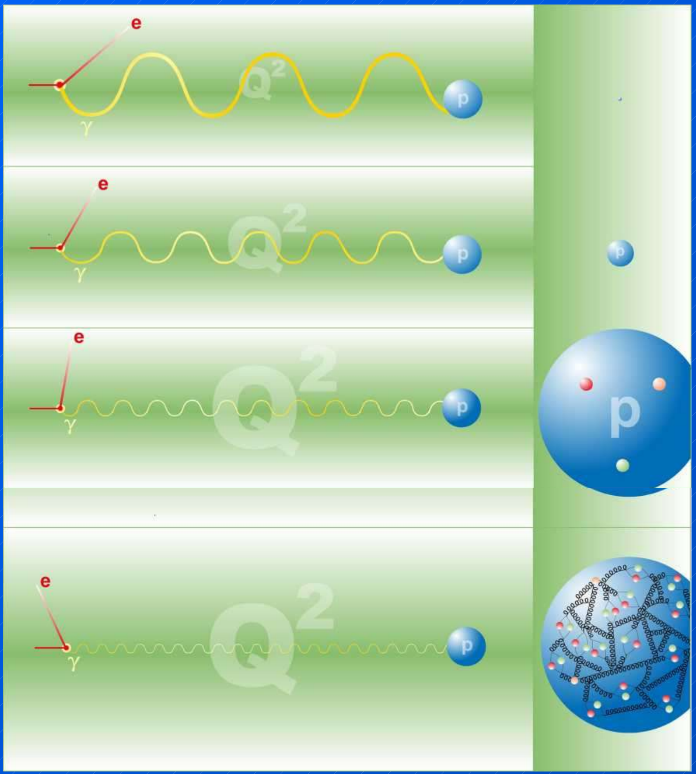


Kinematic variables:

Bjorken-x = fraction of the proton momentum carried by the struck parton

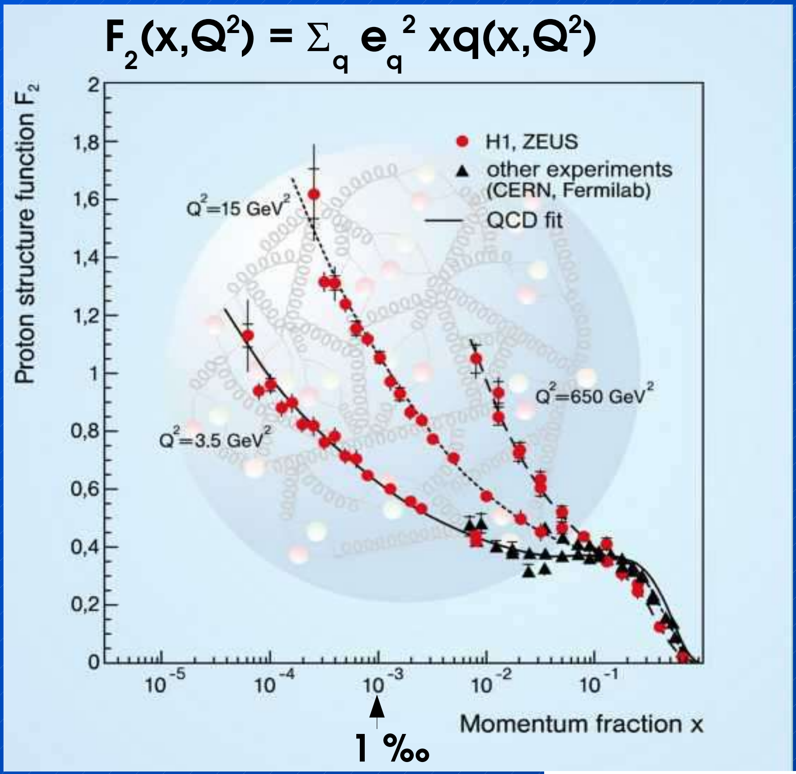
Q^2 = 4-momentum transfer squared in hard interaction ~ 1/transverse resolution power

The Structure of the Proton (2)

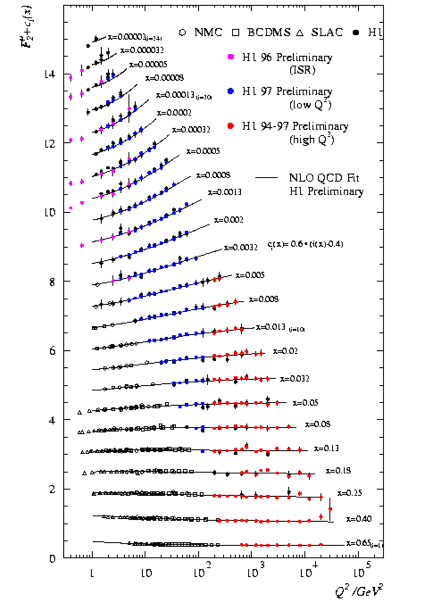


hadronic

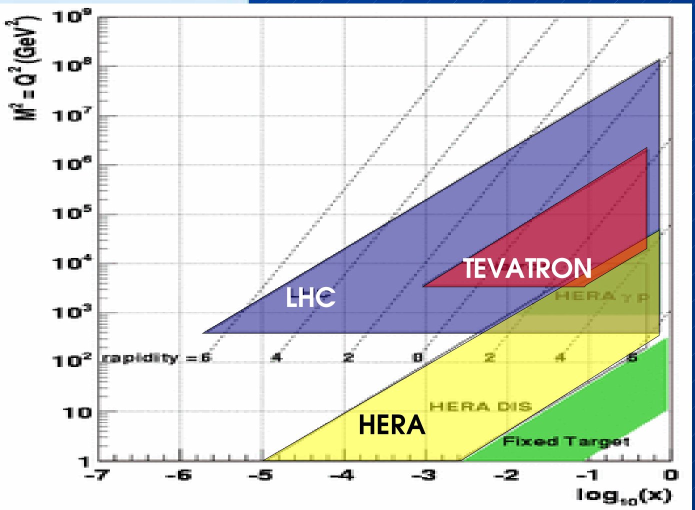
partonic



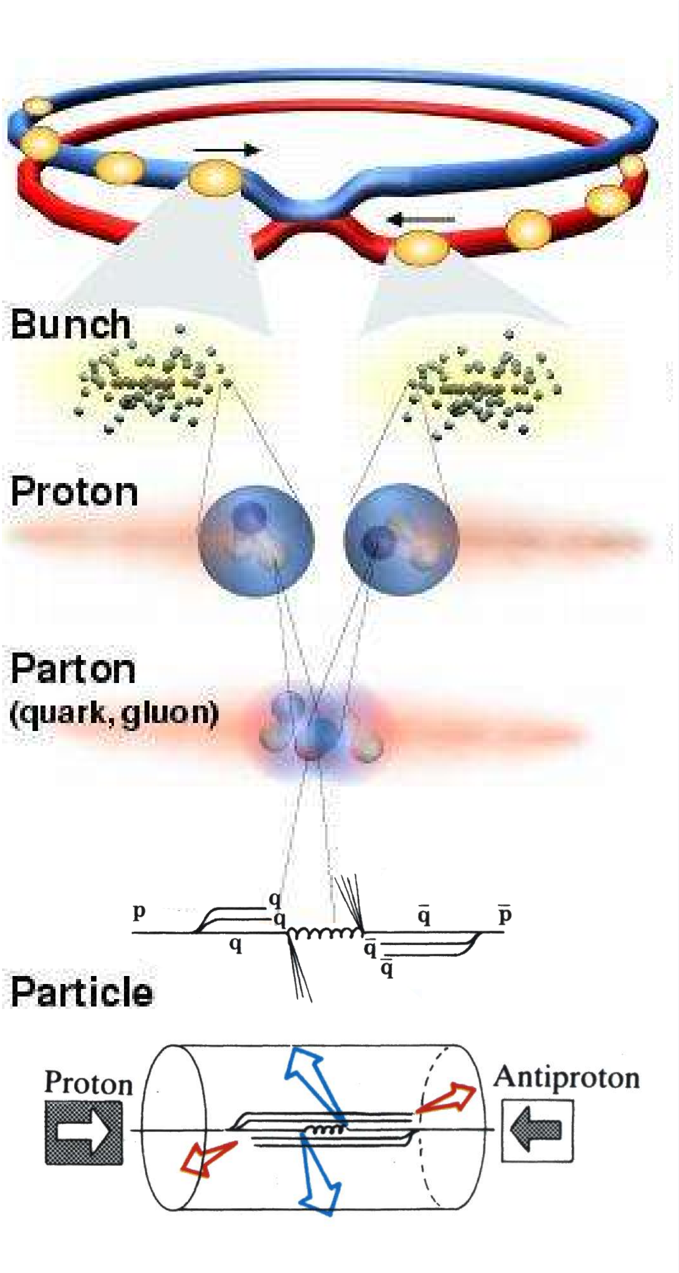
A wealth of data at low Q^2

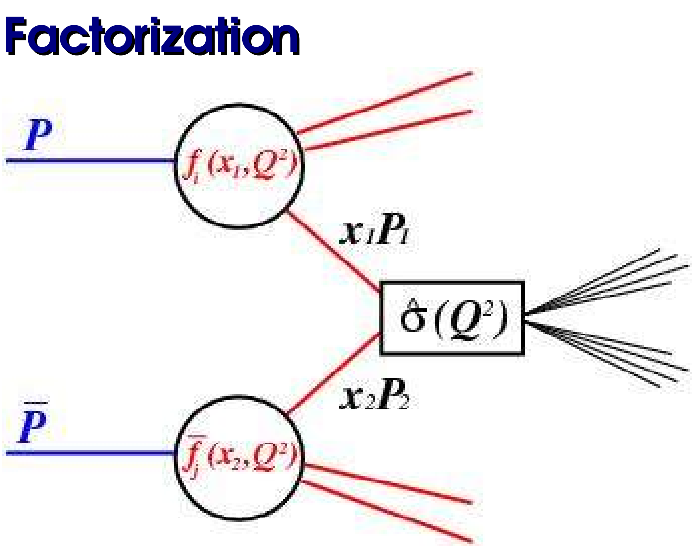


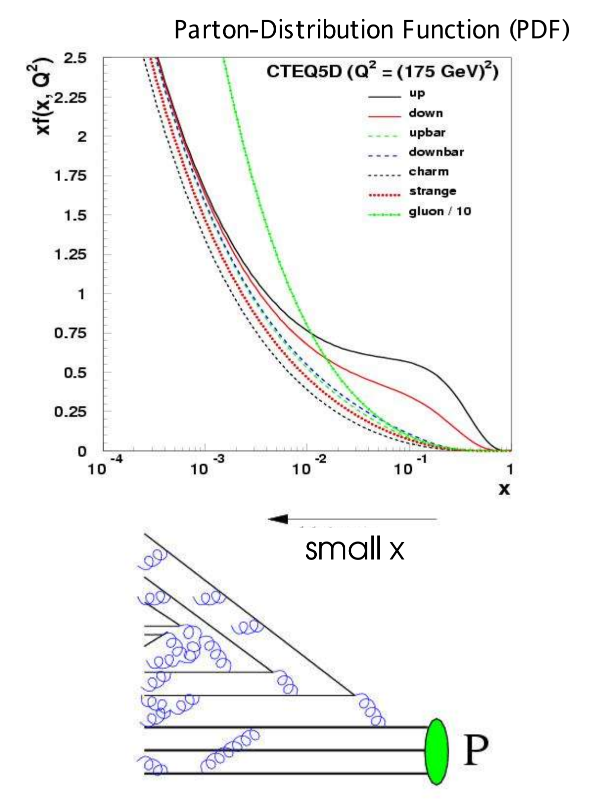
- Partonic structure at high Q^2
- Parton density (F_2) rises towards low x
- Extrapolation of HERA and fixed target measurements towards lower x and higher Q^2
- Need to measure (longitudinal) parton (momentum) distribution functions (PDFs) in every experiment
  parametrisations by CTEQ, MRST ...



Parton-Parton Interactions





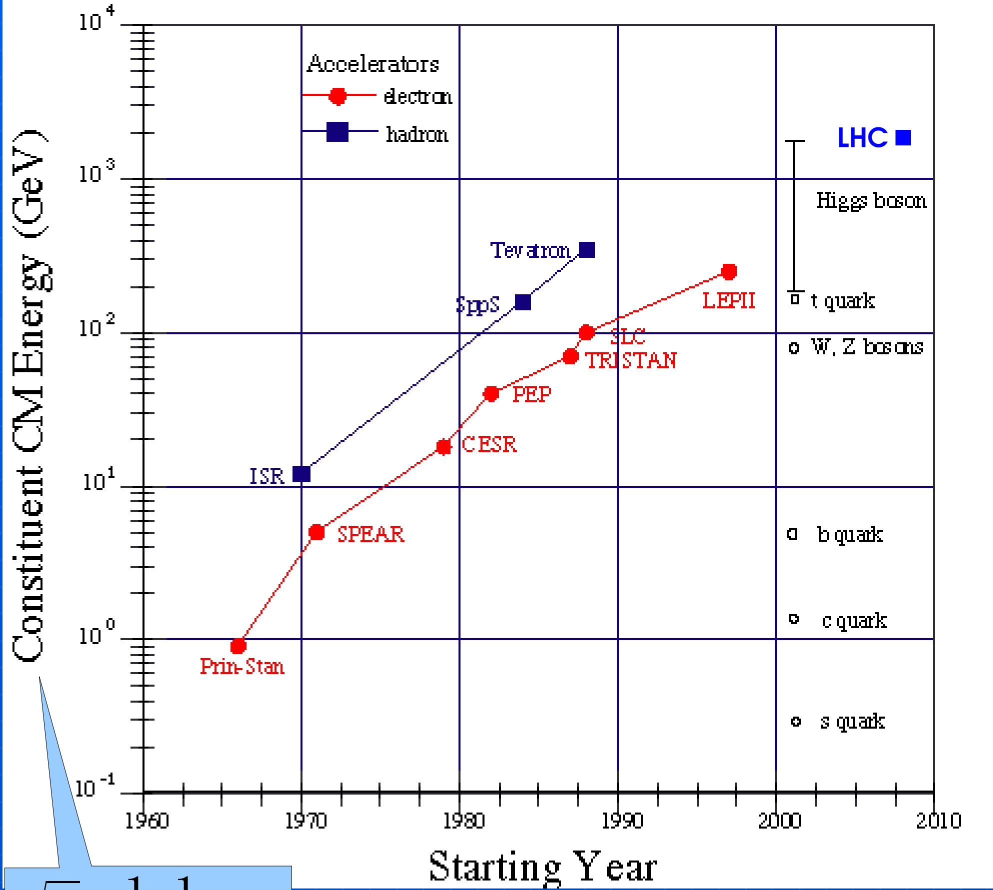


- if parton interaction requires large CMS (valence) quark dominated, e.g. top production at Tevatron
- if parton interaction possible with low CMS gluon-gluon dominated, e.g. top or light Higgs production at LHC

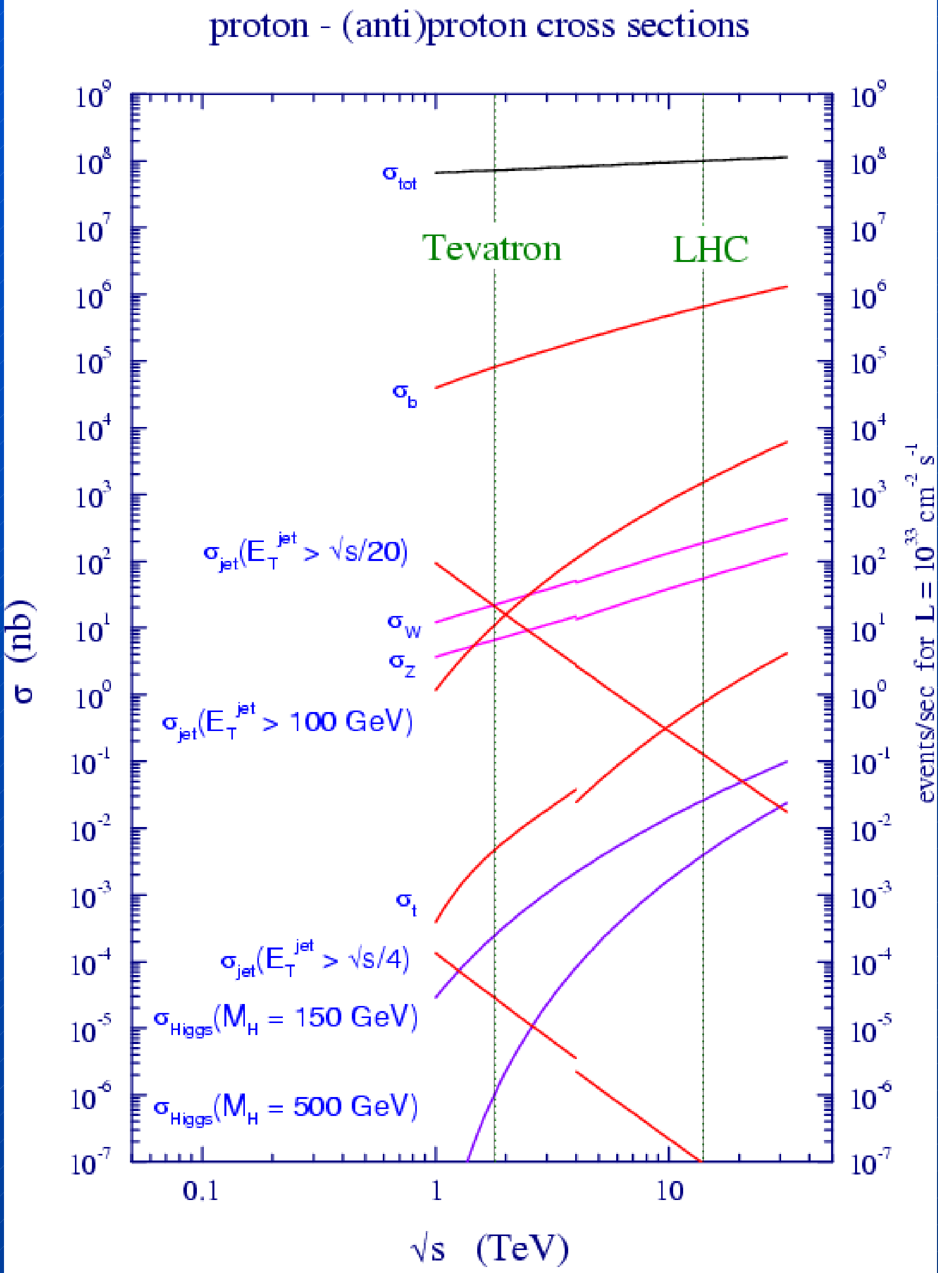
Parton-Parton Interactions

Progress in High Energy Physics depends on advancing the energy frontier :

Livingston plot



$$\sqrt{s'} \approx \frac{1}{2} \cdot \frac{1}{3} \sqrt{s}$$



Kinematics

- **parton-parton center of mass energy**, estimate: $\sqrt{s'} \approx 1/6 * \sqrt{s}$
- partons have longitudinal momentum distribution !
  ⇨ **boost of CMS system** along beam line, a priori unknown

- focus on **transverse quantities**:
  - transverse momentum P_T
  - transverse Energy E_T
  - missing transverse energy MET :

$$MET = \left| -\sum_{vis} \vec{P}_T \right|$$

- **angles** measured in :

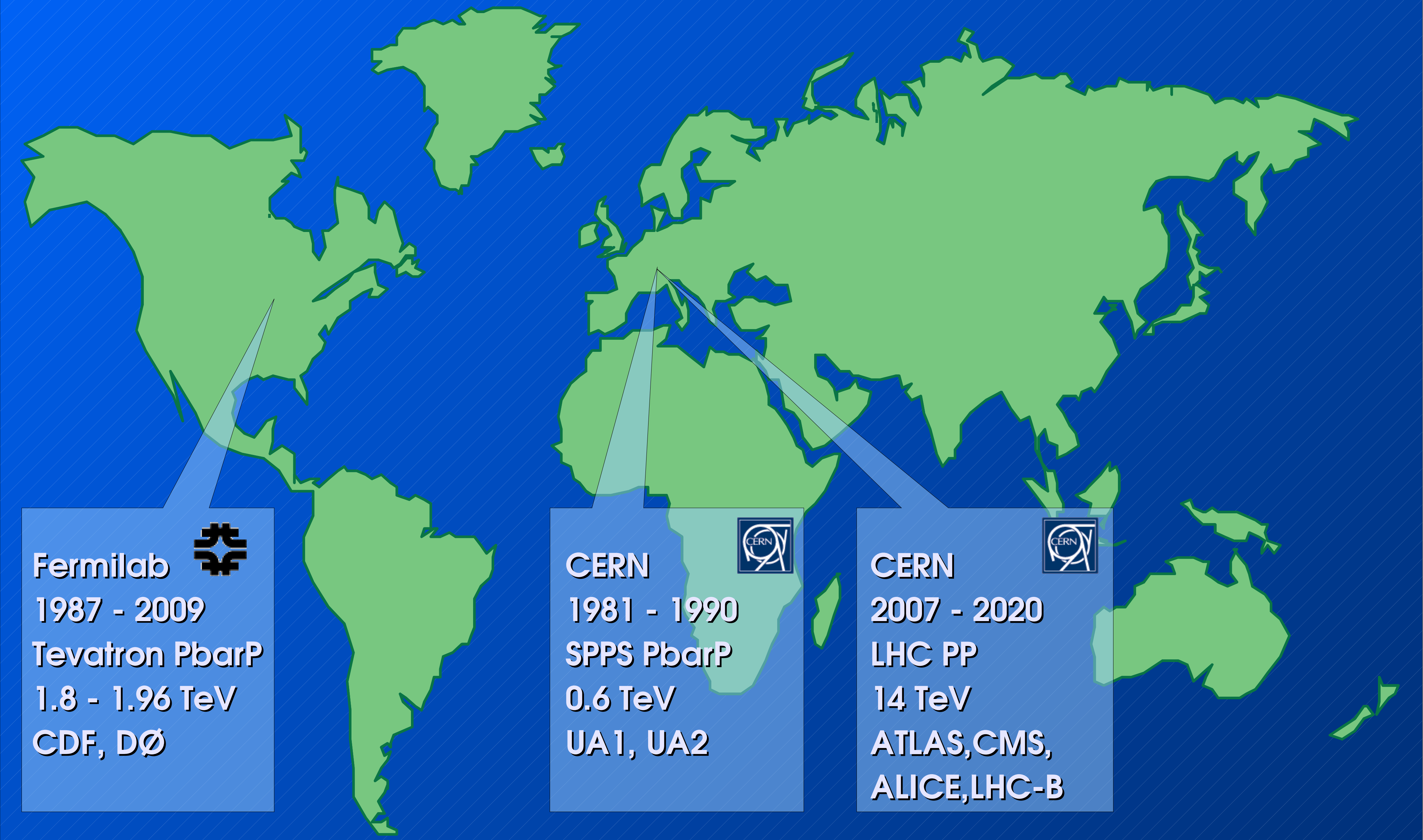
  - **rapidity**: $y = 1/2 \ln \{(E+P_L) / (E-P_L)\}$

    if $m \ll E, P_L$
  - **pseudorapidity**: $\eta = -\ln \tan \theta/2$

rapidity intervals y are Lorentz-invariant

distance measure : $R^2 = (\Delta\phi)^2 + (\Delta\eta)^2$

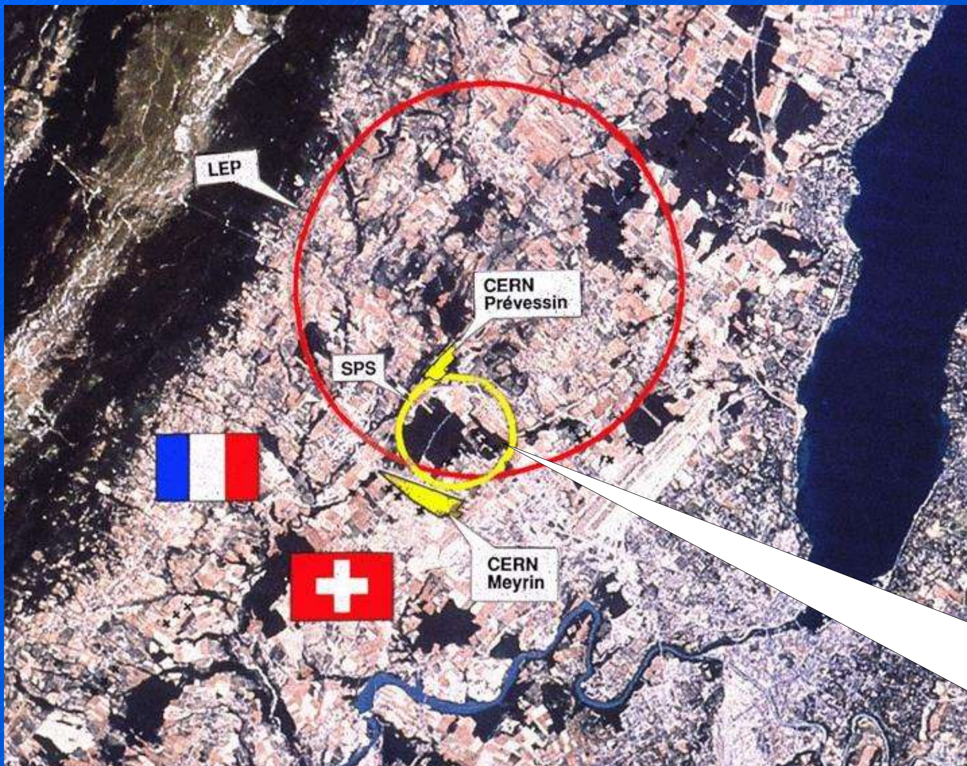
Hadron Colliders

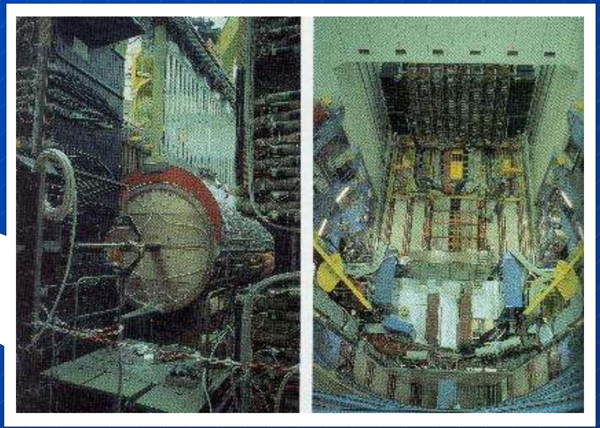


CERN's Proton Synchrotron

proton-synchroton (PS) accelerated first protons to 24 GeV in 1959
... still running today and in the future for the LHC ...

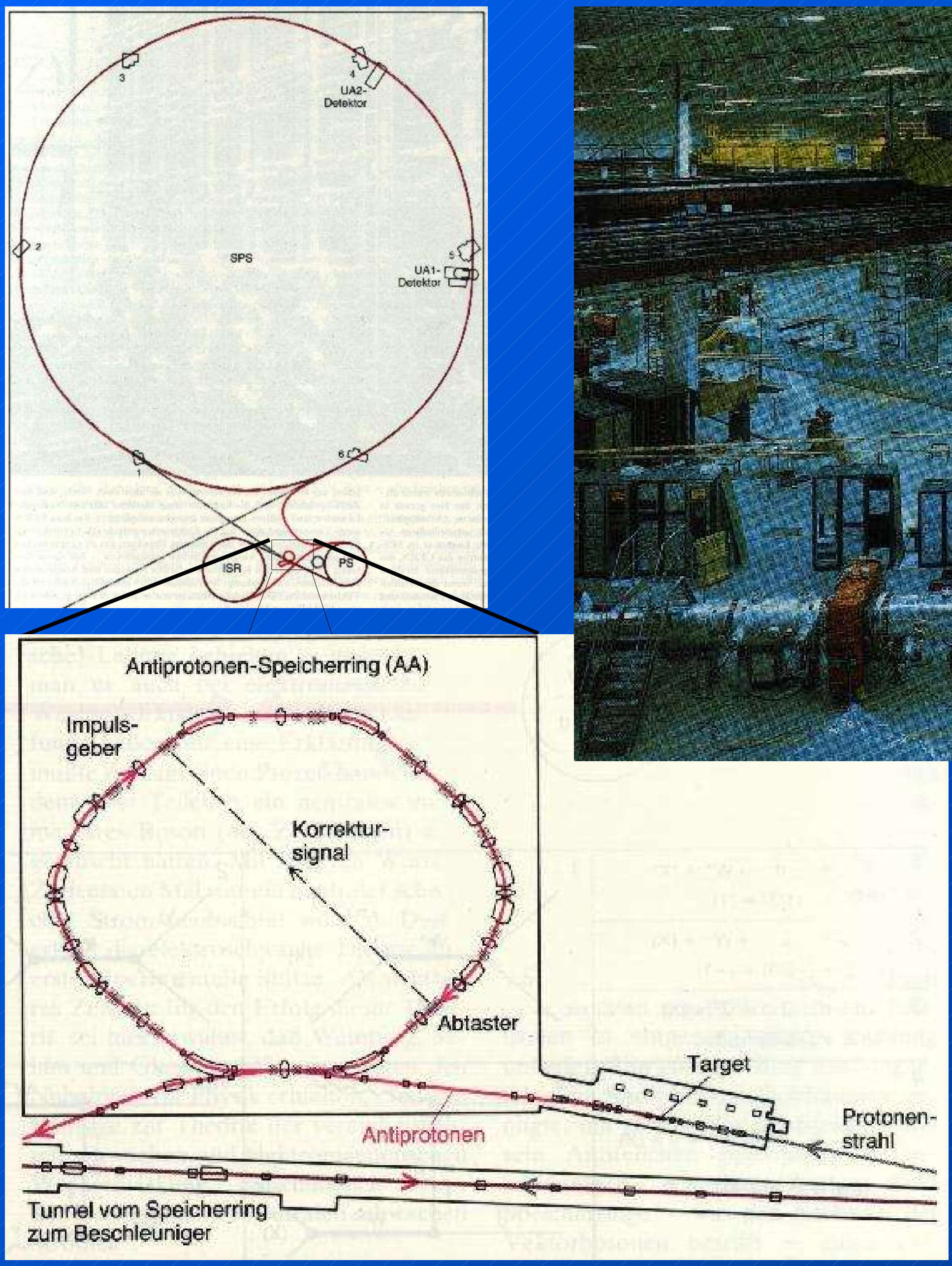
CERN's Super-Proton Synchrotron

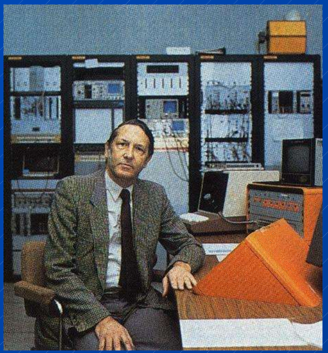




- first p-pbar reactions observed in Juli 1981 ...
  at 270 GeV per beam
- then turned into a proton-antiproton collider (one beampipe)
- collider experiments UA1 and UA2 (Nobel prize for machine and W/Z-discovery)

Stochastic Cooling

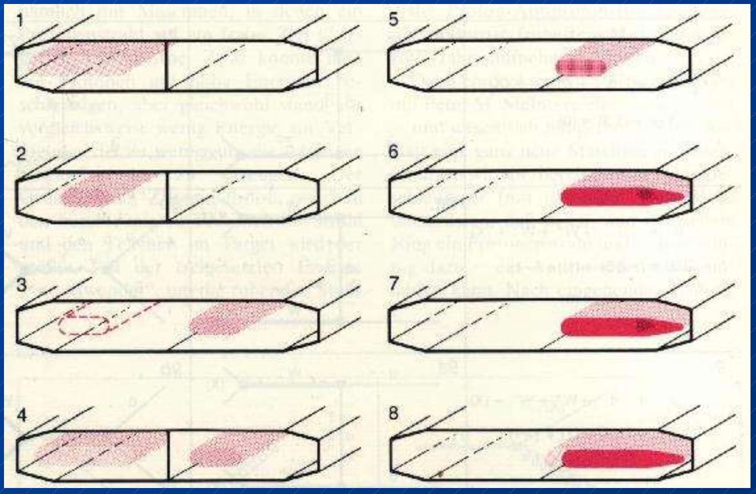




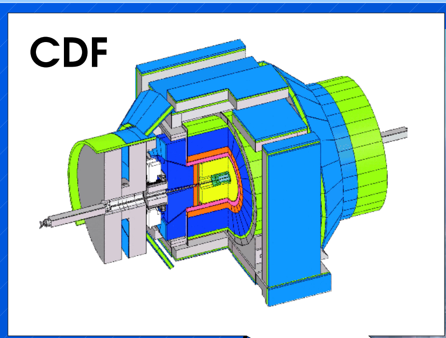
Simon van der Meer

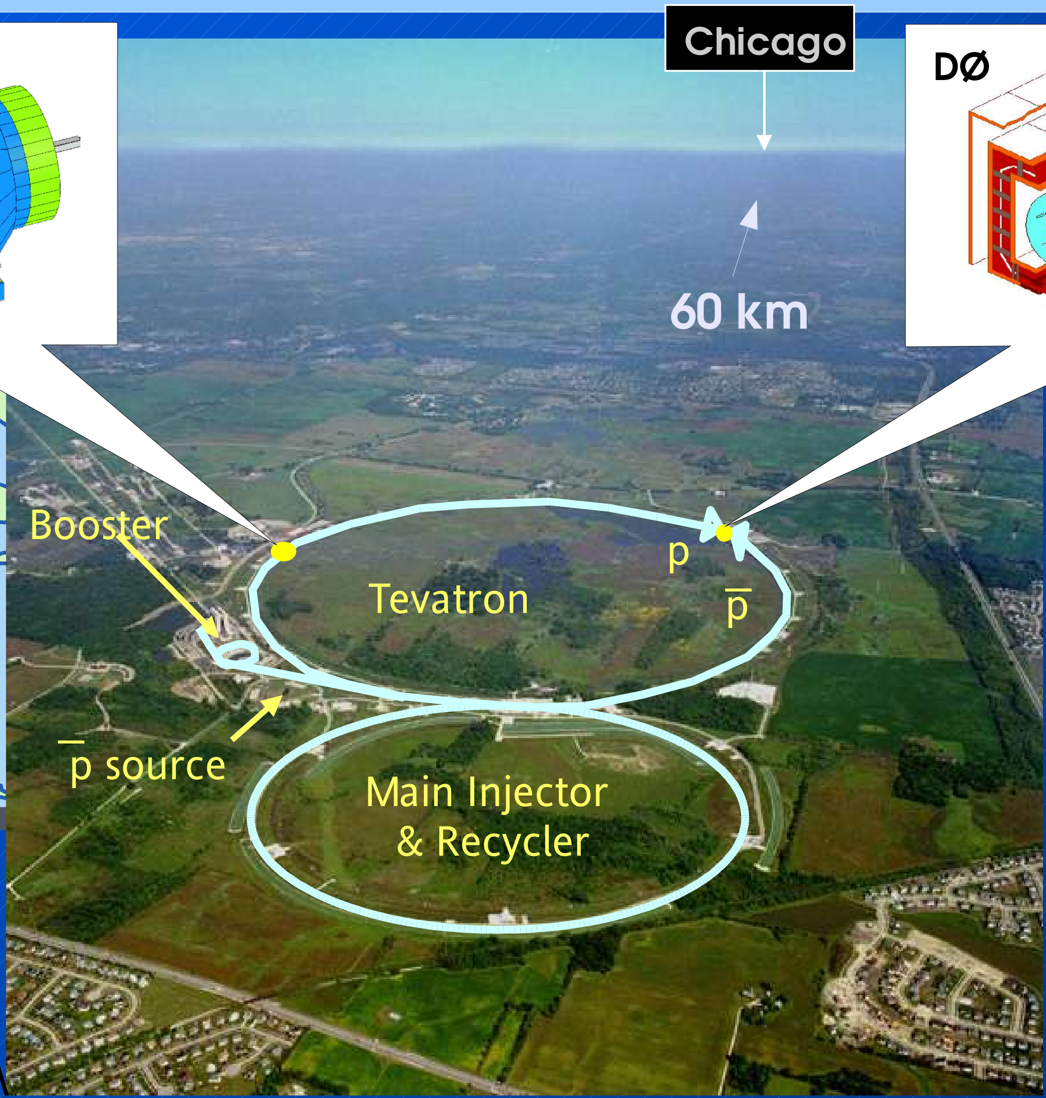
$5 \cdot 10^{11}$ Pbar/beam

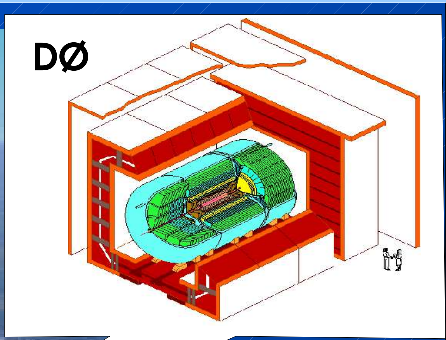
1 Pbar per $3 \cdot 10^{5}$ P



The TEVATRON at Fermilab (1)







$\sqrt{s}$ = 1.96 TeV

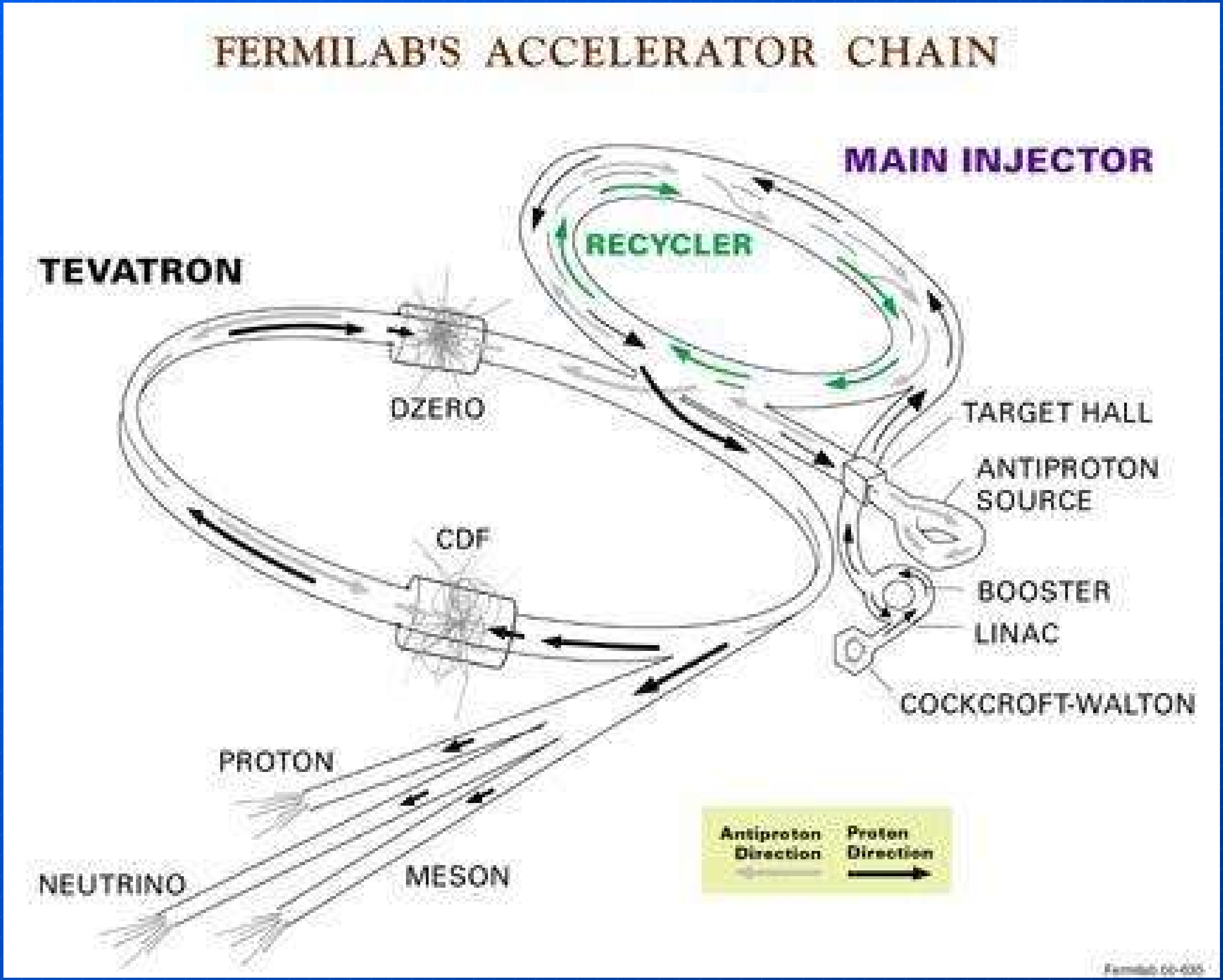
Δt = 396 ns

Run I 1987 (92)-95 L_{int} ~ 125 pb^{-1}

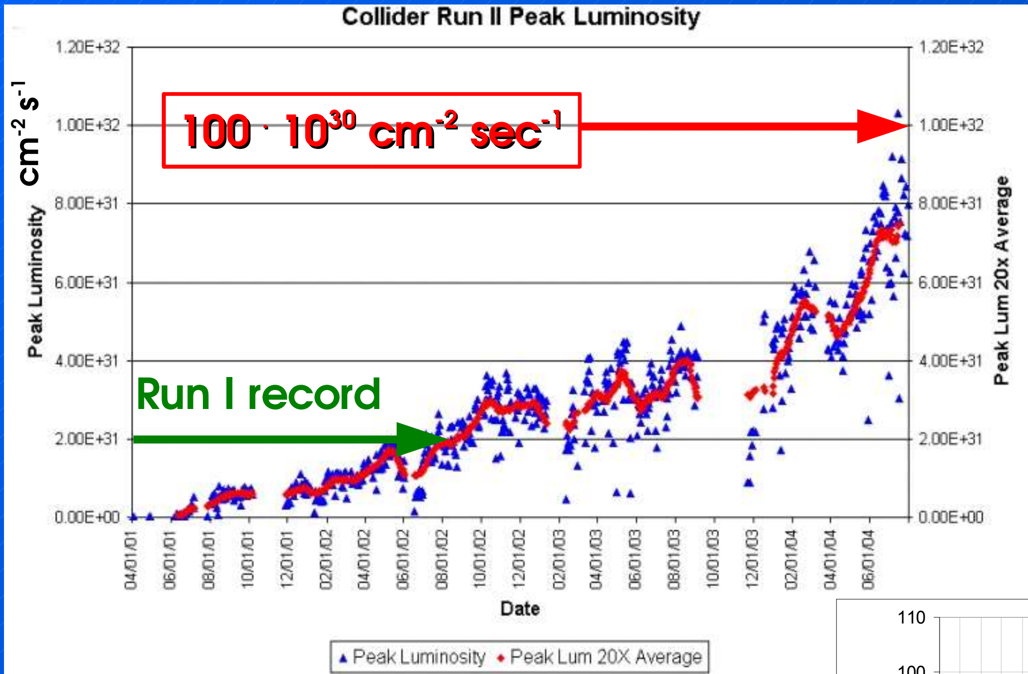
Run II 2001-09(?)

> 40-times larger dataset at increased energy

The TEVATRON at Fermilab (2)

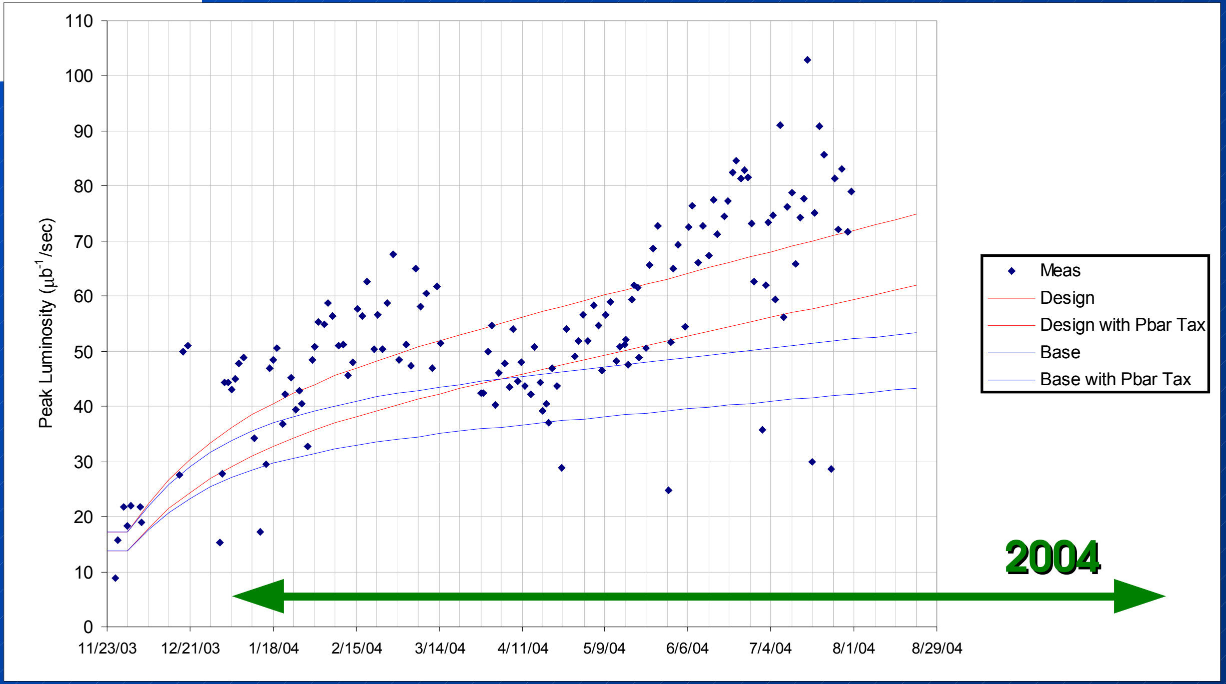


The TEVATRON Performance (1)

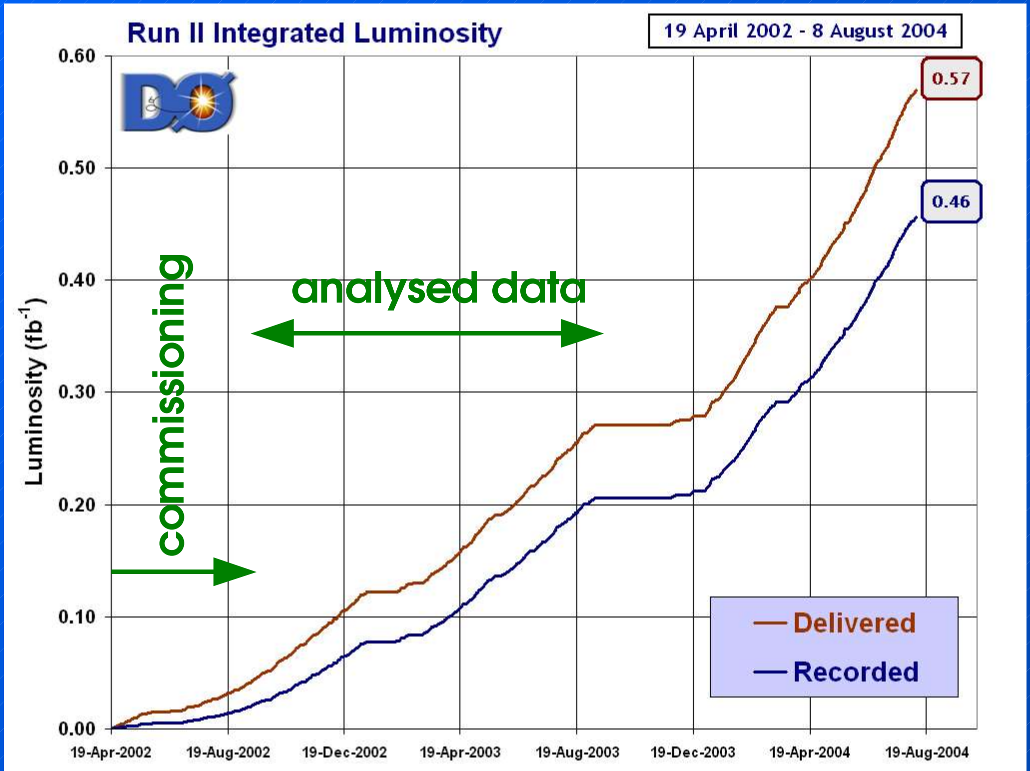


Peak luminosity:

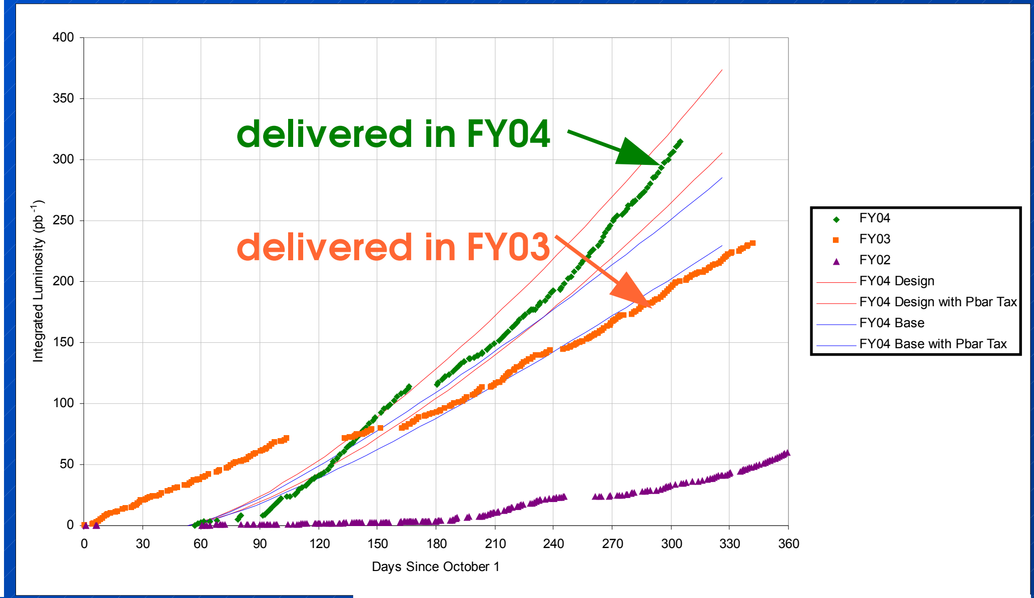
- $1 \cdot 10^{12}$ $\bar{P}$ per beam, \`only' twice SPPS !
- in spring 2002 passed Run I record
- in 2004 often above optimistic design scenario



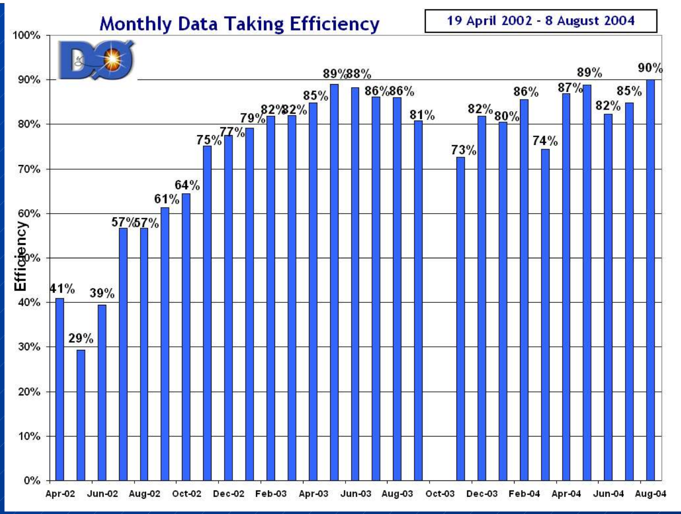
The TEVATRON Performance (2)



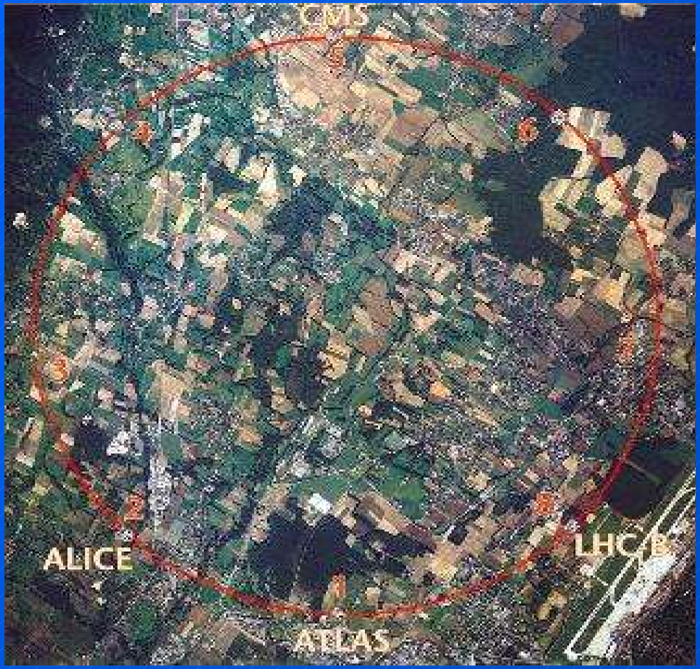
- TEVATRON at upper limit of optimistic scenario



- CDF and DØ ~500 pb^{-1} recorded each
- typical data taking efficiency ~90%
- commissioning is over



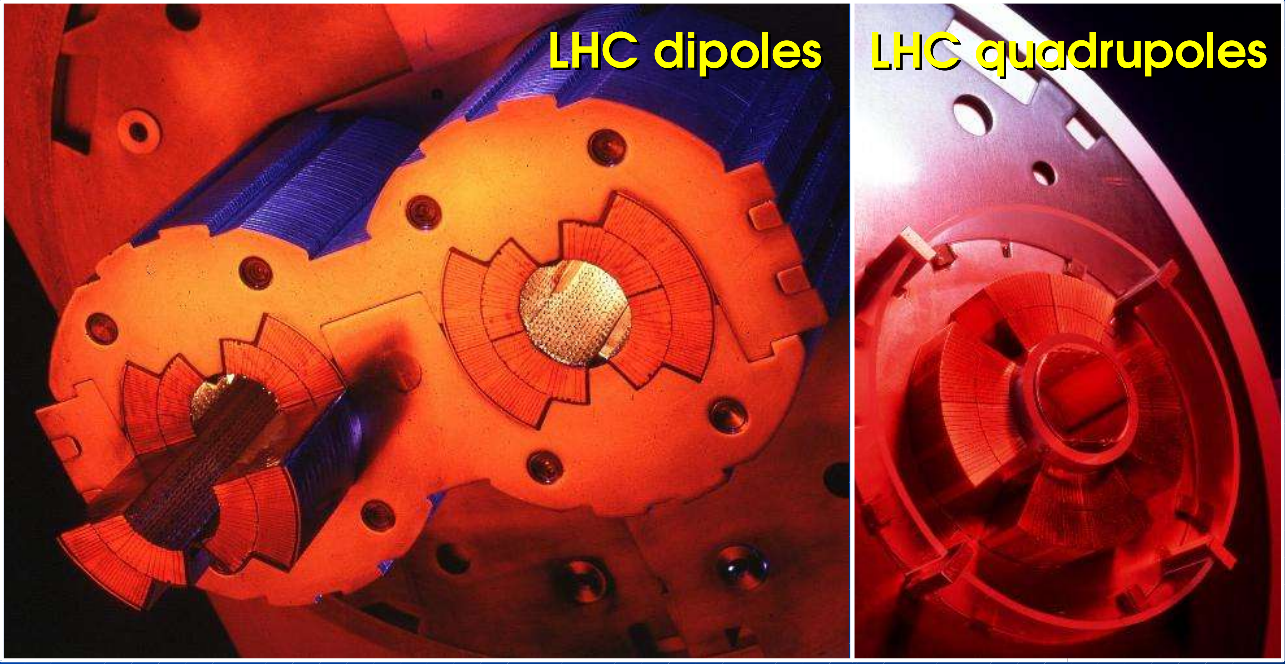
The Large Hadron Collider



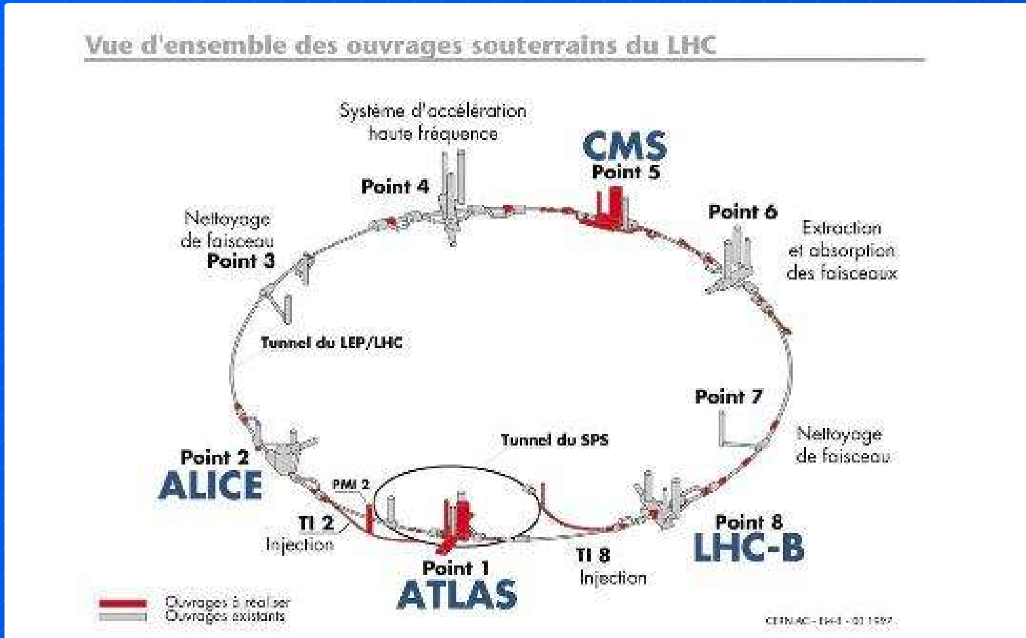
The **L**arge **H**adron **C**ollider:

- proton-proton collisions
- no antiprotons, $3 \cdot 10^{14}$ P
- at 14 TeV center of mass energy
- 40 Mio. collisions per second
- first collisions in 2007
- 4 experiments:
  ATLAS, CMS, ALICE, LHC-B

- two separate beampipes
- two accelerators



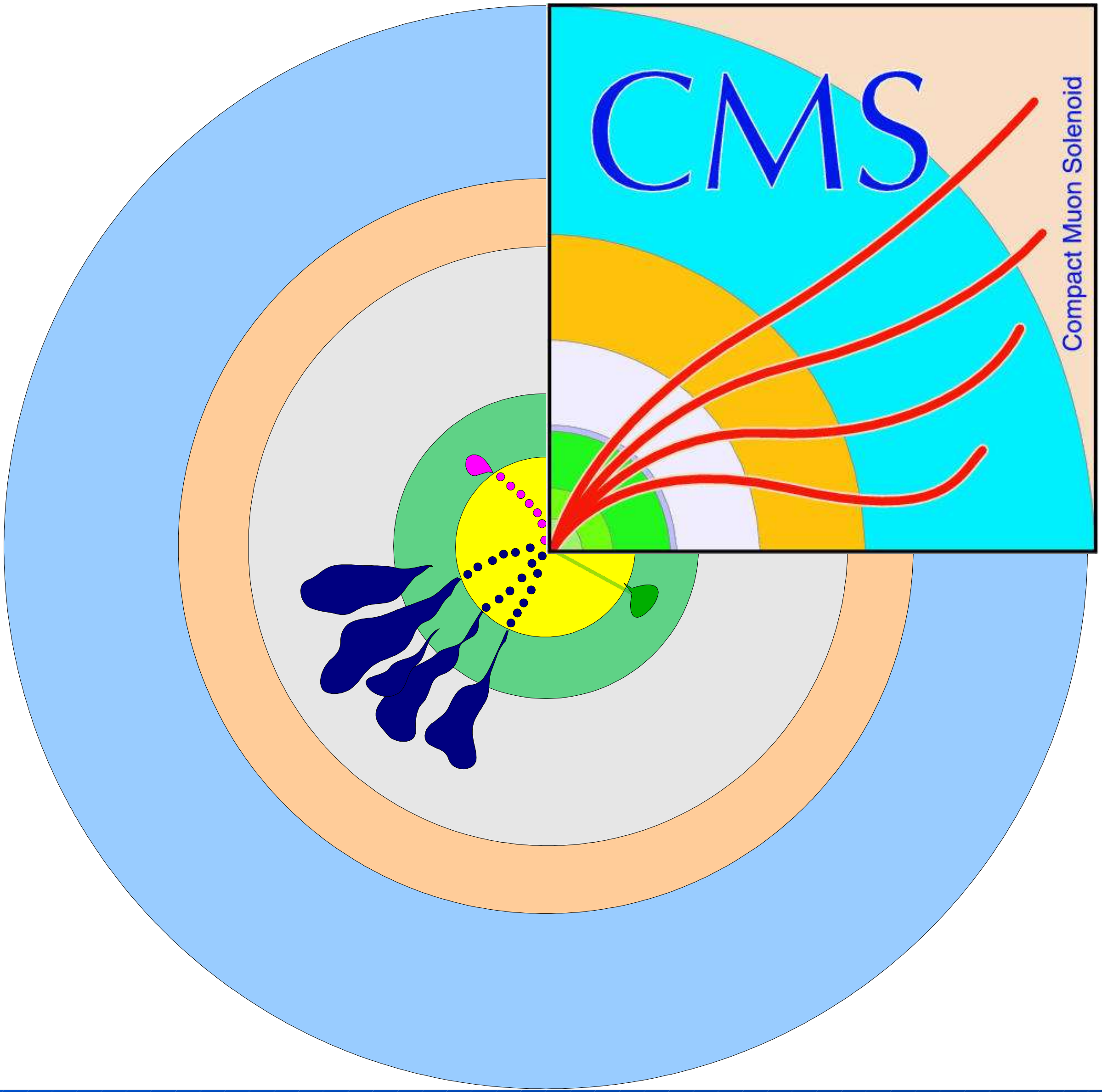
The Large Hadron Collider



Collider Comparison

	SPPS	*Tevatron*	*LHC*
Physics start	1981	1987	2007
Particles	Pbar-p	Pbar-p	p-p
cm energy (TeV)	0.62	1.96	14
Lumi (10^{30} cm^{-2} s^{-1})	6	50-100	0.1 - 1.0 x 10^4
Lumi (fb^{-1} $year^{-1}$)	0.05	0.5	100
Bunch spacing (ns)	3800	396	25
Particles per bunch (10^{10})	P: 15, Pbar: 8	P: 24, Pbar: 3	P: 11.5
Max.no Pbar in accumulator	1.2 x 10^{12}	2.6 x 10^{12}	-
Bunches	6 + 6	36 + 36	2835 + 2835
Circumference (km)	6.9	6.28	26.7
Nr. dipoles	232	774	1232 (main dipoles)
Magnet type	warm	cold, warm iron	cold, cold iron
Peak magnetic field (T)	1.4	4.4	8.3

Particle Identification



Muon (high energy)

(medium ene.)

(low energy)

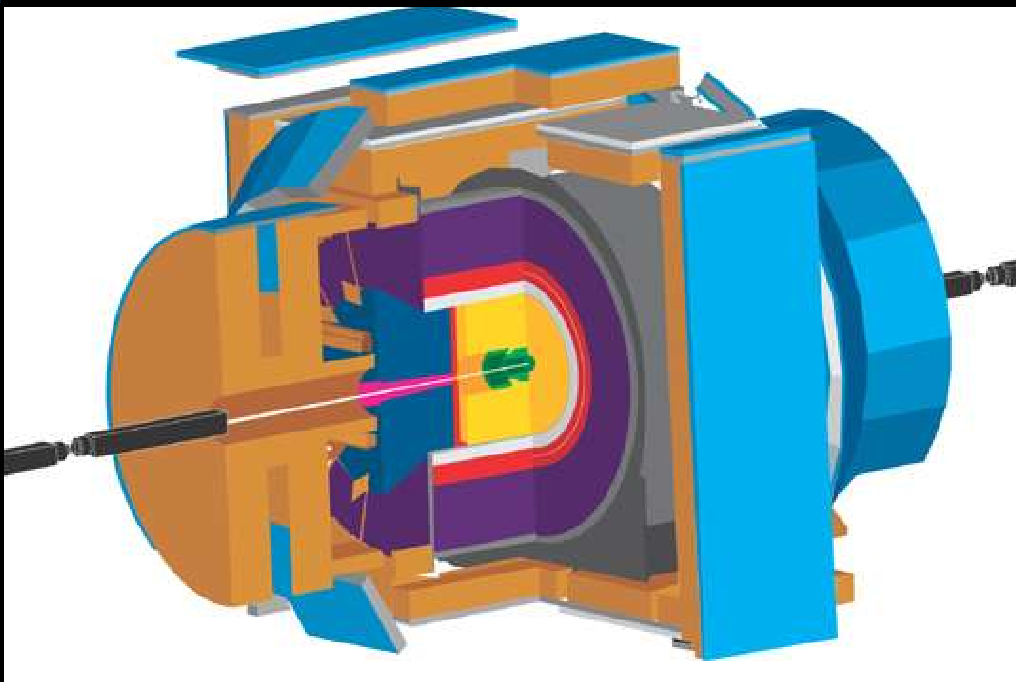
Photon

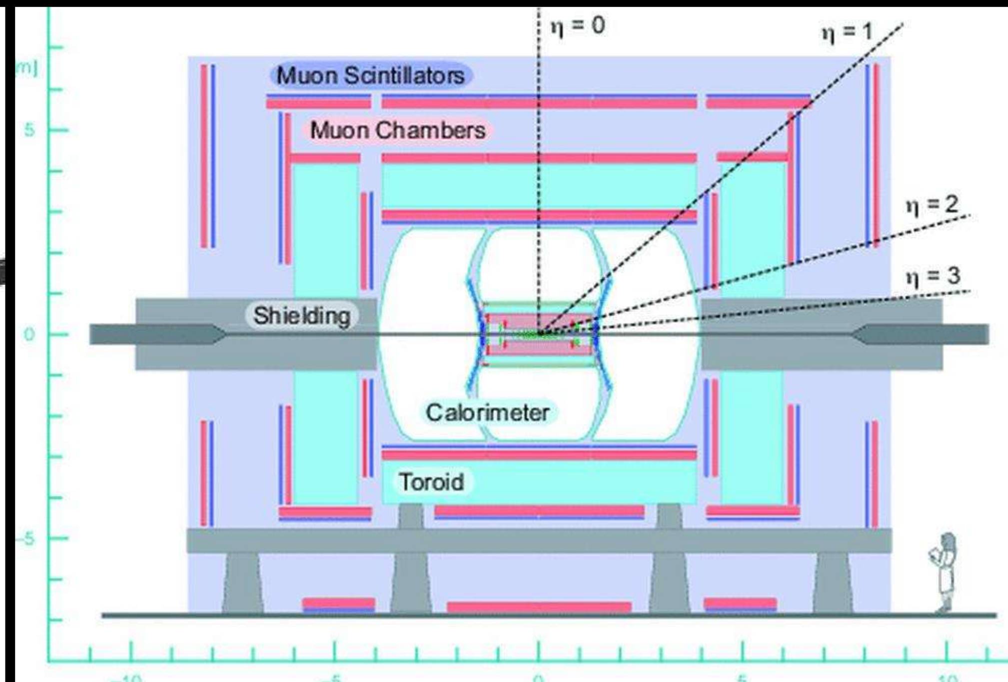
Electron

Quark → Jet

- Tracking detector
- Electron calorimeter
- Hadron calorimeter
- Magnet coil
- Muon chambers

The CDF and DØ Experiment





- new bigger silicon, new drift chamber, TOF
- Upgraded calorimeter and muon system
- Upgraded DAQ/trigger
- Displaced track trigger
- ~750 physicists

- new silicon and fibre tracker
- new ~2 T solenoid
- upgraded muon system
- upgraded (track) trigger/DAQ
- 19 countries, 83 institutes, 664 physicists

The DØ Experiment

compensating
Uranium/Liquid Argon calorimeter

Calorimeter:

Transverse Segmentation

$\Delta\eta \times \Delta\varphi = 0.1 \times 0.1$

(EM3)

$\Delta\eta \times \Delta\varphi = 0.1 \times 0.1$

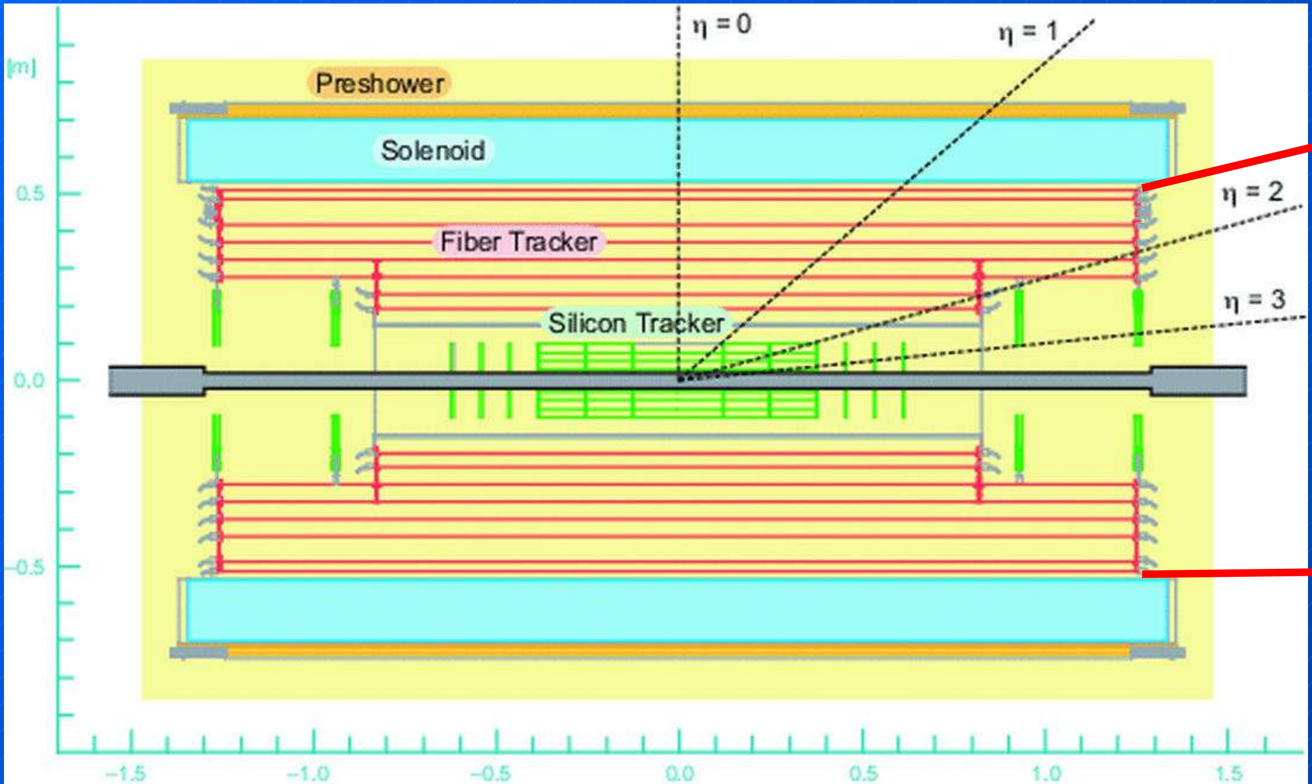
L1/L2 fast trigger readout in
0.2 x 0.2 towers

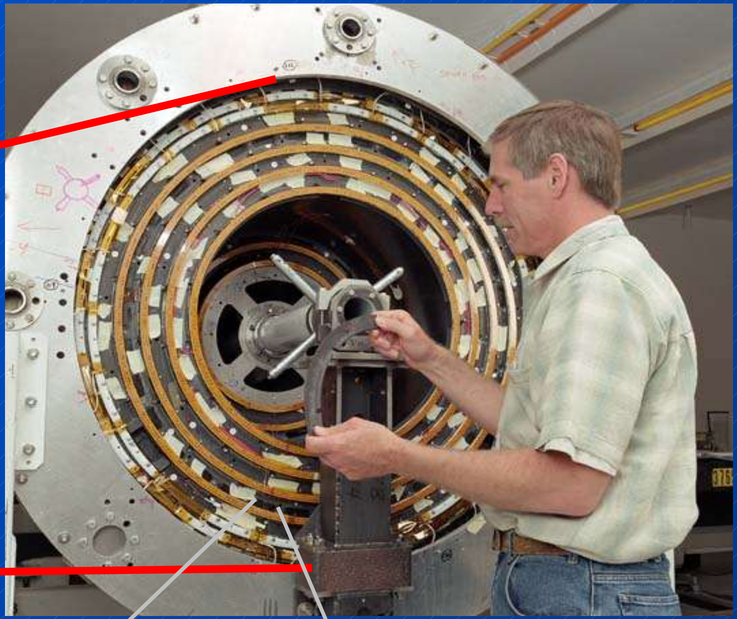
resolutions:

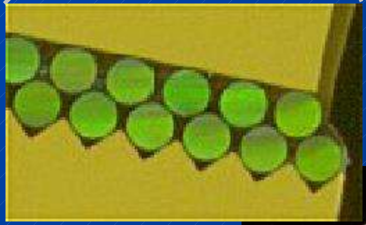
EM: $\sigma_E/E = 15\% / \mathrm{sqrt}(E)$

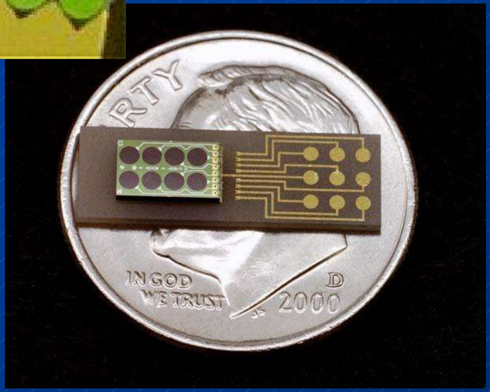
HAD: $\sigma_E/E = 50\% / \mathrm{sqrt}(E)$

The DØ Fiber Tracker



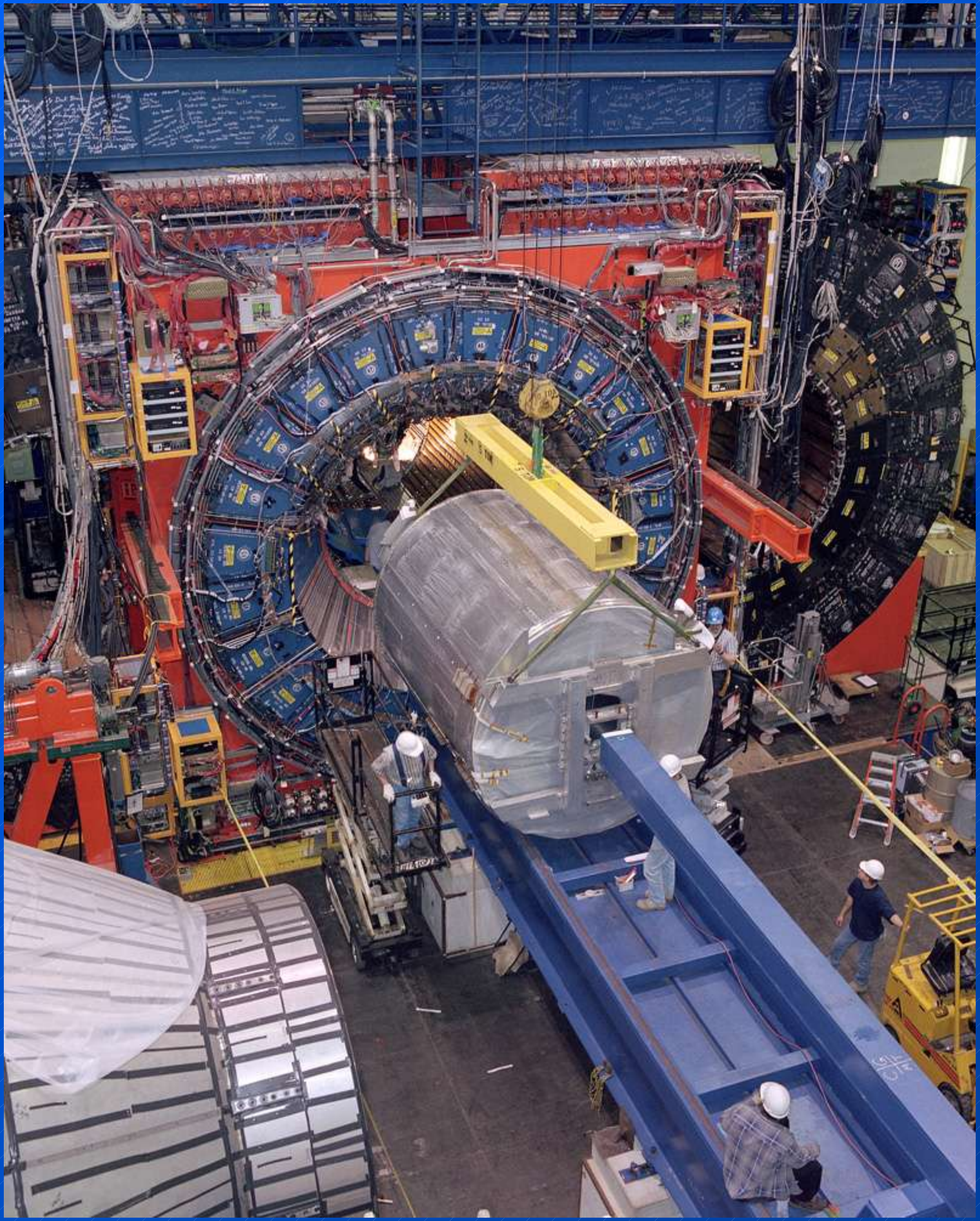






- Central Fiber Tracker:
  - 77k fibers in eight barrels, 800 μm diameter fibers
  - 3° stereo layers in each barrel
  - VLPC readout, ~7 photo-electrons/track at $\eta = 0$

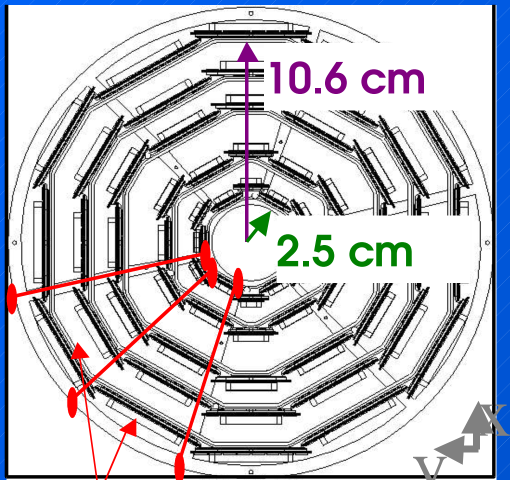
The CDF Experiment

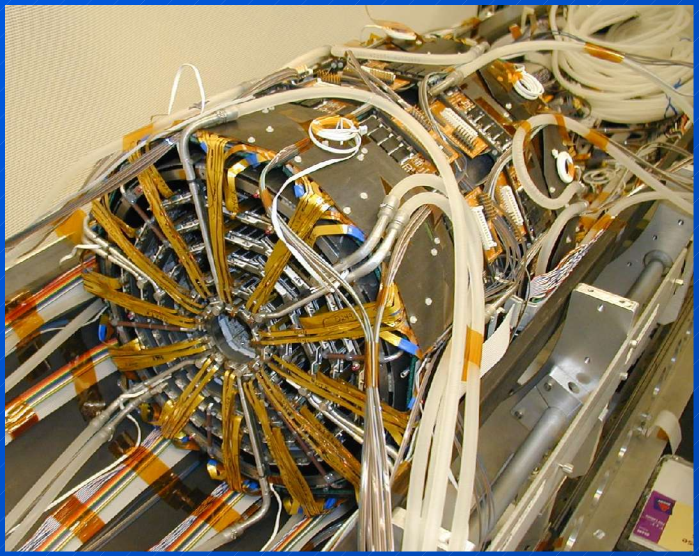


drift chamber (COT)
radius 40 - 137 cm, |z| < 160 cm
60 axial, 24 stereo hits (3°)

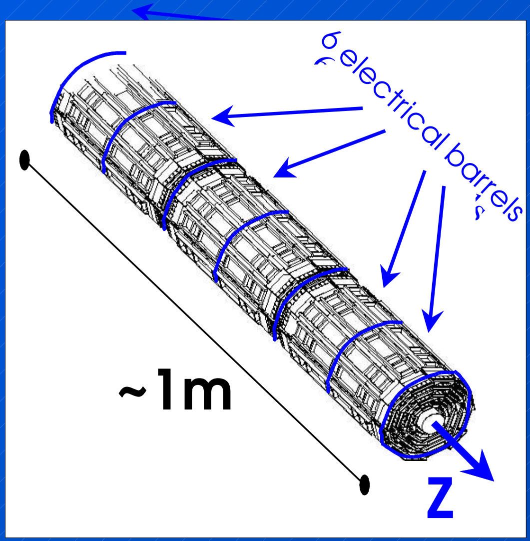
scintillator based calorimeter
EM: $\sigma_E/E = 13.5\% / \text{sqrt}(E)$
HAD: $\sigma_E/E = 50\% / \text{sqrt}(E)$

Baseline Upgrade: SVX II





- 7-8 silicon layers
- 722, 432 channels
- $r\varphi$, rz views
- z^{max} = 45 cm, η^{max} =3
- 1.3 < r < 30 cm

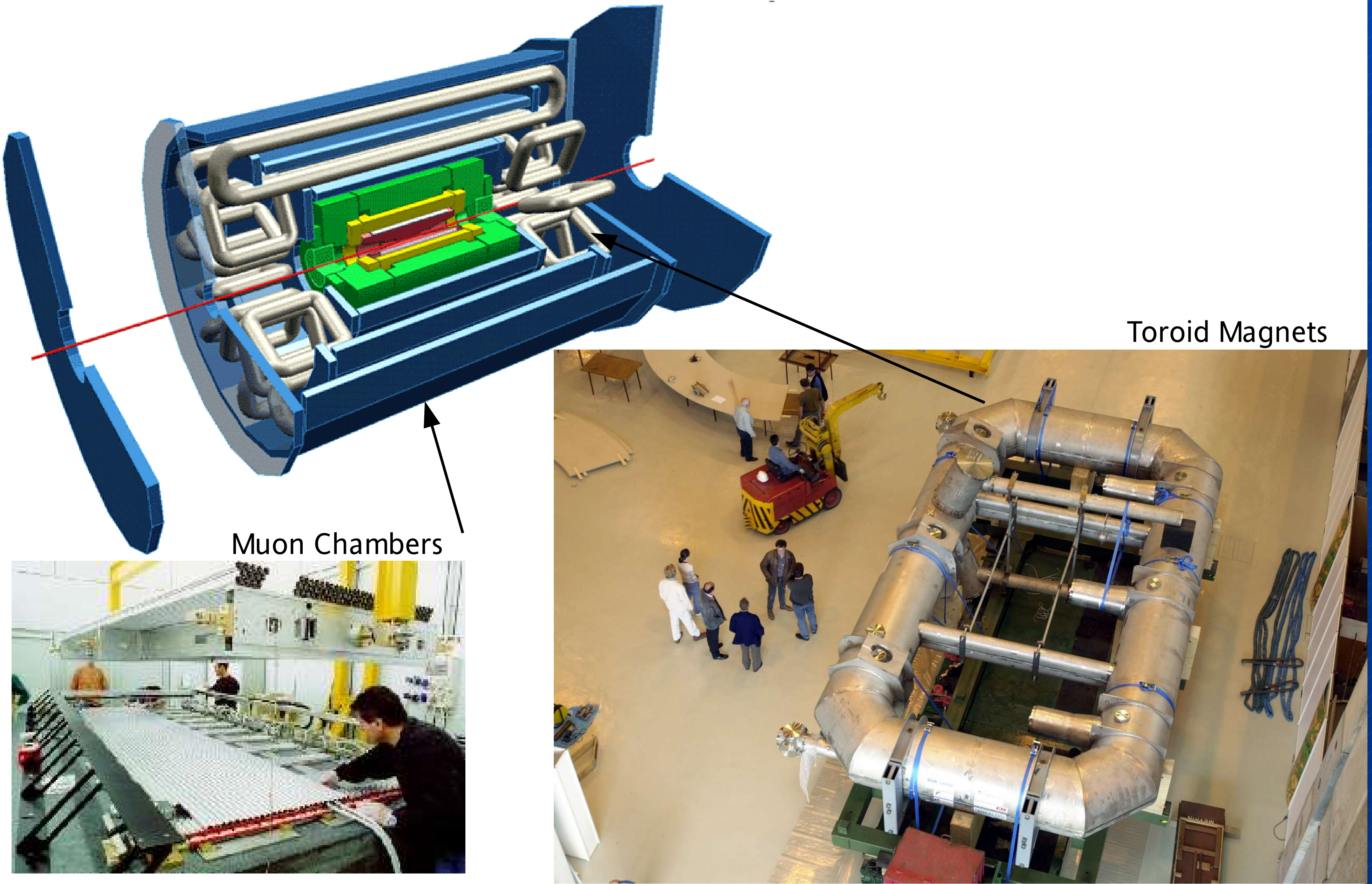


- 5 double-sided layers
  - 5 axial, 3 x 90°, 2 x 1.2°
- Tight alignment tolerances
  - for displaced track trigger
- Highly symmetric
  - 12-fold in φ
  - 6-fold in z

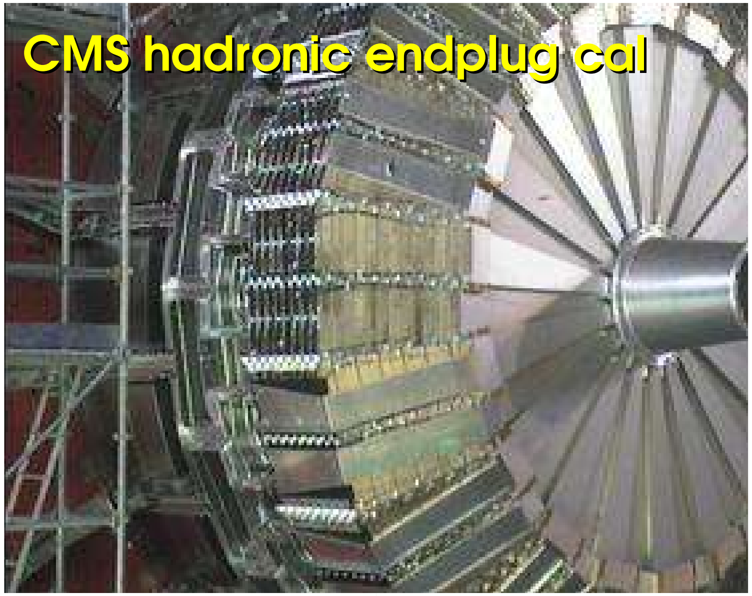
Run II physics goals:
- properties of top quark
- precision Electroweak
- CKM, B_s mixing
- search for new phenomena
- tests of QCD

heavy flavour tagging:
- B reconstruction efficiency
- increased forward acceptance
- improved σ_{d0} (~20 μm)

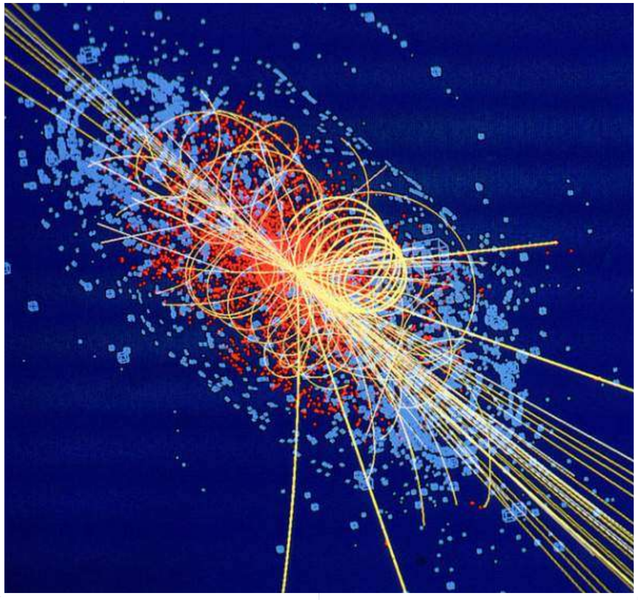
The ATLAS Experiment



The CMS Experiment



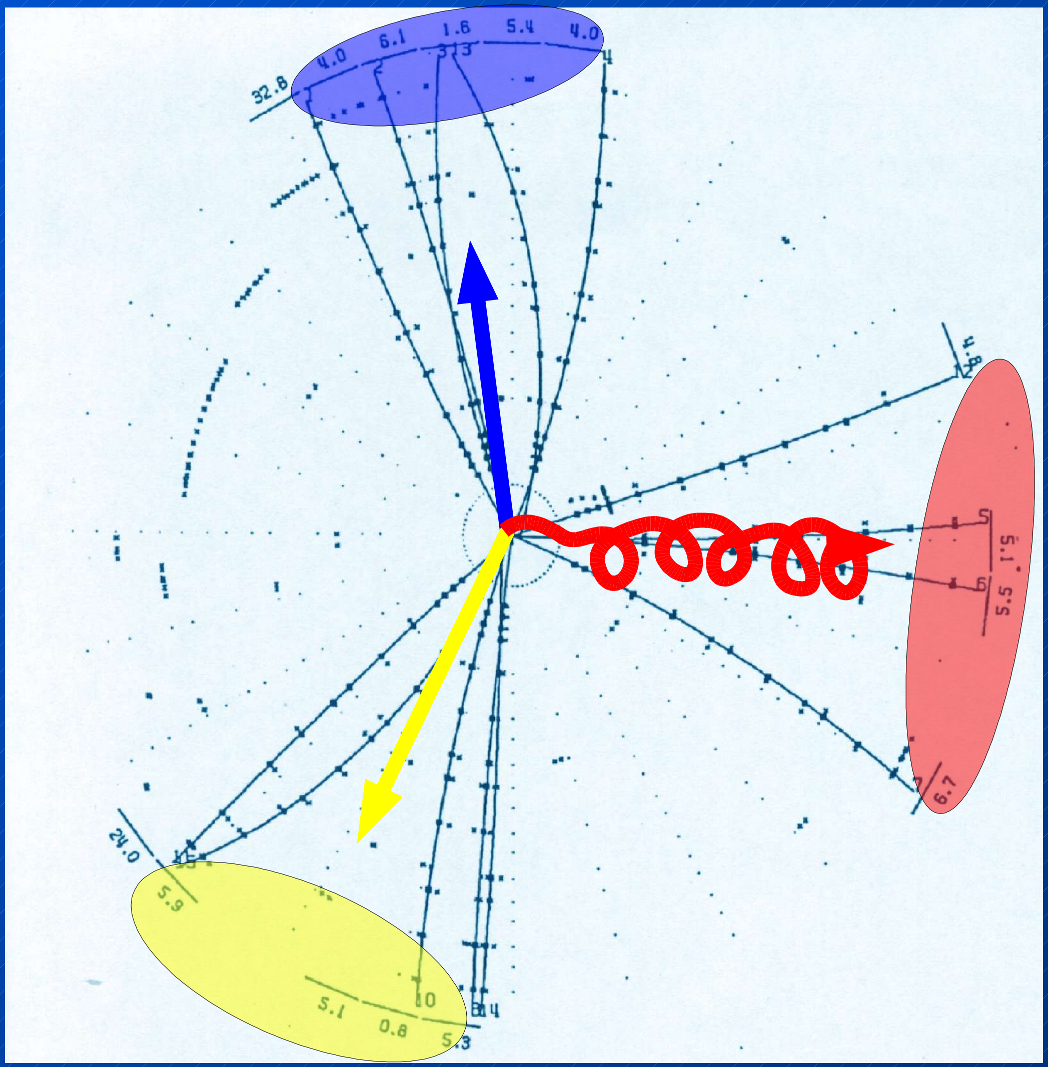
- assembly advanced
- lowering into cavern in 2005
- software and simulation being prepared as well ...



QCD Studies

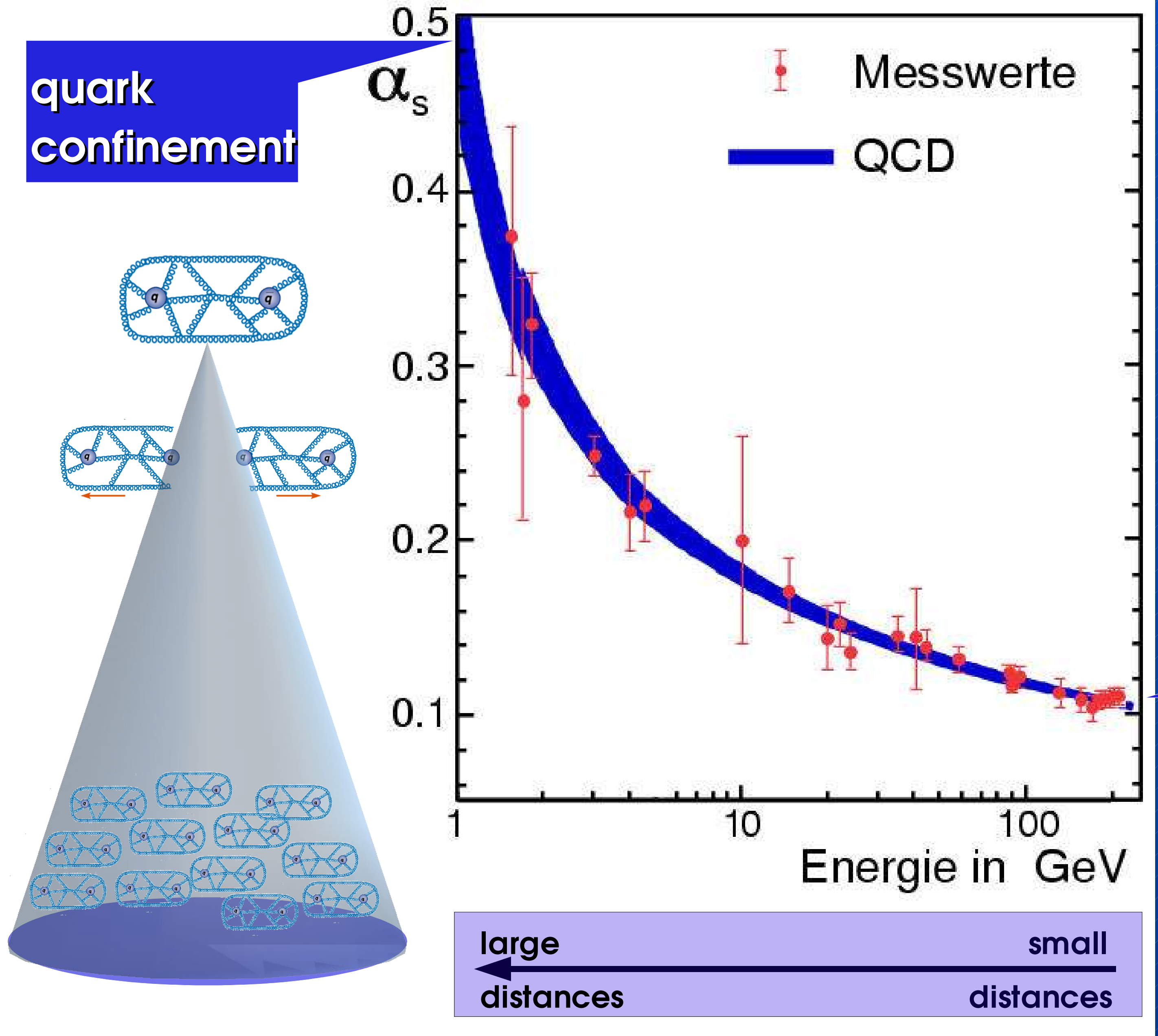
Quantum Chromo Dynamics (QCD)

- Theory of Strong Interactions
- Acts on all quarks
- Mediated via Gluons
  ... discovered in 70ties at PETRA / DESY ...



Jets

Quantum Chromo Dynamics (QCD)



asymptotic freedom

quasi-free quarks

QCD Overview

- Rapidity Gaps/Diffractive/Elastic Physics
- PDF's: double parton interactions, W charge asymmetry
- non-perturbative QCD: jet shapes, W/Z boson p_T spectra
- perturbative QCD, particle cross section, W/Z bosons, prompt photons, jets, top, b/c quarks
- Perturbative QCD with W/Z bosons
- Perturbative QCD with jets: α_s, jet topologies
- ...

... here only a small selection ...

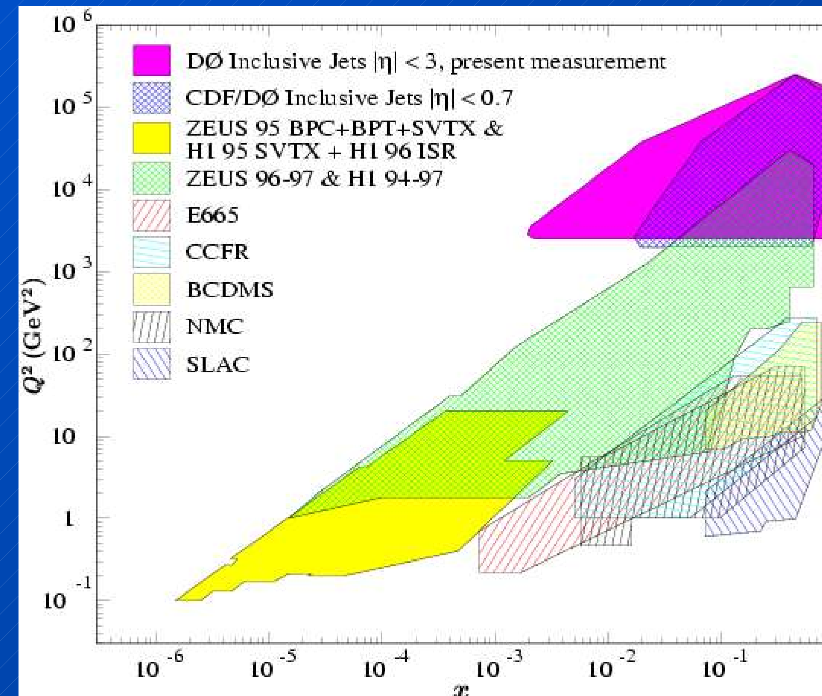
- Inclusive jet cross section
- Dijet mass spectra
- uncorrected dijet $\Delta\varphi$
- jet multiplicities in W+jet events

Inclusive Jet Cross Section (1)

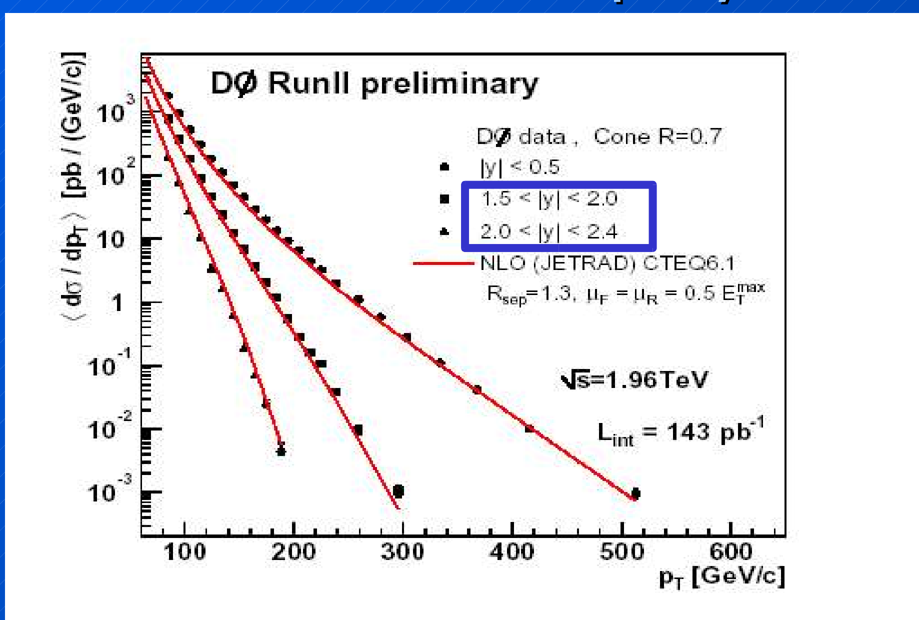
High E_T jets probe large x

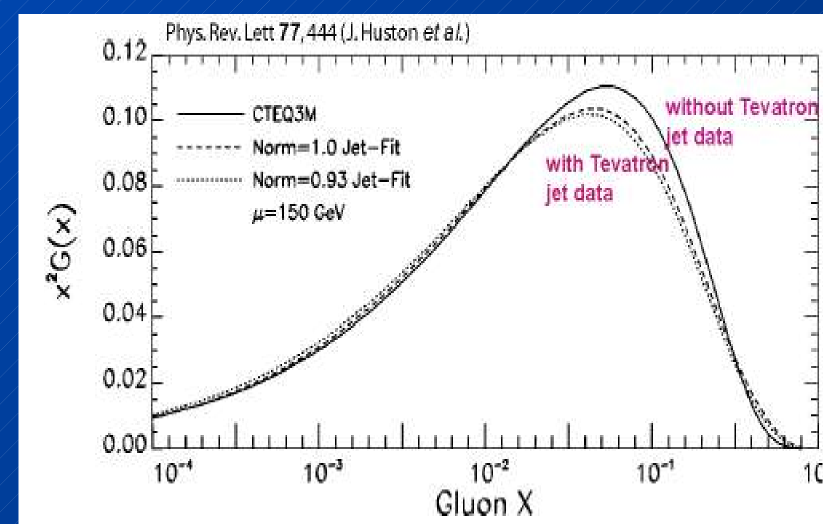
PDF's, especially gluon

PDF: Run II has extended reach in jet E_T

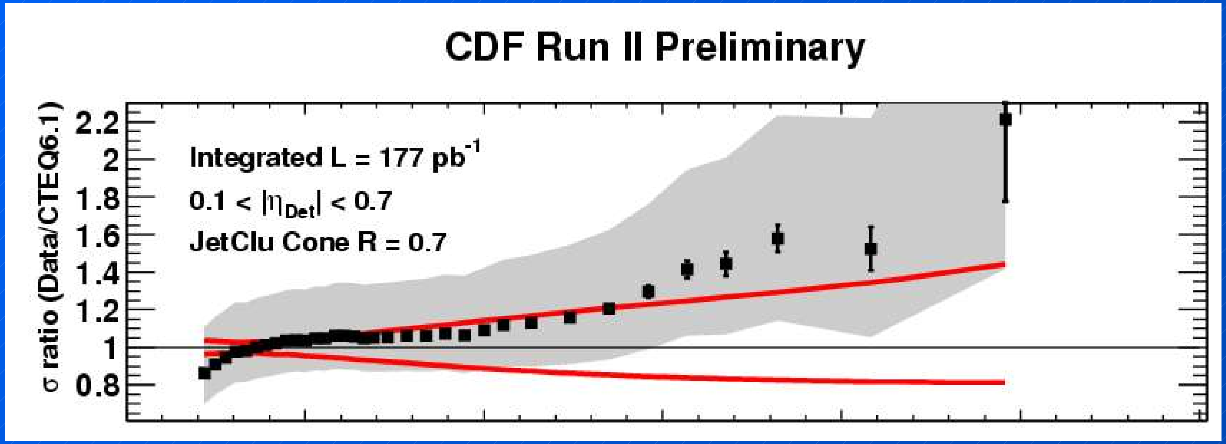


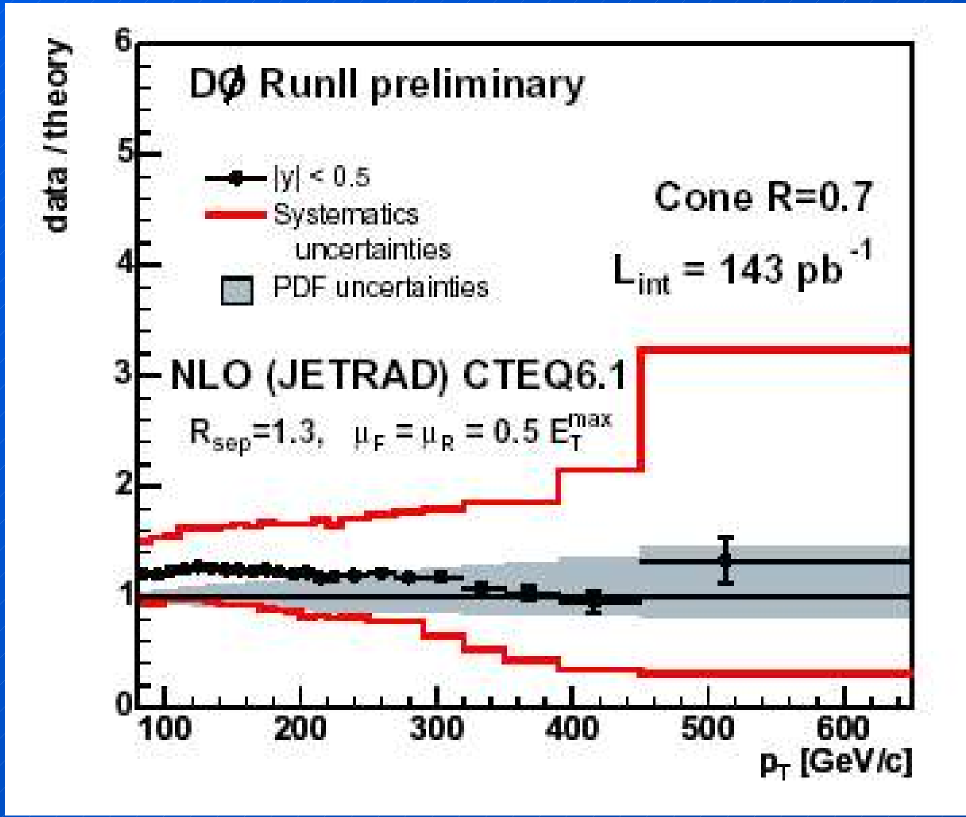
important information is in
the cross section versus rapidity





Inclusive Jet Cross Section (2)





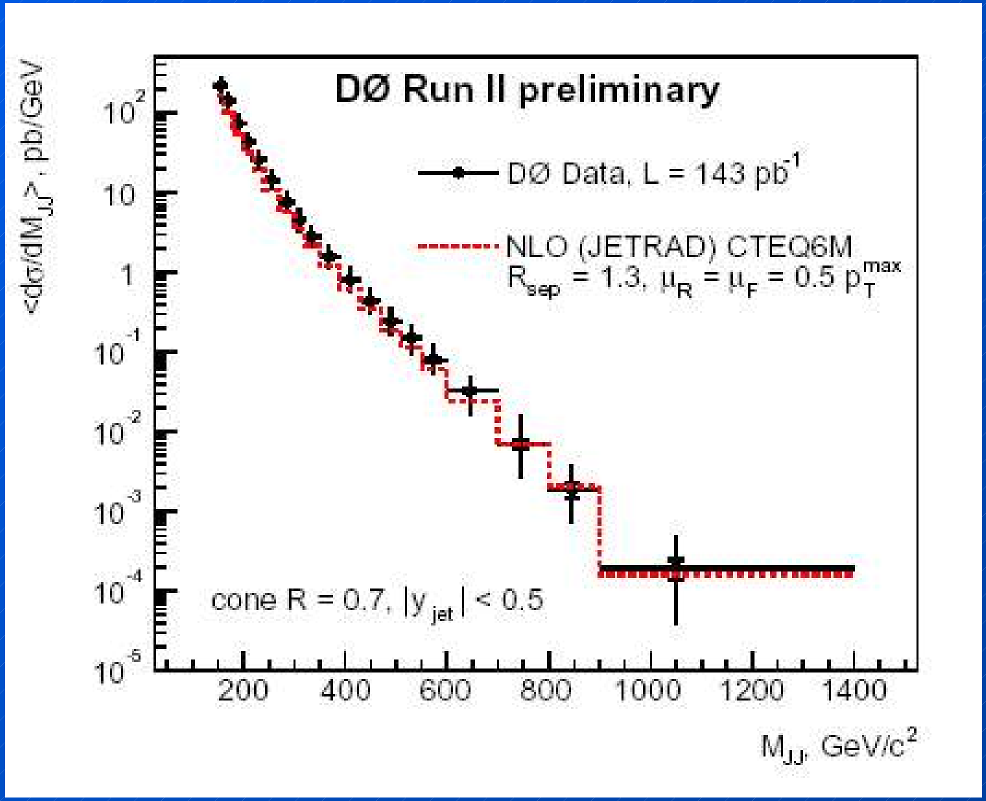
Uncertainties dominated by jet energy scale. Jet energy is systematics dominated in central. Needs data and time to study and understand more precisely.

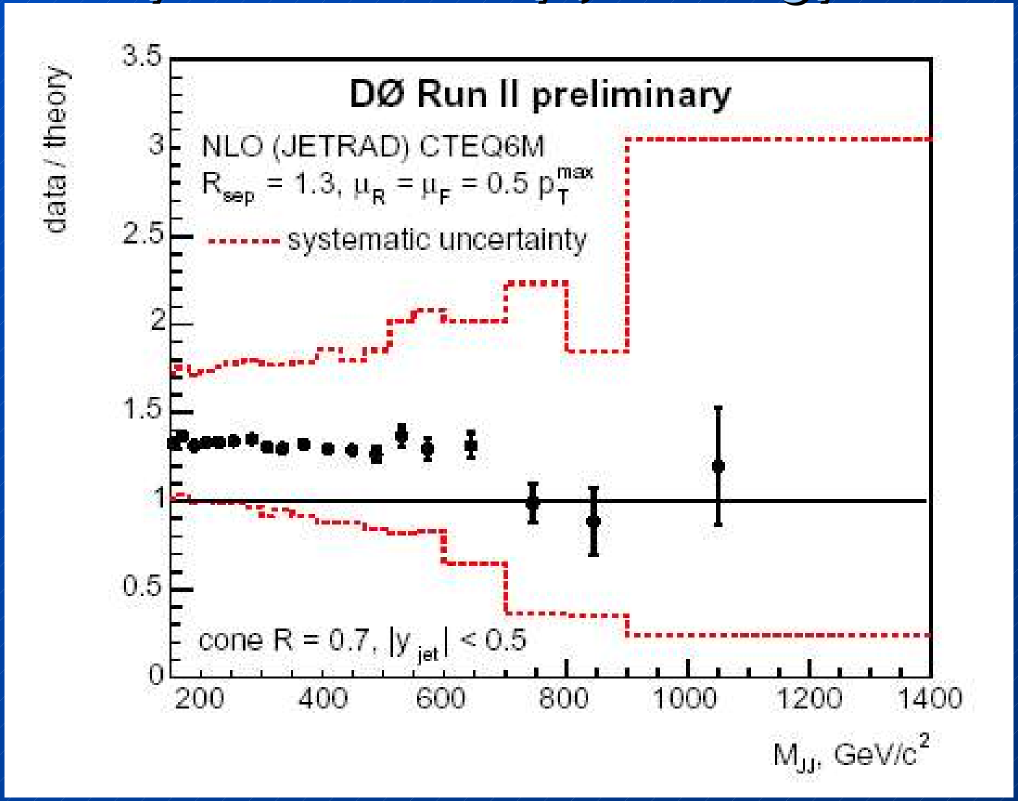
Uncertainty for central jets in central region (GeV)		
	Run I	Run II
CDF	17	30
DØ	14	14

Dijet Cross Section

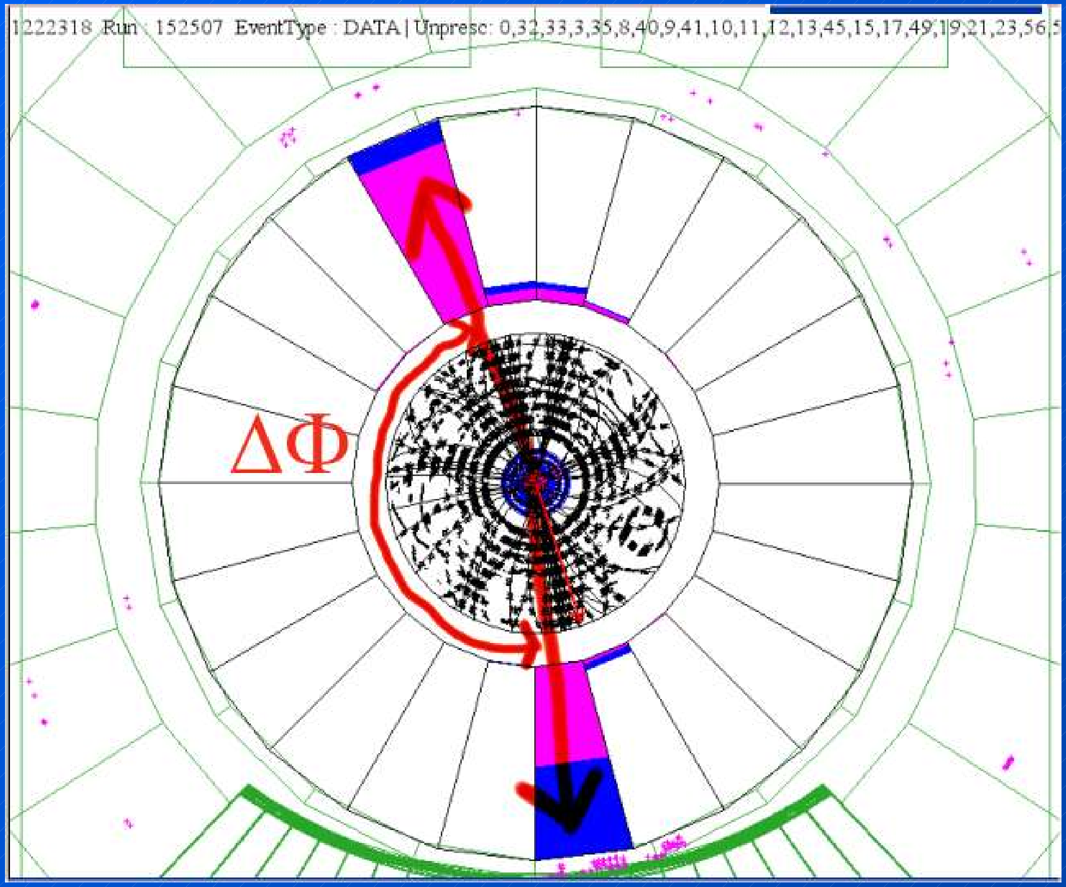
Often used to search for new resonances ...

Uncertainty dominated by jet energy scale





Φ Decorrelation

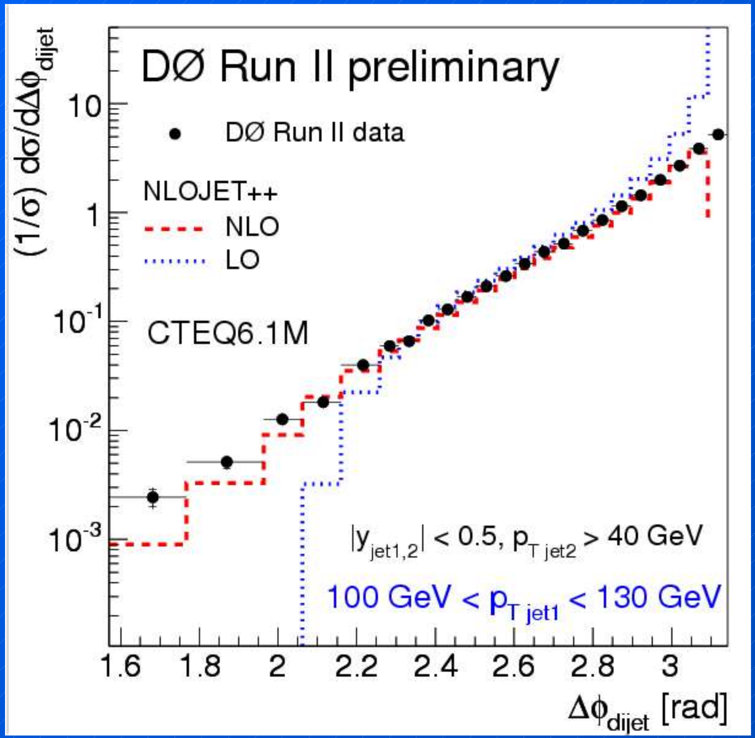


high mass event:
M_{JJ} = 1364 GeV, ET = 633, 666 GeV

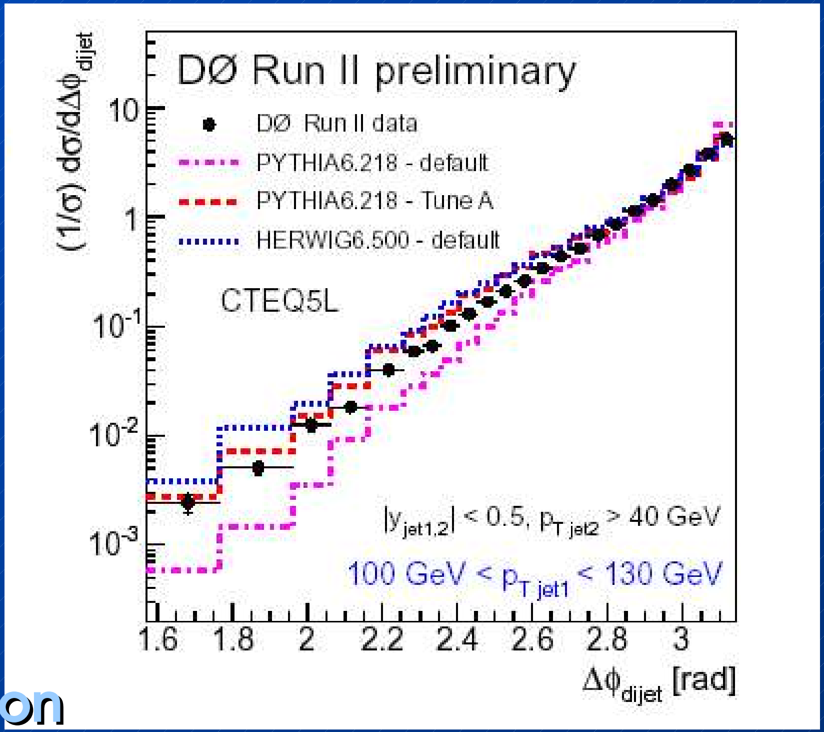
Sensitive to radiation of beyond 2 jets without actually measuring them. Test of pQCD

$\Delta\varphi$ = 180° at LO

Φ Decorrelation

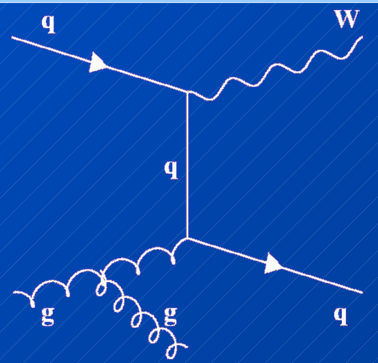


- Run II differential measurement at small $\Delta\eta$
- LO (in 3[rd] jet) perturbative calculation (JETRAD) does not agree
- lots of 4 jet events at smaller $\Delta\varphi$
- NLO calculation pretty good

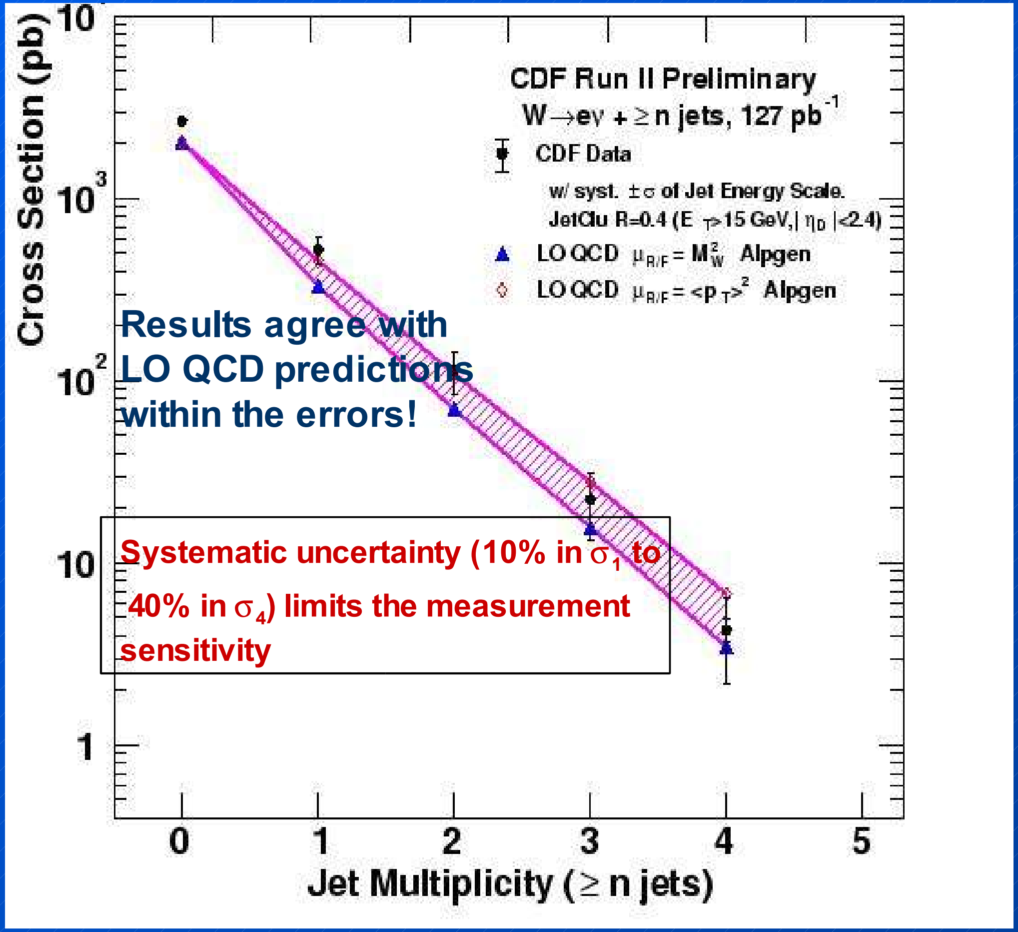


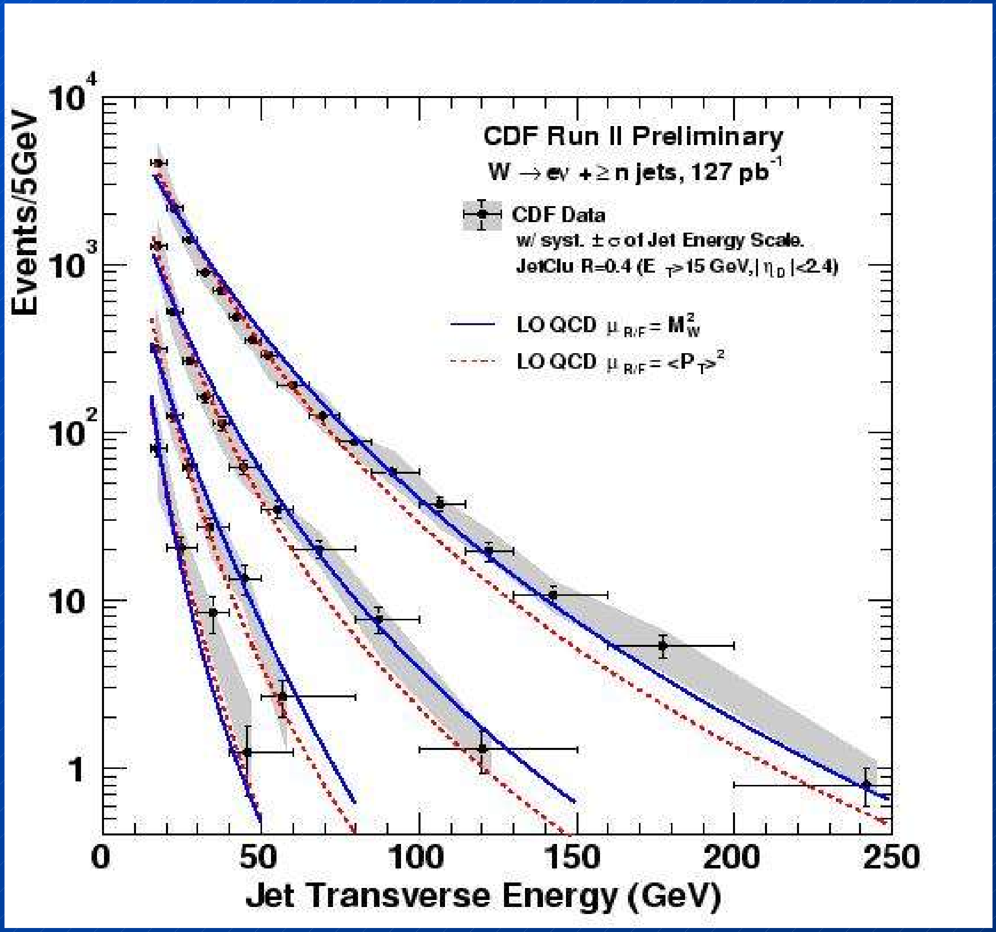
- good agreement with HERWIG and PYTHIA
- tuned PYTHIA (soft underlying event)
- gives best agreement
- NLO calculation is better in intermediate region

W+Jet Events



- Crucial to be able to calculate/understand this process for top & Higgs physics
- ALPGEN LO matrix element interfaced to HERWIG for parton shower
- not more than one parton associated with a reconstructed jet





Summary - Part I

- Brief Summary of the Standard Model
- Structure of the Proton
- Hadron Colliders :
  PS – SPPS – Tevatron – LHC
- The Experiments :
  CDF – DØ – ATLAS – CMS
- Studies of the Strong Interaction (QCD)

Backup Slides

The Fermilab Site

Protons or Antipronos – The Machine

- History of complex !
  - What machines do you already have ?
- Pbarp, single magnetic ring
  - SppbarS at CERN
  - Tevatron at FNAL
- BUT need pbar production and cooling, a limitation
- pp two magnetic rings, but helped by 2 in 1 design
  - (SSC)
  - Large Hardon Collider

Tevatron Luminosity

Frequency

Protons in Bunch

Total Antiprotons

$$L = \frac{f\,N_p\left(B \cdot N_{\bar{p}}\right)}{2\pi\left(\sigma_p^2 + \sigma_{\bar{p}}^2\right)} F\left(\sigma_z / \beta^*\right)$$

Beam Sizes

Beam Shape Form factor at Intersection

Antiprotons

Production

- 120 GeV Protons impact on target
- 8 GeV antiprotons produced, large angles
- focussed using Lithium Lens

Accumulation

- pbars injected into large aperture accelerators
- *Debuncher*
- *Accumulator*

Cooling

- multiple stochastic cooling systems
- different bandwidth systems react to different characteristics of the beam

Acceleration

- *Main Injector* 8 to 150 GeV
- *Tevatron* 150 – 1000 GeV

Recycler Ring

- 8 GeV
- permanent storage ring; magnetic field controlled by mechanical construction of magnets (reliable, less dependent on power glitches !)
- further storage for accumulation of antiprotons
- filled from *Accumulator*
- *Electron Cooling* cooling antiprotons after production and accumulator
- will support several hundred times 10^{10} antiprotons

Both Accumulator and Recycler used to feed single store in TEVATRON !

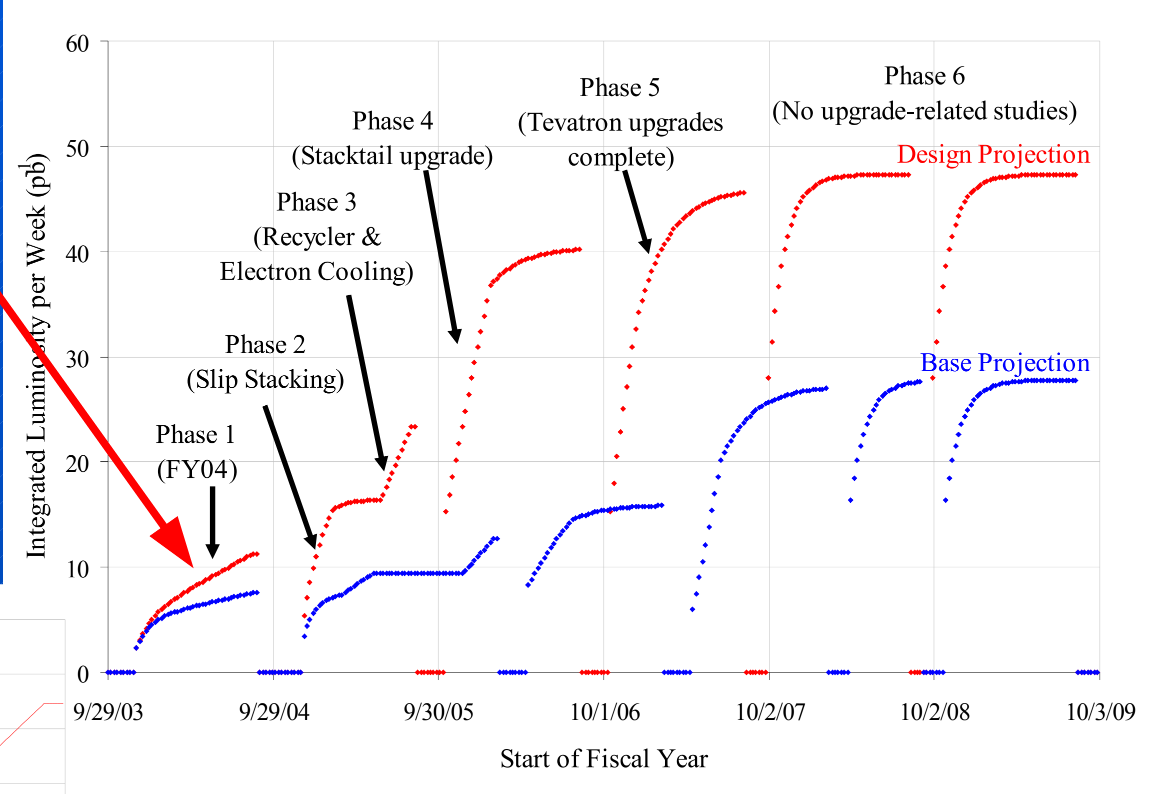
Record Luminosity (> $1 \cdot 10^{32}$ cm^{-2} sec^{-1})

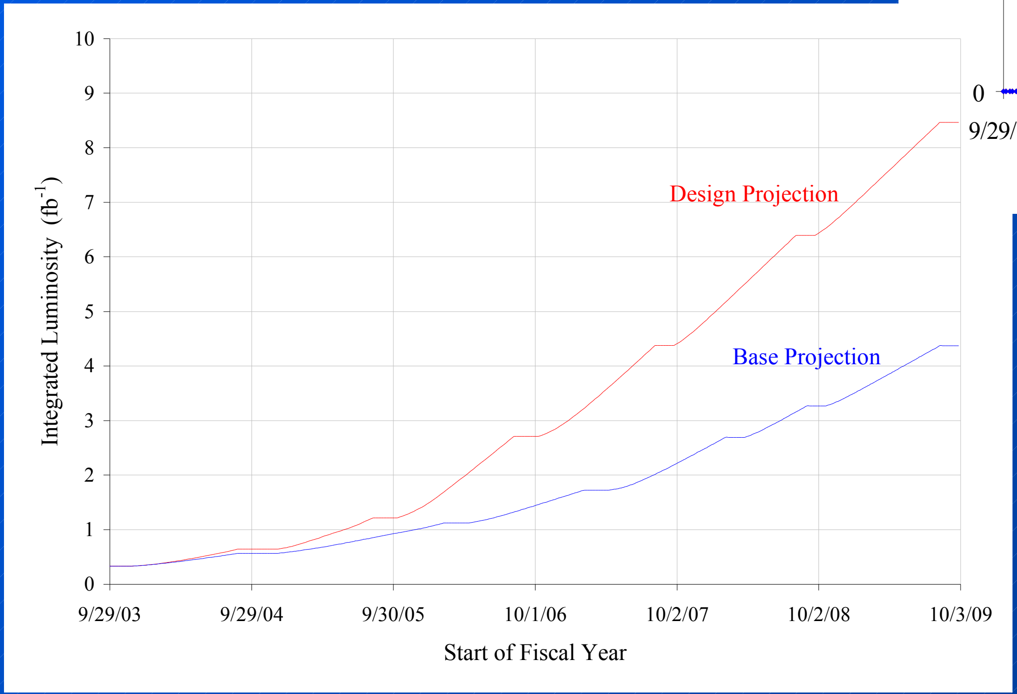
The TEVATRON until 2009

luminosity projection (fb^{-1})

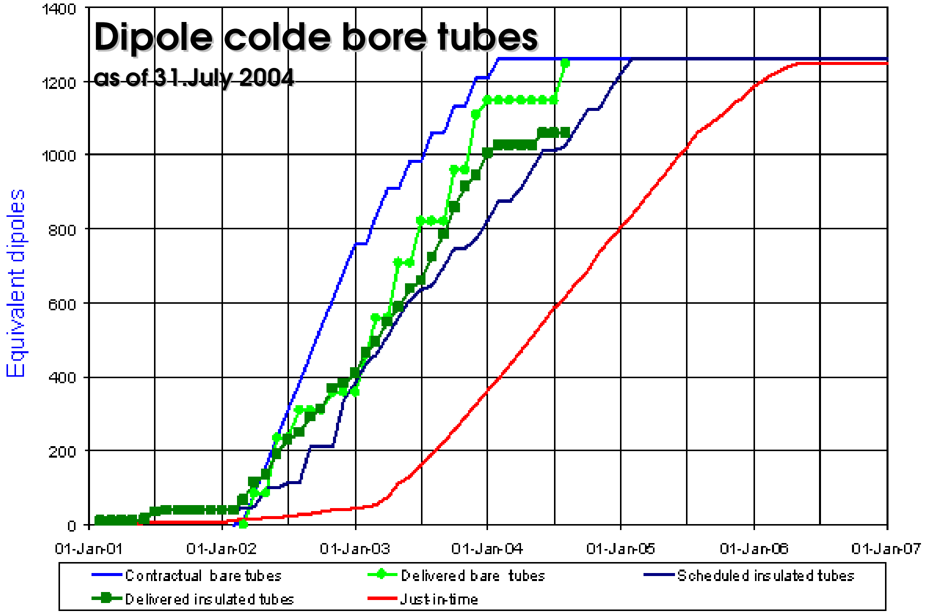
year	baseline	design
2003	0.28	0.3
2004	0.59	0.68
2005	0.98	1.36
2006	1.48	2.24
2007	2.11	3.78
2008	3.25	6.15
2009	4.41	8.57

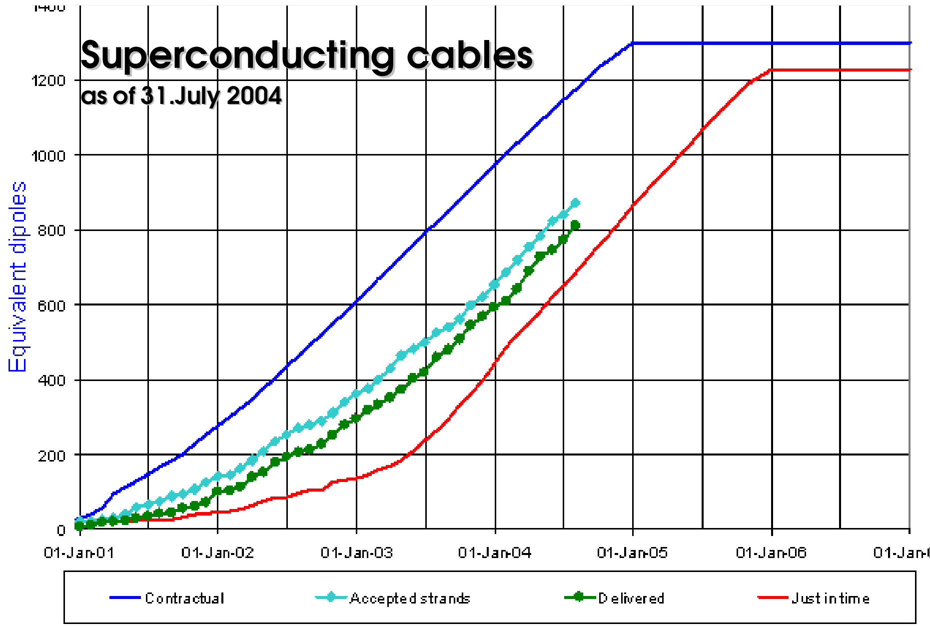
we are here

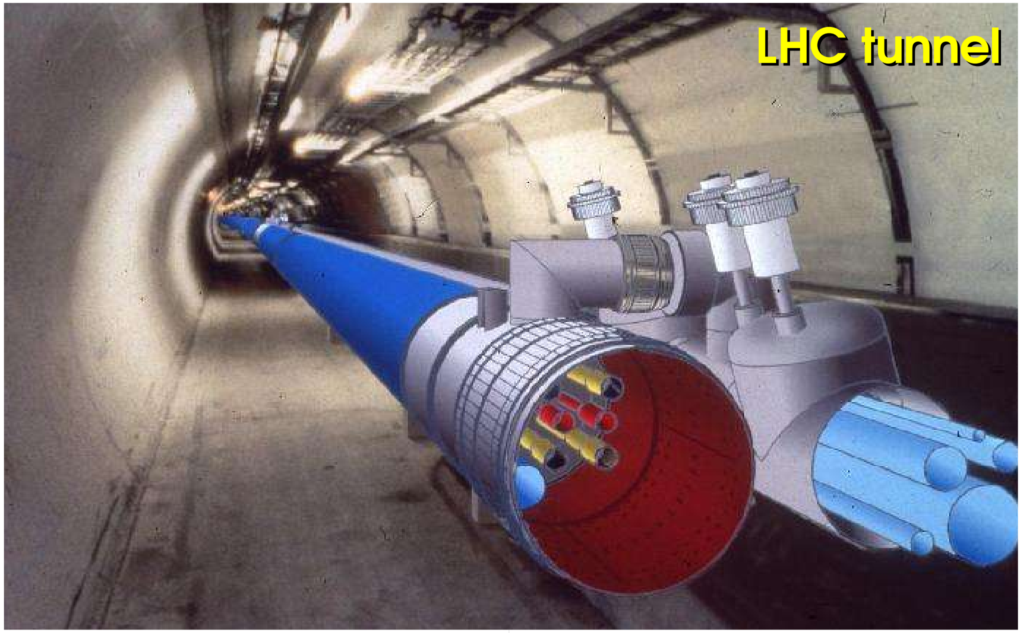




The Large Hadron Collider

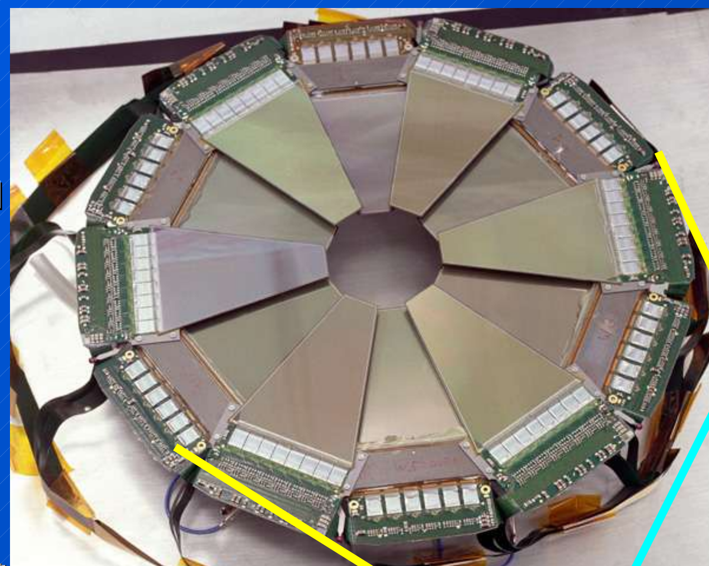


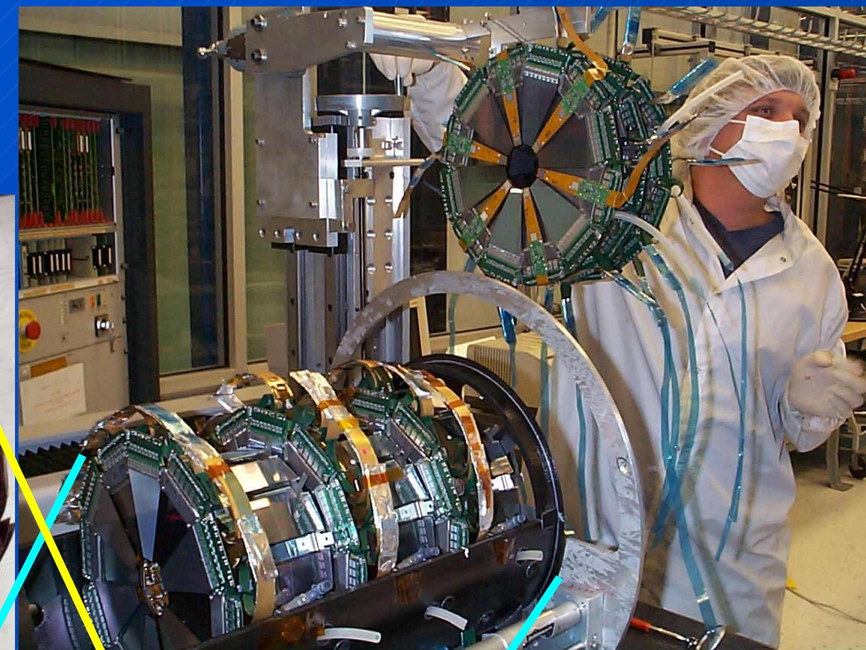




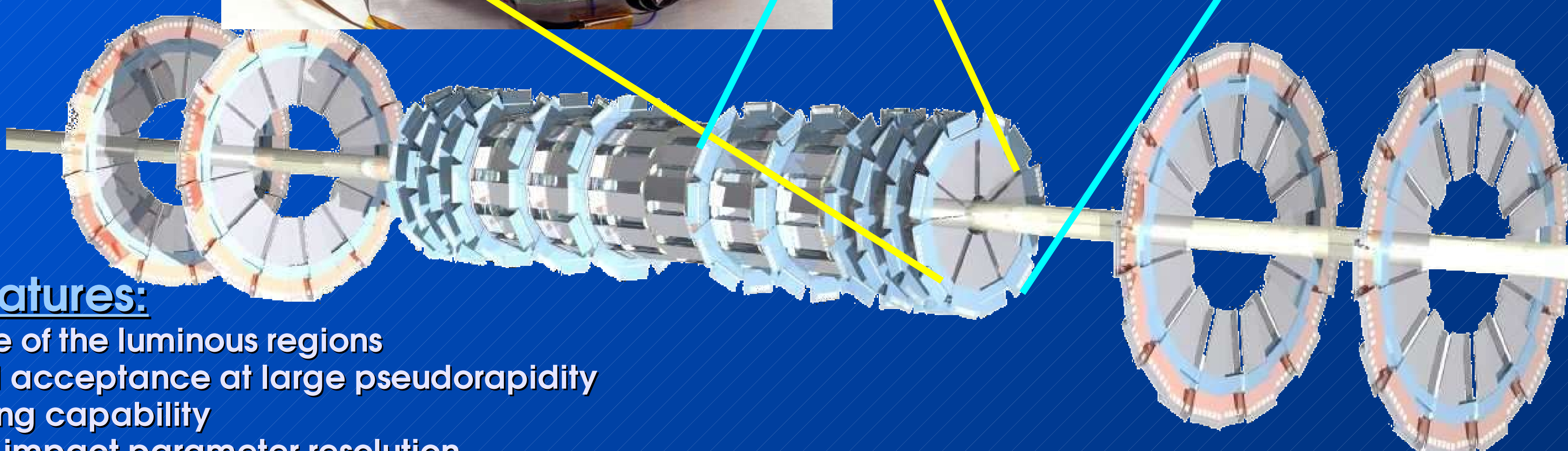
The DØ Fiber Tracker

- **Silicon Microstrip Tracker (SMT):**
  - 6 barrels, 14 disks
  - Tracking out to ~ $\eta = 3$





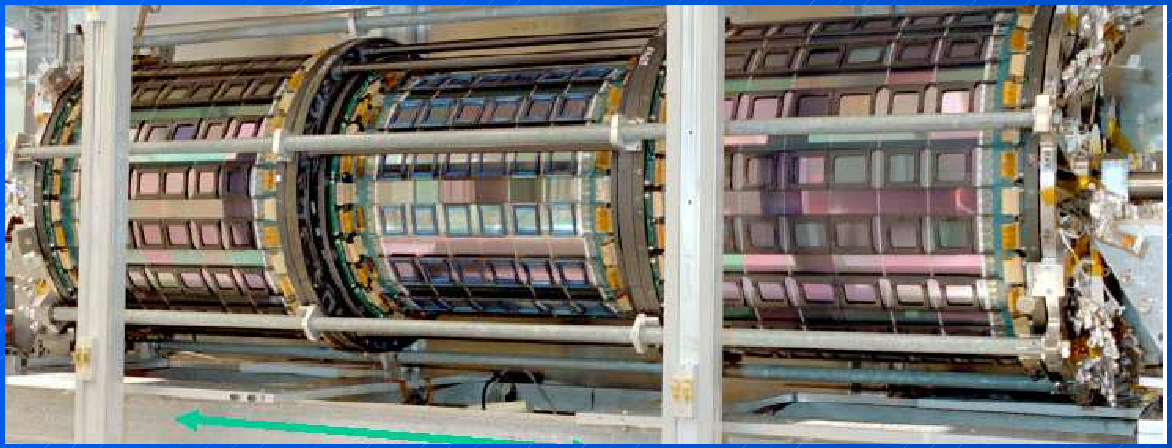
- axial, double-sided small-angle stereo and double-sided 90° detectors
- 800k channels, SVX2 readout

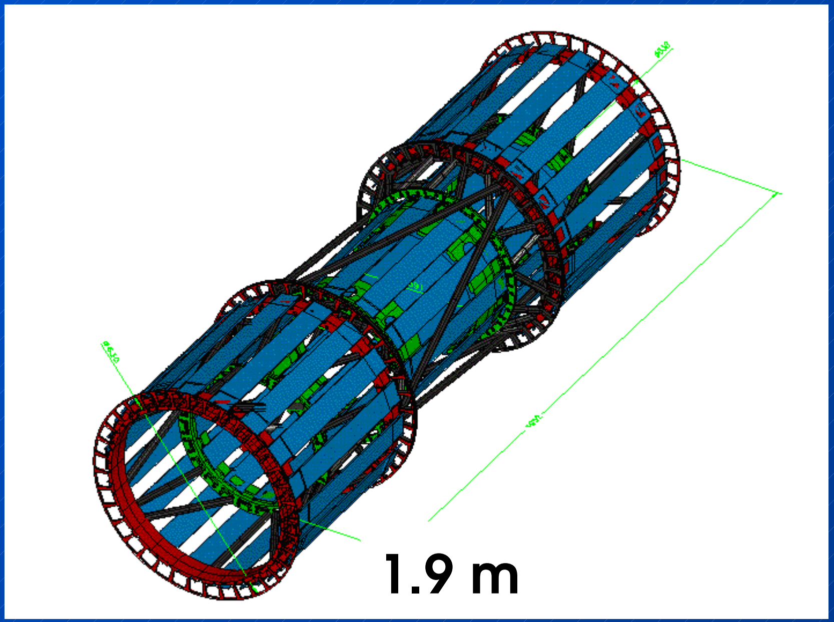


Main features:

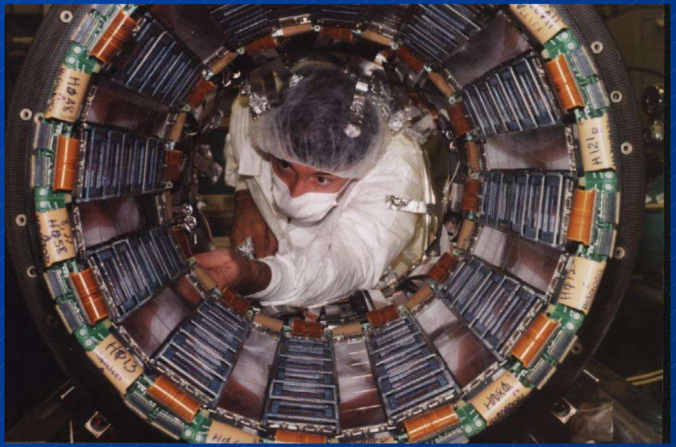
- Coverage of the luminous regions
- Extended acceptance at large pseudorapidity
- 3D Tracking capability
- Excellent impact parameter resolution

Intermediate Silicon Layer (ISL)

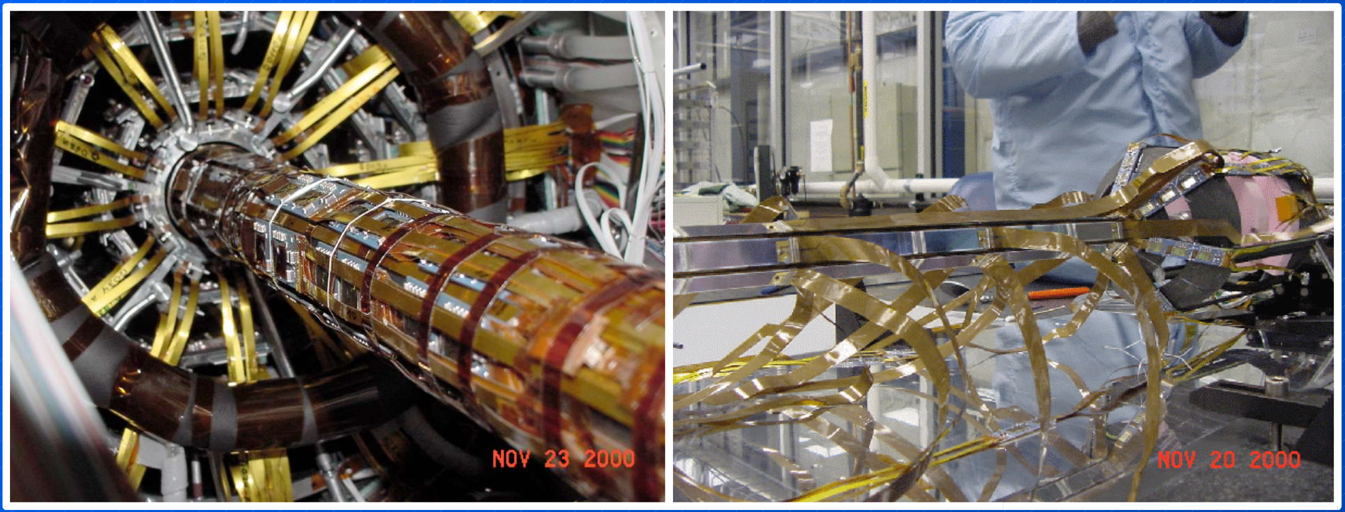


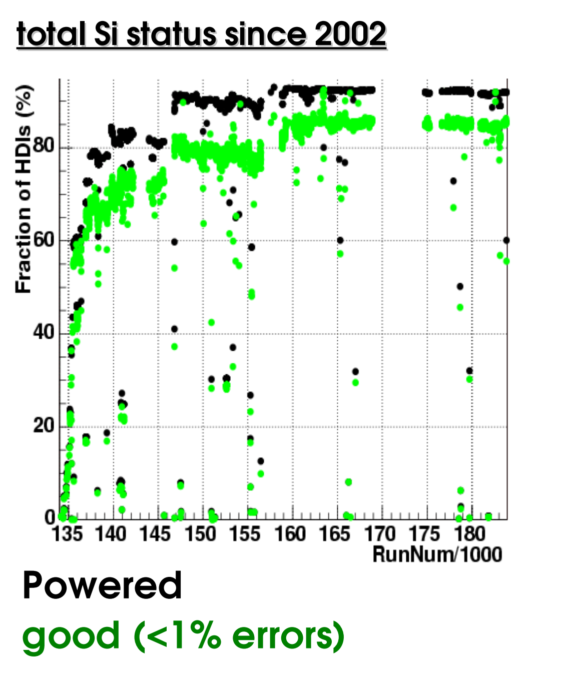


- One central layer ($|\eta|<1$)
  - link tracks from drift chamber to SVXII
- two forward layers ($1<|\eta|<2$)
  - silicon tracking in forward regions
- simple design
  - not used in trigger (less stringent alignment)
  - hybrids mounted off silicon
  - one kind of double-sided sensor flavour



Upgrade more … L00

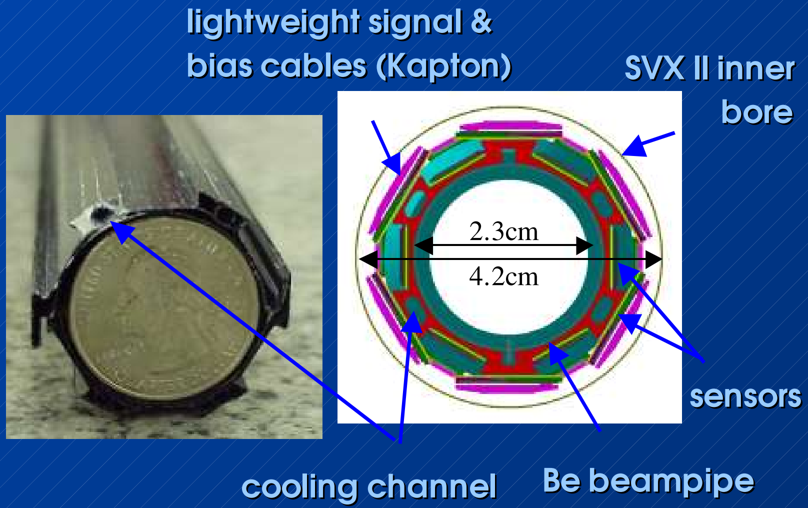




Precision position measurement before scattering

single-sided layer mounted on beam pipe

- 24 μm pitch; every other strip read out
- 0.6 % X_0 (no cooling) –
  1.0% X_0 (cooling, 15% of phi)
- actively cooled
- electronics at either end, large radii
- rad hard silicon; capable of 500V bias
  (will outlast SVXII inner layer)



Comparison between ATLAS and CMS

	ATLAS	CMS
Magnet(s)	` air-core' toroid + inner solenoid 2 independent magnet systems Calorimeter outside of solenoid	solenoid 1 magnet Calorimeter inside solenoid
Tracking detector	Si pixel + strips (3 + 4 layers) TRD for particle ID, B = 2 T $\sigma/p_T \sim 5 \cdot 10^{-4}\, p_T \oplus 0.01$	Si pixel + strips (2 + 10 layers) B = 4 T $\sigma/p_T \sim 1.5 \cdot 10^{-4} p_T \oplus 0.005$
EM calorimeter	Pb liquid argon $\sigma/E \sim 10\% / \sqrt{E}$ Presampler + longitudinal segmentation	$PbWO_4$ crystals $\sigma/E \sim 3\text{-}5\%/\sqrt{E}$ No longitudinal segmentation
Hadron calorimeter	Fe-scintillator (barrel) + Cu liquid Argon (endcaps) (10 λ) $\sigma/E \sim 50\% / \sqrt{E} \oplus 0.03$	Cu scintillator (5.8 λ + catcher) $\sigma/E \sim 65\% / \sqrt{E} \oplus 0.05$
Muons	Air ⇨ $\sigma/p_T \sim 7\%$ at 1 TeV Standalone	Fe ⇨ $\sigma/p_T \sim 5\%$ at 1 TeV Combined with central tracker

Run II Jet Algorithm

- Use Run II cone algorithm
- Combine particles in a R=0.7 cone
- Use the four vector of every tower as a seed
- Rerun using the midpoints between pairs of jets as seed
- Overlapping jets merged if the overlap area contains more than 50% of lower p_T jet, otherwise particles assigned to nearest jet.

Both experiments now using same algorithm

Reduced sensitivity to soft radiation

E-scheme recombination

$$P^J = (E^J, p^J) = \sum_{i = all\ clusters \in jet} (E^i, p^i_x, p^i_y, p^i_z)$$

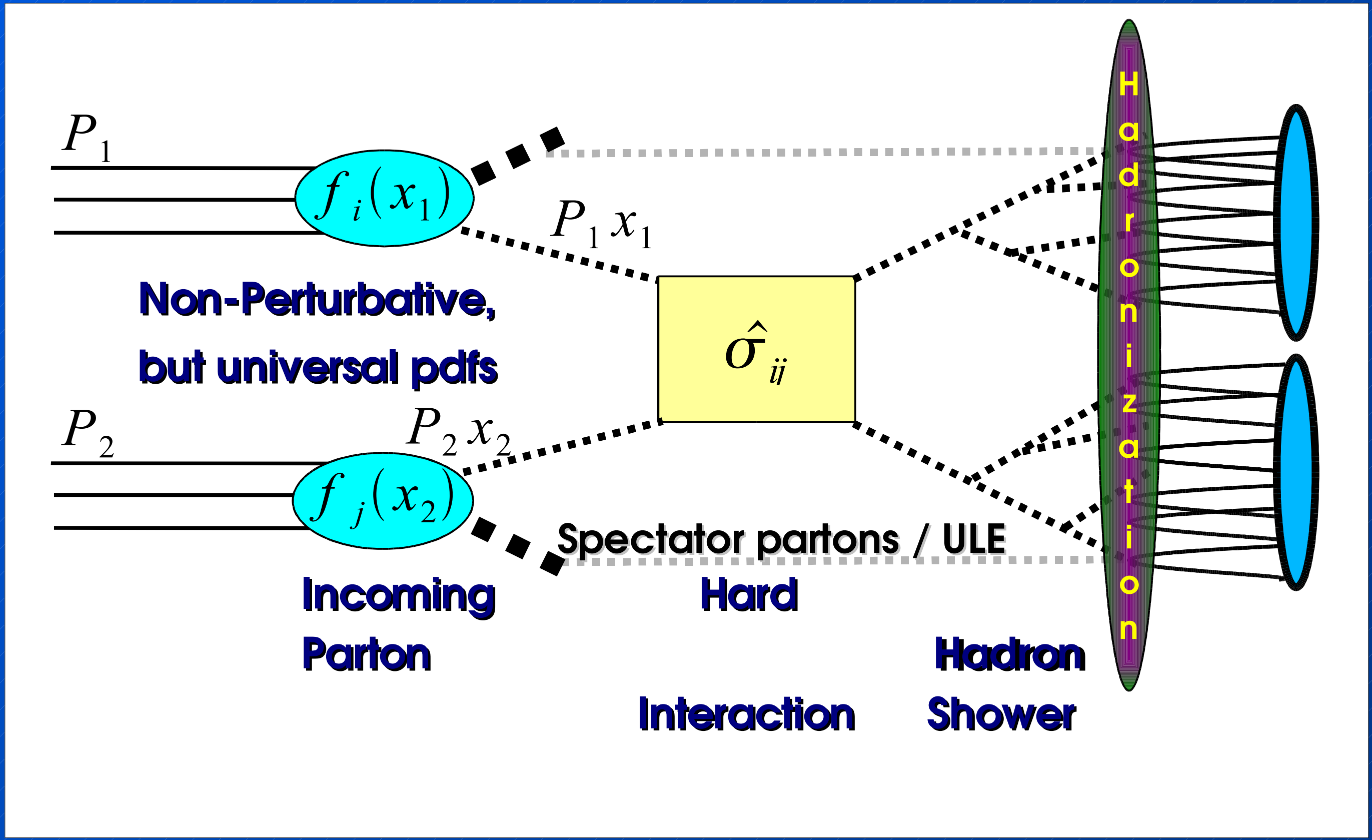
$$p^J_T = \sqrt{(p^J_x)^2 + (p^J_y)^2}$$

$$y^J = \frac{1}{2} \ln \frac{E^J + p^J_z}{E^J - p^J_z}$$

$$\phi^J = \tan^{-1} \frac{p^J_y}{p^J_x}$$

Soft Underlying Event

The "**underlying event**" consists of hard initial & final-state radiation plus the "beam-beam remnants" and possible multiple parton interactions.



Learn through studies of min bias events, Jet events. Look at distributions/correlations of charged particles with $\eta<1$, $p_T>500$ MeV

Also, studies of mini-jets in min bias events

Luminosity Measurement

event rate of inelastic events (luminosity detectors)

$$L = \frac{1}{\sigma_{eff}} \cdot \frac{dN}{dt}$$

need to know effective cross section for this process

$$\sigma_{eff} = \epsilon \cdot A \cdot \sigma_{inelastic}$$

Efficiency and geometrical acceptance
(studied and determined using Monte Carlos)

$$\sigma_{inelastic} \equiv \sigma_{total} - \sigma_{elastic}$$

Measured separately by several experiments
(CDF, E811)

	Run II	Run I
Inelastic	60.7 +- 2.4 mb	59.23 +- 2.3 mb
Single diffraction	9.6 +- 0.5 mb	9.6 +- 0.5 mb
Double diffraction	7.0 +- 2.0 mb	7.0 +- 2.0 mb

last two are part of the inelastic cross section
but with difference acceptance ...

Total Proton-Antiproton Cross-Section (1)

1) $$\sigma_T = \frac{1}{L}(R_{el} + R_{inel})$$

$L = \text{integrated luminosity}$

2) $$\sigma_T = \frac{4\pi}{k} \Im F(0)$$ (optical theorem)

$$t = \frac{-s}{2}(1-\cos\Theta) \Rightarrow d\cos\Theta = \frac{2}{s}dt$$

$$s = 4k^2$$

$$\sigma_T^2 = \frac{16\pi^2}{k^2} \cdot \frac{\Im F(0)^2}{\Im F(0)^2 + \Re F(0)^2} \cdot \underbrace{(\Im F(0)^2 + \Re F(0)^2)}_{|F(0)|^2}$$

$$|F(0)|^2 = \left(\frac{d\sigma}{d\Omega}\right)_{el}^{\Theta=0} = \frac{1}{L}\frac{1}{2\pi}\frac{4k^2}{2}\left(\frac{dR_{el}}{dt}\right)_{t=0}$$

3) $$\sigma_T^2 = \frac{16\pi(Ac)^2}{(1+\rho^2)} \cdot \frac{1}{L}(dR_{el}/dt)_{t=0}$$ with $$\rho = \frac{\Re F(0)}{\Im F(0)}$$

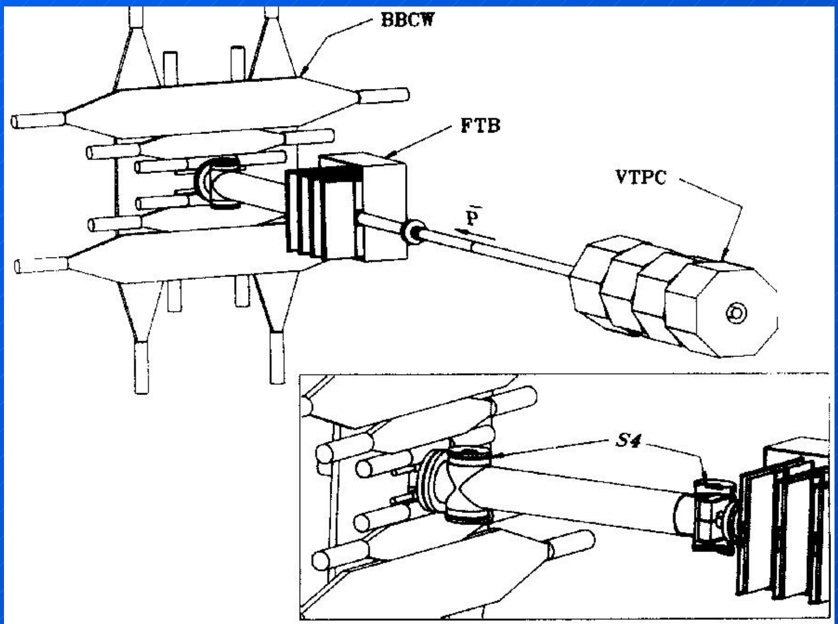
4) dividing 3) by 1) gives :

$$\sigma_T = \frac{16\pi(Ac)^2}{(1+\rho^2)} \cdot \frac{(dR_{el}/dt)_{t=0}}{R_{el} + R_{inel}}$$

... total cross-section can be determined from event rates, i.e. without knowing the luminosity !!!

Total Proton-Antiproton Cross-Section (2)

inelastic proton-antiproton events identified by TPC, telescopes, drift chambers and scintillators up to I   I < 6.7



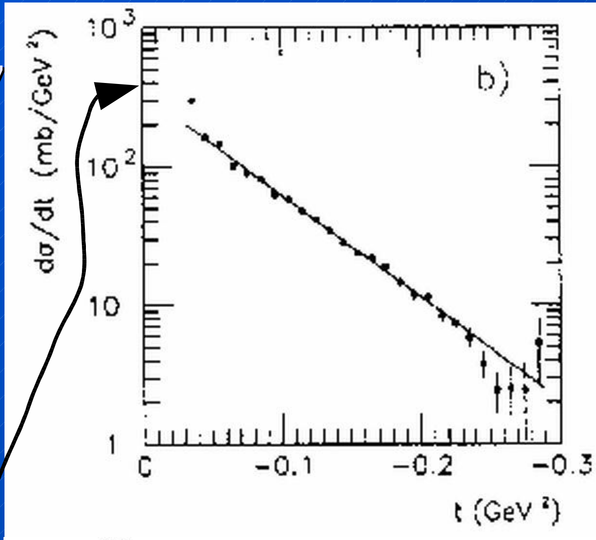
CDF total proton-antriproton cross-section at $\sqrt{s}$= 1800 GeV:

Phys.Rev D 50, 5550 (1994) & Phys.Rev D 50, 5518 (1994)

=0.15 from UA4/2 Phys.Rev.Lett. 68, 2433 (1992)

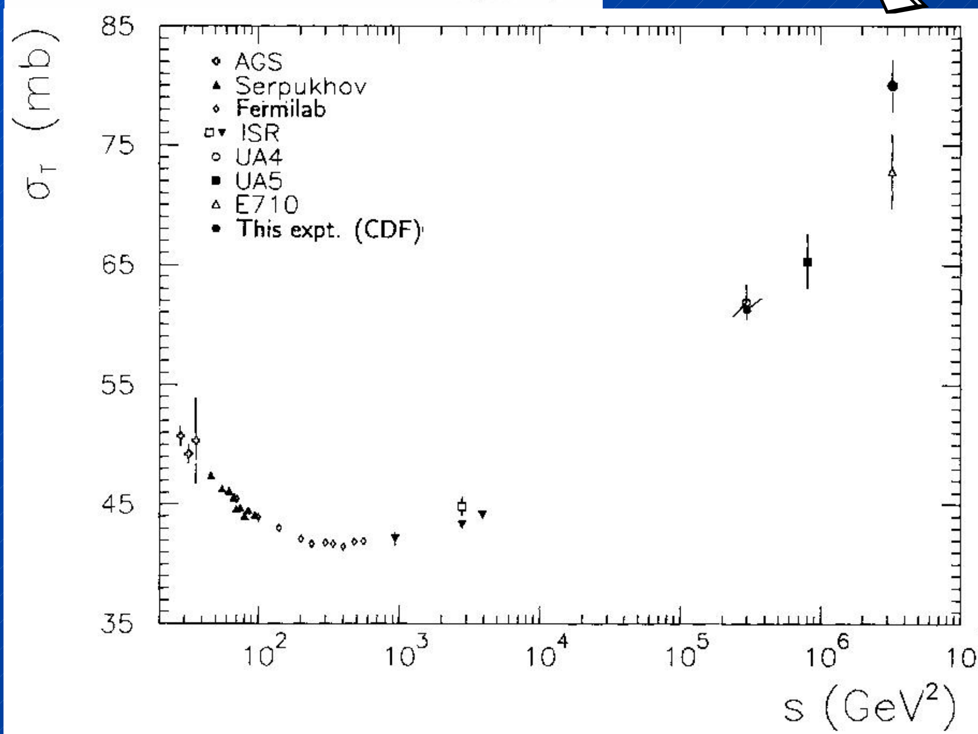
total inelastic	240 982	±2 967
total elastic	78 691	±1 463
total	319 673	±3 308
$(dE_{el} / dt)_{t=0}$	1 336 532	±40 943

$$\Rightarrow \sigma_T^{p\bar{p}}(\sqrt{s}=1800\,GeV)=80.03\pm2.24\,mb$$



here shown is the differential t-distribution; needed is only the event rate at the t=0 intercept; t is measured using roman pot detectors.

CDF result on total proton-antiproton cross-section



Optical Theorem (1)

- Consider beam of particles incident in z-direction (plane wave) on a spinless particle (neglect time-dependence $e^{i\omega t}$ here)
  $k = 1/\tilde{\lambda}$ with $2\pi\tilde{\lambda}$ = deBroglie wavelength
- At sufficiently large radial distances $kr >> 1$:
  $\psi_i = e^{ikz} = \frac{i}{2kr} \sum_l (2l+1) \left[(-1)^l e^{-ikr} - e^{ikr} \right] P_l(\cos\Theta)$
- scattering center can alter phase and amplitude of the outgoing lth partial waves by $2\delta_l$ and $2\eta_l$ ($1 > \eta_l > 0$), respectively:
  $\psi_{total} = \frac{i}{2kr} \sum_l (2l+1) \left[(-1)^l e^{-ikr} - \eta_l \, e^{2i\delta_l} \, e^{ikr} \right] P_l(\cos\Theta)$
- scattered wave = total - incoming :
  $\psi_{scatt} = \psi_{total} - \psi_i = \frac{e^{ikr}}{kr} \sum_l (2l+1) \frac{(\eta_l e^{2i\delta_l} - 1)}{2i} P_l(\cos\Theta)$
  $= \frac{e^{ikr}}{r} F(\Theta)$
  with the scatt. amplitude $F(\Theta) = \frac{1}{k} \sum_l (2l+1) \left(\frac{\eta_l e^{2i\delta_l} - 1}{2i} \right) P_l(\cos\Theta)$
  ... elastically scattered wave, since k taken to be same for incoming and outgoing ...
- scattered outgoing flux in solid angle $d\Omega$, at radius r
  $v_o \psi_{scatt} \psi^*_{scatt} \, r^2 \, d\Omega = v_o |F(\Theta)|^2 d\Omega$
  where v_o is velocity of outgoing particle relative to scattering center
  THIS IS the product of the scattering cross-section and the incident flux ($= v_i \psi_i \psi_i^* = v_i$); elastic scattering $\Rightarrow v_i = v_o$
  $v_o d\sigma = v_o |F(\Theta)|^2 d\Omega$
  $\Leftrightarrow$
  $\left(\frac{d\sigma}{d\Omega}\right)_{el} = |F(\Theta)|^2$

Optical Theorem (2)

- Legendre polynomials P_l obey orthogonality condition :
  $\int P_l \, P_{l'} d\Omega = \frac{4\pi\delta_{l,l'}}{2l+1}$
  total elastic scattering cross-section, integrated over angle
  $\sigma_{el.} = 4\pi\lambda \sum_l (2l+1) \left| \frac{\eta_l e^{2i\delta_l} - 1}{2i} \right|^2$
- if no absorption of incoming wave ... $(\eta_l = 1)$:
  $\sigma_{el.} = 4\pi\lambda \sum_l (2l+1) \sin^2 \delta_l$
- if $\eta_l < 1$ get reaction cross-section from probability conservation:
  $\sigma_r = \int (|\psi_{in}|^2 - |\psi_{out}|^2) r^2 d\Omega$
  ψ_{in} is first term of ..., ψ_{out} is second term of ...
  $\Rightarrow \sigma_r = \pi\lambda^2 \sum_l (2l+1)(1-\eta_l^2)$
- total cross-section :
  $\sigma_{tot} = \sigma_{el} + \sigma_r = \pi\lambda^2 \sum_l (2l+1)\, 2\, (1 - \eta_l \cos 2\delta_l)$
- since $P_l(1) = 1$ for all l and $\cos(\Theta = 0) = 1$
  $\mathrm{Im}\, F(0) = \frac{1}{2k} \sum_l (2l+1)\, (1 - \eta_l \cos 2\delta_l)$
- $\Rightarrow$ $\boxed{\mathrm{Im}\, F(0) = \frac{k}{4\pi} \sigma_{tot}}$

... total cross-section is related to the imaginary part of the forward elastic scattering amplitude ...