Hadron Collider Physics
- Experimental Overview – Part I -

Arnulf Quadt
Outline

• Part I  Introduction to Hadron Colliders
  Studies of the Strong Interaction (QCD)

• Part II  Electroweak Physics
  Top Physics

• Part III  Search for the Standard Model Higgs Boson
  Search for the Higgs beyond the Standard Model
  Search for New Phenomena
Part I

• Brief Summary of the Standard Model

• Structure of the Proton

• Hadron Colliders:
  - PS – SPPS – Tevatron – LHC

• The Experiments:
  - CDF – DØ – ATLAS – CMS

• Studies of the Strong Interaction (QCD)
The Standard Model

- 2 * 3 * 2 fundamental particles (fermions)
- 4 fundamental forces + Higgs boson
- Gauge theories with local gauge invariance
The Structure of the Proton (1)

Protons (hadrons) are not elementary particles
- they have substructure (partons = quarks and gluons)
- not well defined parton energy, but energy distribution
- various parton flavours can collide
- data analysis more complex than $e^+e^-$,
  QCD always involved

Why do we collide them?
- much less synchrotron radiation losses
- not limited by cavity power
- but by ring size / magnet strength
- essentially no strict kinematic limit,
  only luminosity limited
- protons are easy to produce in vast quantities
- antiproton production? (cooling techniques)

Kinematic variables:
Bjorken-$x =$ fraction of the proton momentum carried by the struck parton
$Q^2 =$ 4-momentum transfer squared in hard interaction
  $~ 1/$transverse resolution power

$$\Delta E = \frac{4\pi\alpha\hbar c}{3R} \left( \frac{E}{m} \right)^4$$
The Structure of the Proton (2)

Partonic structure at high $Q^2$

Parton density ($F_2$) rises towards low $x$

Extrapolation of HERA and fixed target measurements towards lower $x$ and higher $Q^2$

Need to measure (longitudinal) parton (momentum) distribution functions (PDFs) in every experiment parametrisations by CTEQ, MRST ...
Parton-Parton Interactions

**Factorization**

- if parton interaction requires large CMS (valence) quark dominated, e.g. top production at Tevatron
- if parton interaction possible with low CMS gluon-gluon dominated, e.g. top or light Higgs production at LHC
Parton-Parton Interactions

Progress in High Energy Physics depends on advancing the energy frontier:

\[ \sqrt{s} \approx \frac{1}{2} \cdot \frac{1}{3} \sqrt{s} \]

Livingston plot

- ISR
- SPEAR
- PEP
- CESR
- CERN
- TRISTAN
- LEP II
- Tevatron
- LHC

Proton- (anti)proton cross sections

- \( \sigma_{\text{W}} \)
- \( \sigma_{\text{Z}} \)
- \( \sigma_{\text{t}} \)
- \( \sigma_{\text{\mu}}(E_{\text{T}} > \sqrt{s}/20) \)
- \( \sigma_{\text{\mu}}(E_{\text{T}} > 100 \text{ GeV}) \)
- \( \sigma_{\text{\mu}}(M_{\text{Higgs}} = 150 \text{ GeV}) \)
- \( \sigma_{\text{\mu}}(M_{\text{Higgs}} = 500 \text{ GeV}) \)

\( \sqrt{s} \) (TeV)

\( \sigma \) (nb)
Kinematics

- parton-parton center of mass energy, estimate: $\sqrt{s'} \approx 1/6 \times \sqrt{s}$
- partons have longitudinal momentum distribution!
  ◊ boost of CMS system along beam line, a priori unknown

- focus on transverse quantities:
  - transverse momentum $P_T$
  - transverse Energy $E_T$
  - missing transverse energy MET:

\[
MET = \left| \sum_{\text{vis}} \vec{P}_T \right|
\]

- angles measured in:
  - rapidity: $y = 1/2 \ln \left\{ \frac{(E + P_L)}{(E - P_L)} \right\}$
    if $m \ll E, P_L$
  - pseudorapidity: $\eta = - \ln \tan \theta/2$

rapidity intervals $y$ are Lorentz-invariant
distance measure: $R^2 = (\Delta \phi)^2 + (\Delta \eta)^2$
Hadron Colliders

Fermilab
1987 - 2009
Tevatron PbarP
1.8 - 1.96 TeV
CDF, DØ

CERN
1981 - 1990
SPPS PbarP
0.6 TeV
UA1, UA2

CERN
2007 - 2020
LHC PP
14 TeV
ATLAS, CMS,
ALICE, LHC-B
proton-synchrotron (PS) accelerated first protons to 24 GeV in 1959
... still running today and in the future for the LHC ...
CERN's Super-Proton Synchrotron

- first p-pbar reactions observed in Juli 1981 ... at 270 GeV per beam
- then turned into a proton-antiproton collider (one beampipe)
- collider experiments UA1 and UA2 (Nobel prize for machine and W/Z-discovery)
Stochastic Cooling

Simon van der Meer

5 \cdot 10^{11} \text{ Pbar/beam}

1 \text{ Pbar per } 3 \cdot 10^5 \text{ P}

- Impulsgeber
- Korrektursignal
- Abtaster
- Antiprotonen-Speicherring (AA)
- Tunnel vom Speicherring zum Beschleuniger

- Target
- Protonenstrahl

[Diagram of a circular accelerator with labels and arrows indicating the beam path]
The TEVATRON at Fermilab (1)

Main Injector & Recycler

Tevatron

CDF

DØ

Chicago

60 km

$\sqrt{s} = 1.96 \text{ TeV}$

$\Delta t = 396 \text{ ns}$

Run I 1987 (92)-95 $L_{\text{int}} \sim 125 \text{ pb}^{-1}$

Run II 2001-09(?)

> 40-times larger dataset at increased energy
The TEVATRON at Fermilab (2)
The TEVATRON Performance (1)

Peak luminosity:
- $1 \cdot 10^{12} \overline{P}$ per beam, `only' twice SPPS!
- In spring 2002 passed Run I record
- In 2004 often above optimistic design scenario
CDF and DØ ~500 pb\(^{-1}\) recorded each

- typical data taking efficiency ~90%
- commissioning is over

The TEVATRON at upper limit of optimistic scenario

analysed data
delivered in FY04
delivered in FY03

Delivered
Recorded
The Large Hadron Collider:
- proton-proton collisions
- no antiprotons, $3 \cdot 10^{14}$ P
- at 14 TeV center of mass energy
- 40 Mio. collisions per second
- first collisions in 2007
- 4 experiments: ATLAS, CMS, ALICE, LHC-B

- two separate beampipes
- two accelerators
The Large Hadron Collider
## Collider Comparison

<table>
<thead>
<tr>
<th></th>
<th>SPPS</th>
<th>Tevatron</th>
<th>LHC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Particles</td>
<td>Pbar-p</td>
<td>Pbar-p</td>
<td>p-p</td>
</tr>
<tr>
<td>cm energy (TeV)</td>
<td>0.62</td>
<td>1.96</td>
<td>14</td>
</tr>
<tr>
<td>Lumi ($10^{30}$ cm$^{-2}$ s$^{-1}$)</td>
<td>6</td>
<td>50-100</td>
<td>0.1 - 1.0 x 10$^4$</td>
</tr>
<tr>
<td>Lumi (fb$^{-1}$ year$^{-1}$)</td>
<td>0.05</td>
<td>0.5</td>
<td>100</td>
</tr>
<tr>
<td>Bunch spacing (ns)</td>
<td>3800</td>
<td>396</td>
<td>25</td>
</tr>
<tr>
<td>Particles per bunch ($10^{10}$)</td>
<td>P: 15, Pbar: 8</td>
<td>P: 24, Pbar: 3</td>
<td>P: 11.5</td>
</tr>
<tr>
<td>Max.no Pbar in accumulator</td>
<td>$1.2 \times 10^{12}$</td>
<td>$2.6 \times 10^{12}$</td>
<td>-</td>
</tr>
<tr>
<td>Bunches</td>
<td>6 + 6</td>
<td>36 + 36</td>
<td>2835 + 2835</td>
</tr>
<tr>
<td>Circumference (km)</td>
<td>6.9</td>
<td>6.28</td>
<td>26.7</td>
</tr>
<tr>
<td>Nr. dipoles</td>
<td>232</td>
<td>774</td>
<td>1232 (main dipoles)</td>
</tr>
<tr>
<td>Magnet type</td>
<td>warm</td>
<td>cold, warm iron</td>
<td>cold, cold iron</td>
</tr>
<tr>
<td>Peak magnetic field (T)</td>
<td>1.4</td>
<td>4.4</td>
<td>8.3</td>
</tr>
</tbody>
</table>
Particle Identification

Muon (high energy)
(medium ene.)
(low energy)

Photon

Electron

Quark → Jet

Tracking detector
Electron calorimeter
Hadron calorimeter
Magnet coil
Muon chambers
The CDF and DØ Experiment

- new bigger silicon, new drift chamber, TOF
- Upgraded calorimeter and muon system
- Upgraded DAQ/trigger
- Displaced track trigger
- ~750 physicists

- new silicon and fibre tracker
- new ~2 T solenoid
- upgraded muon system
- upgraded (track) trigger/DAQ
- 19 countries, 83 institutes, 664 physicists
The DØ Experiment

Calorimeter:
Transverse Segmentation
\[ \Delta \eta \times \Delta \phi = 0.1 \times 0.1 \]

(EM3)
\[ \Delta \eta \times \Delta \phi = 0.1 \times 0.1 \]

L1/L2 fast trigger readout in
0.2 x 0.2 towers

resolutions:
EM: \[ \sigma_E / E = 15\% / \sqrt{E} \]
HAD: \[ \sigma_E / E = 50\% / \sqrt{E} \]
Central Fiber Tracker:
- 77k fibers in eight barrels, 800 \( \mu \text{m} \) diameter fibers
- 3° stereo layers in each barrel
- VLPC readout, \(~7\) photo-electrons/track at \( \eta = 0 \)
The CDF Experiment

drift chamber (COT)
- radius 40 - 137 cm, $|z| < 160$ cm
- 60 axial, 24 stereo hits (3°)

scintillator based calorimeter
- EM: $\sigma_E/E = 13.5\% / \sqrt{E}$
- HAD: $\sigma_E/E = 50\% / \sqrt{E}$
Baseline Upgrade: SVX II

- 7-8 silicon layers
- 722, 432 channels
- \( r_\phi, rz \) views
- \( z^{\text{max}} = 45 \) cm, \( \eta^{\text{max}} = 3 \)
- \( 1.3 < r < 30 \) cm

Run II physics goals:
- properties of top quark
- precision Electroweak
- CKM, \( B_s \) mixing
- search for new phenomena
- tests of QCD

Heavy flavour tagging:
- \( B \) reconstruction efficiency
- increased forward acceptance
- improved \( \sigma_{d0} \) (\( \sim 20 \mu m \))

- 5 double-sided layers
  - 5 axial, 3 x 90°, 2 x 1.2°
- Tight alignment tolerances
  - for displaced track trigger
- Highly symmetric
  - 12-fold in \( \phi \)
  - 6-fold in \( z \)

Note wedge symmetry
The ATLAS Experiment

ATLAS Parameters:
- Weight: 7000 t
- Height: 22 m
- Length: 42 m
- Magnet (solenoid): 2 Tesla

Trigger: 100 Mio. events/sec. → select 100
Computing: 100 Mio. events per year

Hadron calorimeter, Electromagnet, Transition Radiation Detector, Pixel Detector, Toroid Magnets, Muon Chambers
The CMS Experiment

CMS return yoke and solenoid cryostat

CMS hadronic endplug cal

- assembly advanced
- lowering into cavern in 2005
- software and simulation being prepared as well...
QCD Studies
Quantum Chromo Dynamics (QCD)

- Theory of Strong Interactions
- Acts on all quarks
- Mediated via Gluons

... discovered in 70ties at PETRA / DESY ...

Jets
Quantum Chromo Dynamics (QCD)

quark confinement

asymptotic freedom

quasi-free quarks

\[ \alpha_s \]

Messwerte

QCD

Energie in GeV

large distances

small distances
Rapidity Gaps/Diffractive/Elastic Physics

- PDF's: double parton interactions, W charge asymmetry
- non-perturbative QCD: jet shapes, W/Z boson \( p_T \) spectra
- perturbative QCD, particle cross section, W/Z bosons, prompt photons, jets, top, b/c quarks

Perturbative QCD with W/Z bosons

Perturbative QCD with jets: \( \alpha_s \), jet topologies

- Inclusive jet cross section
- Dijet mass spectra
- uncorrected dijet \( \Delta\phi \)
- jet multiplicities in W+jet events

... here only a small selection ...
Inclusive Jet Cross Section (1)

High $E_T$ jets probe large $x$
PDF’s, especially gluon
PDF: Run II has extended reach in jet $E_T$

Important information is in the cross section versus rapidity
Uncertainties dominated by jet energy scale. Jet energy is systematics dominated in central. Needs data and time to study and understand more precisely.

<table>
<thead>
<tr>
<th>Uncertainty for central jets in central region (GeV)</th>
<th>Run I</th>
<th>Run II</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDF</td>
<td>17</td>
<td>30</td>
</tr>
<tr>
<td>DØ</td>
<td>14</td>
<td>14</td>
</tr>
</tbody>
</table>
Dijet Cross Section

Often used to search for new resonances ...

Uncertainty dominated by jet energy scale
Φ Decorrelation

high mass event:
\[ M_{jj} = 1364 \text{ GeV}, \ ET = 633, 666 \text{ GeV} \]

Sensitive to radiation of beyond 2 jets without actually measuring them. Test of pQCD

\[ \Delta \phi = 180^\circ \text{ at LO} \]
**Decorrelation**

- Run II differential measurement at small $\Delta \eta$
- LO (in 3rd jet) perturbative calculation (JETRAD) does not agree
- lots of 4 jet events at smaller $\Delta \phi$
- NLO calculation pretty good

- good agreement with HERWIG and PYTHIA
- tuned PYTHIA (soft underlying event)
- gives best agreement
- NLO calculation is better in intermediate region
W+Jet Events

- Crucial to be able to calculate/understand this process for top & Higgs physics
- ALPGEN LO matrix element interfaced to HERWIG for parton shower
- not more than one parton associated with a reconstructed jet

Systematic uncertainty (10% in $\sigma_1$ to 40% in $\sigma_4$) limits the measurement sensitivity.

Results agree with LO QCD predictions within the errors!

![Graph showing cross-section vs. jet multiplicity and events vs. jet transverse energy](image)
Summary - Part I

- Brief Summary of the Standard Model

- Structure of the Proton

- Hadron Colliders:
  - PS – SPPS – Tevatron – LHC

- The Experiments:
  - CDF – DØ – ATLAS – CMS

- Studies of the Strong Interaction (QCD)
The Fermilab Site
• **History of complex !**
  - **What machines do you already have ?**

• **Pbarp, single magnetic ring**
  - **SppbarS at CERN**
  - **Tevatron at FNAL**

• **BUT need pbar production and cooling, a limitation**

• **pp two magnetic rings, but helped by 2 in 1 design**
  - **(SSC)**
  - **Large Hardon Collider**
Tevatron Luminosity

\[ L = \frac{\int N_p (B \cdot N_p)}{2\pi (\sigma_p^2 + \sigma_{p^*}^2)} F\left(\sigma_z / \beta^*\right) \]
Antiprotons

Cooling
- multiple stochastic cooling systems
- different bandwidth systems react to
different characteristics of the beam

Acceleration
- Main Injector 8 to 150 GeV
- Tevatron 150 – 1000 GeV

Recycler Ring
- 8 GeV
- permanent storage ring; magnetic field controlled
  by mechanical construction of magnets
  (reliable, less dependent on power glitches !)
- further storage for accumulation of antiprotons
- filled from Accumulator
- Electron Cooling cooling antiprotons after
  production and accumulator
- will support several hundred times $10^{10}$ antiprotons

Production
- 120 GeV Protons impact on target
- 8 GeV antiprotons produced, large angles
- focussed using Lithium Lens

Accumulation
- pbars injected into large aperture accelerators
- Debuncher
- Accumulator

Both Accumulator and Recycler used to feed
single store in TEVATRON !

Record Luminosity ($>1 \times 10^{32} \text{ cm}^{-2} \text{ sec}^{-1}$)
The TEVATRON until 2009

<table>
<thead>
<tr>
<th>Year</th>
<th>Baseline</th>
<th>Design</th>
</tr>
</thead>
<tbody>
<tr>
<td>2003</td>
<td>0.28</td>
<td>0.3</td>
</tr>
<tr>
<td>2004</td>
<td>0.59</td>
<td>0.68</td>
</tr>
<tr>
<td>2005</td>
<td>0.98</td>
<td>1.36</td>
</tr>
<tr>
<td>2006</td>
<td>1.48</td>
<td>2.24</td>
</tr>
<tr>
<td>2007</td>
<td>2.11</td>
<td>3.78</td>
</tr>
<tr>
<td>2008</td>
<td>3.25</td>
<td>6.15</td>
</tr>
<tr>
<td>2009</td>
<td>4.41</td>
<td>8.57</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Phase</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase 1</td>
<td>(FY04)</td>
</tr>
<tr>
<td>Phase 2</td>
<td>(Slip Stacking)</td>
</tr>
<tr>
<td>Phase 3</td>
<td>(Recycler &amp; Electron Cooling)</td>
</tr>
<tr>
<td>Phase 4</td>
<td>(Stacktail upgrade)</td>
</tr>
<tr>
<td>Phase 5</td>
<td>(Tevatron upgrades complete)</td>
</tr>
<tr>
<td>Phase 6</td>
<td>(No upgrade-related studies)</td>
</tr>
</tbody>
</table>

we are here

Integrated Luminosity per Week (pb⁻¹)

Start of Fiscal Year

Integrated Luminosity (fb⁻¹)

Start of Fiscal Year
The Large Hadron Collider

LHC cavities

Dipole cold bore tubes
as of 31 July 2004

Superconducting cables
as of 31 July 2004

LHC tunnel

Superconducting cables
as of 31 July 2004

Dipole cold bore tubes
as of 31 July 2004

LHC tunnel
The DØ Fiber Tracker

• **Silicon Microstrip Tracker (SMT):**
  - 6 barrels, 14 disks
  - Tracking out to $\eta = 3$

- axial, double-sided
small-angle stereo and
double-sided 90°
detectors
- 800k channels,
SVX2 readout

**Main features:**
- Coverage of the luminous regions
- Extended acceptance at large pseudorapidity
- 3D Tracking capability
- Excellent impact parameter resolution
Intermediate Silicon Layer (ISL)

- One central layer ($|\eta| < 1$)
  - link tracks from drift chamber to SVXII
- Two forward layers ($1 < |\eta| < 2$)
  - silicon tracking in forward regions
- Simple design
  - not used in trigger (less stringent alignment)
  - hybrids mounted off silicon
  - one kind of double-sided sensor flavour
Precision position measurement before scattering
single-sided layer mounted on beam pipe
- 24 μm pitch; every other strip read out
- 0.6 % $X_0$ (no cooling) –
  1.0% $X_0$ (cooling, 15% of phi)
- actively cooled
- electronics at either end, large radii
- rad hard silicon; capable of 500V bias
  (will outlast SVXII inner layer)
# Comparison between ATLAS and CMS

<table>
<thead>
<tr>
<th></th>
<th>ATLAS</th>
<th>CMS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Magnet(s)</strong></td>
<td><code>air-core</code> toroid + inner solenoid 2 independent magnet systems</td>
<td>solenoid 1 magnet</td>
</tr>
<tr>
<td></td>
<td>Calorimeter outside of solenoid</td>
<td>Calorimeter inside solenoid</td>
</tr>
<tr>
<td><strong>Tracking detector</strong></td>
<td>Si pixel + strips (3 + 4 layers) TRD for particle ID, B = 2 T</td>
<td>Si pixel + strips (2 + 10 layers) B = 4 T</td>
</tr>
<tr>
<td></td>
<td>$\sigma/p_T \sim 5 \times 10^{-4} ; p_T \oplus 0.01$</td>
<td>$\sigma/p_T \sim 1.5 \times 10^{-4} p_T \oplus 0.005$</td>
</tr>
<tr>
<td><strong>EM calorimeter</strong></td>
<td>Pb liquid argon $\sigma/E \sim 10% / \sqrt{E}$</td>
<td>PbWO4 crystals $\sigma/E \sim 3-5% / \sqrt{E}$</td>
</tr>
<tr>
<td></td>
<td>Presampler + longitudinal segmentation</td>
<td>No longitudinal segmentation</td>
</tr>
<tr>
<td><strong>Hadron calorimeter</strong></td>
<td>Fe-scintillator (barrel) + Cu liquid Argon (endcaps) (10 $\lambda$)</td>
<td>Cu scintillator (5.8 $\lambda$ + catcher)</td>
</tr>
<tr>
<td></td>
<td>$\sigma/E \sim 50% / \sqrt{E} \oplus 0.03$</td>
<td>$\sigma/E \sim 65% / \sqrt{E} \oplus 0.05$</td>
</tr>
<tr>
<td><strong>Muons</strong></td>
<td>Air $\Rightarrow \sigma/p_T \sim 7%$ at 1 TeV</td>
<td>Fe $\Rightarrow \sigma/p_T \sim 5%$ at 1 TeV</td>
</tr>
<tr>
<td></td>
<td>Standalone</td>
<td>Combined with central tracker</td>
</tr>
</tbody>
</table>
Run II Jet Algorithm

- Use Run II cone algorithm
- Combine particles in a R=0.7 cone
- Use the four vector of every tower as a seed
- Rerun using the midpoints between pairs of jets as seed
- Overlapping jets merged if the overlap area contains more than 50% of lower $p_T$ jet, otherwise particles assigned to nearest jet.

Both experiments now using same algorithm

Reduced sensitivity to soft radiation

E-scheme recombination

$$P^J = (E^J, p^J) = \sum_{i = \text{all clusters } \in \text{jet}} (E^i, p_x^i, p_y^i, p_z^i)$$

$$p_T^J = \sqrt{(p_x^J)^2 + (p_y^J)^2}$$

$$y^J = \frac{1}{2} \ln \frac{E^J + p_y^J}{E^J - p_y^J}$$

$$\phi^J = \tan^{-1} \frac{p_y^J}{p_x^J}$$
The “underlying event” consists of hard initial & final-state radiation plus the “beam-beam remnants” and possible multiple parton interactions.

Learn through studies of min bias events, Jet events. Look at distributions/correlations of charged particles with $\eta<1$, $p_T>500$ MeV.

Also, studies of mini-jets in min bias events.
Luminosity Measurement

\[ L = \frac{1}{\sigma \text{eff}} \cdot \frac{dN}{dt} \]

- Event rate of inelastic events (luminosity detectors)

- Need to know effective cross section for this process

\[ \sigma \text{eff} = \epsilon \cdot A \cdot \sigma \text{inelastic} \]

- Efficiency and geometrical acceptance (studied and determined using Monte Carlos)

\[ \sigma \text{inelastic} \equiv \sigma \text{total} - \sigma \text{elastic} \]

- Measured separately by several experiments (CDF, E811)

<table>
<thead>
<tr>
<th></th>
<th>Run II</th>
<th>Run I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inelastic</td>
<td>60.7 (\pm) 2.4 mb</td>
<td>59.23 (\pm) 2.3 mb</td>
</tr>
<tr>
<td>Single diffraction</td>
<td>9.6 (\pm) 0.5 mb</td>
<td>9.6 (\pm) 0.5 mb</td>
</tr>
<tr>
<td>Double diffraction</td>
<td>7.0 (\pm) 2.0 mb</td>
<td>7.0 (\pm) 2.0 mb</td>
</tr>
</tbody>
</table>

- Last two are part of the inelastic cross section but with different acceptance...
1) \[ \sigma_T = \frac{1}{L} \left( R_{el} + R_{inel} \right) \]

\( L = \text{integrated luminosity} \)

2) \[ \sigma_T = \frac{4 \pi}{k} |F(0)| \] (optical theorem)

\[ \sigma_T^2 = \frac{16 \pi^2}{k^2} \frac{|F(0)|^2}{|F(0)|^2 + \Re F(0)^2} \left( |F(0)|^2 + \Re F(0)^2 \right) \]

\[ |F(0)|^2 = \left( \frac{d \sigma}{d \Omega} \right)_{\theta=0} \]

\[ t = \frac{-s}{2} (1 - \cos \Theta) \Rightarrow d \cos \Theta = \frac{2}{s} dt \]

\( s = 4k^2 \)

3) \[ \sigma_T^2 = \frac{16 \pi (A c)^2}{(1 + \rho^2)} \cdot \frac{1}{L} \left| \frac{dR_{el}/dt}{dt} \right|_{t=0} \]

\[ \rho = \frac{\Re F(0)}{|F(0)|} \]

4) dividing 3) by 1) gives:

\[ \sigma_T = \frac{16 \pi (A c)^2}{(1 + \rho^2)} \cdot \frac{1}{R_{el} + R_{inel}} \left| \frac{dR_{el}/dt}{dt} \right|_{t=0} \]

... total cross-section can be determined from event rates, i.e. without knowing the luminosity !!!
Total Proton-Antiproton Cross-Section (2)

Inelastic proton-antiproton events identified by TPC, telescopes, drift chambers and scintillators up to $|t| < 6.7$.

CDF result on total proton-antiproton cross-section here shown is the differential $t$-distribution; needed is only the event rate at the $t=0$ intercept; $t$ is measured using Roman pot detectors.

CDF total proton-antiproton cross-section at $\sqrt{s} = 1800$ GeV:


= 0.15 from UA4/2 Phys. Rev. Lett. 68, 2433 (1992)

<table>
<thead>
<tr>
<th>Component</th>
<th>Value</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total inelastic</td>
<td>240,982</td>
<td>±2,967</td>
</tr>
<tr>
<td>Total elastic</td>
<td>78,691</td>
<td>±1,463</td>
</tr>
<tr>
<td>Total</td>
<td>319,673</td>
<td>±3,308</td>
</tr>
<tr>
<td>$(dE_{vis}/dt)</td>
<td>t=0$</td>
<td>1,336,532</td>
</tr>
</tbody>
</table>

$$\Rightarrow \sigma_{pp}^{T}(\sqrt{s} = 1800 \text{GeV}) = 80.03 \pm 2.24 \text{mb}$$
Optical Theorem (1)

- Consider beam of particles incident in z-direction (plane wave) on a spinless particle (neglect time-dependence $e^{i\omega t}$ here)
  
  $$ k = \frac{1}{\tilde{\lambda}} \text{ with } 2\pi \tilde{\lambda} = \text{deBroglie wavelength} $$

- At sufficiently large radial distances $kr >> 1$:
  
  $$ \psi_i = e^{ikz} = \frac{i}{2kr} \sum_l (2l + 1) \left( (-1)^l e^{-ikr} - e^{ikr} \right) P_l(\cos \Theta) $$

- Scattering center can alter phase and amplitude of the outgoing $l$th partial waves by $2\delta_l$ and $2\eta_l$ ($1 > \eta_l > 0$), respectively:
  
  $$ \psi_{total} = \frac{i}{2kr} \sum_l (2l + 1) \left[ (-1)^l e^{-ikr} - \eta_l e^{2i\delta_l} e^{ikr} \right] P_l(\cos \Theta) $$

- Scattered wave = total - incoming:
  
  $$ \psi_{scatt} = \psi_{total} - \psi_i = \frac{e^{ikr}}{kr} \sum_l (2l + 1) \left( \eta_l e^{2i\delta_l} - 1 \right) P_l(\cos \Theta) $$

  with the scatt. amplitude

  $$ F(\Theta) = \frac{1}{k} \sum_l (2l + 1) \left( \frac{\eta_l e^{2i\delta_l} - 1}{2i} \right) P_l(\cos \Theta) $$

... elastically scattered wave, since $k$ taken to be same for incoming and outgoing ...

- Scattered outgoing flux in solid angle $d\Omega$, at radius $r$
  
  $$ v_o \psi_{scatt} \psi_{scatt}^* r^2 d\Omega = v_o |F(\Theta)|^2 d\Omega $$

  where $v_o$ is velocity of outgoing particle relative to scattering center

  **THIS IS** the product of the scattering cross-section and the incident flux ($= v_i \psi_i \psi_i^* = v_i$); elastic scattering $\Rightarrow v_i = v_o$

  $$ v_o d\sigma = v_o |F(\Theta)|^2 d\Omega $$

  $\Leftrightarrow$

  $$ \left( \frac{d\sigma}{d\Omega} \right)_{el} = |F(\Theta)|^2 $$
Optical Theorem (2)

- Legendre polynomials $P_l$ obey orthogonality condition:
  \[
  \int P_l P_{l'} d\Omega = \frac{4\pi \delta_{l,l'}}{2l+1}
  \]

- Total elastic scattering cross-section, integrated over angle:
  \[
  \sigma_{el.} = 4\pi \lambda \sum_l (2l + 1) \left| \frac{\eta e^{i\delta_l} - 1}{2i} \right|^2
  \]

- If no absorption of incoming wave ... ($\eta_l = 1$):
  \[
  \sigma_{el.} = 4\pi \lambda \sum_l (2l + 1) \sin^2 \delta_l
  \]

- If $\eta_l < 1$ get reaction cross-section from probability conservation:
  \[
  \sigma_r = \int (|\psi_{in}|^2 - |\psi_{out}|^2) r^2 d\Omega
  \]
  $\psi_{in}$ is first term of ..., $\psi_{out}$ is second term of ...
  \[
  \Rightarrow \sigma_r = \pi \lambda^2 \sum_l (2l + 1)(1 - \eta^2_l)
  \]

- Total cross-section:
  \[
  \sigma_{tot} = \sigma_{el.} + \sigma_r = \pi \lambda^2 \sum_l (2l + 1) 2 (1 - \eta_l \cos 2\delta_l)
  \]

- Since $P_l(1) = 1$ for all $l$ and $\cos(\Theta = 0) = 1$
  \[
  \text{Im } F(0) = \frac{1}{2k} \sum_l (2l + 1) (1 - \eta_l \cos 2\delta_l)
  \]

- \[
  \Rightarrow \text{Im } F(0) = \frac{k}{4\pi} \sigma_{tot}
  \]

... total cross-section is related to the imaginary part of the forward elastic scattering amplitude ...