1.13 Hyperbolic functions

It is often convenient to represent certain combinations of exponential functions by separate functions. The **hyperbolic cosine** and **hyperbolic sine** functions, denoted by cosh and sinh respectively, are defined by the following formulae.

Hyperbolic functions

$$\cosh x = \tfrac{1}{2}(e^x + e^{-x}), \qquad \sinh x = \tfrac{1}{2}(e^x - e^{-x}). \tag{1.26}$$

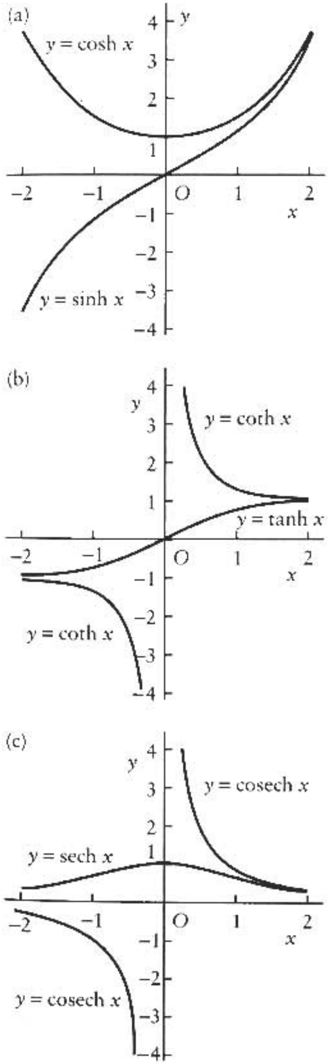

Fig. 1.32 Graphs of the hyperbolic functions.

Since

$$\cosh(-x) = \tfrac{1}{2}(e^{-x} + e^{-(-x)}) = \tfrac{1}{2}(e^{-x} + e^{x}) = \cosh x,$$

it follows that the graph of cosh x is symmetrical about the y axis, that is, cosh x is an *even* function. By a similar argument, it can be shown that sinh x is on *odd* function of x. Graphs of the two functions are shown in Fig. 1.32a.

From the definitions (1.26)

$$\cosh x + \sinh x = e^x, \qquad \cosh x - \sinh x = e^{-x}.$$

The remaining hyperbolic functions are defined in a similar manner to their trigonometric counterparts. Thus

$$\tanh x = \frac{\sinh x}{\cosh x}, \qquad \coth x = \frac{\cosh x}{\sinh x},$$
$$\operatorname{sech} x = \frac{1}{\cosh x}, \qquad \operatorname{cosech} x = \frac{1}{\sinh x}. \tag{1.27}$$

Graphs of tanh x, coth x, sech x, and cosech x are shown in Fig. 1.32.

From the definitions, a number of identities follow which parallel those for trigonometric functions but with important sign differences. Some are derived below.

(a) $\cosh^2 x + \sinh^2 x = \cosh 2x$,

(b) $\cosh^2 x - \sinh^2 x = 1$. **(1.28)**